

A study of the constitution of Grade 8 mathematics within the
context of the Revised National Curriculum Statement in five
Western Cape schools.

A dissertation submitted in part fulfillment of the requirements for the degree of
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by

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DECLARATION

This work has not been previously submitted in whole, or in part, for the award of any degree. It is my own work. Each significant contribution to, and quotation in, this dissertation from the work, or works, of other people has been attributed, and has been cited and referenced.

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ABSTRACT

This dissertation is an investigation into the constitution of school mathematics within the context of the Revised National Curriculum Statement in a selection of Grade 8 mathematics lessons in five working-class schools in the Western Cape Province of South Africa. The study is located within the broad framework of the sociology of education, specifically drawing on Bernstein's (1996) sociological theory of education and his pedagogic device. This study focuses on the way in which the content of the evaluative rule of the pedagogic device is realised in the particular selection of schools. My theoretical framework relies on the work of Davis (2010a, 2010b, 2010c, 2011a, 2011b, 2011c, 2012, 2013a & 2013b) and Bernstein (*ibid.*). These theoretical resources were drawn on to describe and analyse the mathematical activity in the five schools as well as serving as a means for generating analytical resources for describing the constitution of mathematics. In my analysis I present an account of the computational activity of teachers and their learners and the regulation of mathematical activity in fifteen Grade 8 mathematics lessons. I use these descriptions of computational activity to discuss the realisation of content against a general background of curriculum reform that has de-emphasised explicit use of formal definitions. I explore what mathematical content was recognised and constituted in relation to topics announced by teachers and use the mathematics encyclopaedia as a resource to ascertain the content that substitutes for formal mathematical definitions, axioms and propositions.

The results of this study show that (1) for the majority of lessons observed there is a disjuncture between the realised content and the mathematics encyclopaedic content associated with the announced topic; (2) that computational resources are the dominant means of regulating the computational activity and serve as substitutes for the explicit use of formal mathematical definitions and propositions. Furthermore, this study shows that (3) Grade 8 topics are realised in terms of very elementary content i.e. basic arithmetic and counting which may provide a possible explanation for the dismal performance of South African students in regional, national and international testing. This study also illustrates that (4) in a general milieu of curriculum reform, where there is a distinct de-emphasis of formal definition of mathematical objects and processes in favour of more empirical means of justification, there is a tendency to blur the distinction between descriptions of mathematical objects and descriptions of everyday objects and that the consequences for independent learner work remains uncertain.

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LIST OF ABBREVIATIONS

AMESA	The Association for Mathematics Education of South Africa
C2005	Curriculum 2005
CAPS	Curriculum and Assessment Policy Statement
CASME	Centre for the Advancement of Science and Mathematics Education
DEC	Department of Education and Culture
DoE	Department of Education
DET	Department of Education and Training (South Africa, pre- 1994)
GET	General Education and Training
HoR	House of Representatives
HSRC	Human Sciences Research Council
ID	Instructional Discourse
LO	Learning Outcome
MALATI	Mathematics Learning and Teaching Initiative
MEP	Mathematics Education Project
MST	National Strategy for Mathematics, Science and Technology
NCTM	National Council of Teachers of Mathematics
NECC	National Education Crisis (Co-ordinating) Committee
NEPI	National Education Policy Investigation
NGO	Non-Governmental Organisation
NSF	National Science Foundation
OBE	Outcomes Based Education
OHP	Overhead Projector
ORF	Official Recontextualising Field
PEI	President's Education Initiative
PRF	Pedagogic Recontextualising Field
RD	Regulative Discourse
RNCS	Revised National Curriculum Statement
RUMEP	Rhodes University Mathematics Education Project
RUMEUS	Research Unit for Mathematics Education at the University of Stellenbosch
WCED	Western Cape Education Department

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University of Cape Town

Chapter 1

Introduction

1.1 The problematic

The general problematic that this research project engages with is that of the constitution of mathematics in the pedagogic situations of schooling. Of particular interest to this project is what is constituted as mathematics and how in pedagogic situations in the General Education and Training (GET) phase of schooling, where we find that the use of explicit definitions of mathematical objects as well as the explicit study of mathematical propositions, have been de-emphasised. Curriculum reform since 1997 in South Africa brought along with it a fundamental shift in the way it prescribed that teaching and learning ought to change at the level of the curriculum, texts for teaching and pedagogy, by de-emphasising both the study of formal definitions of mathematical objects and processes as well as presenting mathematics in a less formal way.

A description of the ways in which the curriculum recontextualises mathematics in an educational context in which the explicit study of definitions and propositions are no longer required, along with descriptions of the recontextualisations of mathematics in textbooks and pedagogic situations in lessons, is one way to approach the problem of the constitution of mathematics in contemporary schooling contexts in South Africa.

1.2 Pedagogy and curriculum reform

In his theory of pedagogic discourse, Bernstein (2000, 1996, 1975) describes how knowledge is transformed into communication in pedagogic contexts by way of the *pedagogic device*. The device describes the general principles underlying the transformation of knowledge into pedagogic communication through three hierarchically-related ‘rules’: namely the *distributive*, the *recontextualising* and the *evaluative rules*. The distributive rules are concerned with the distribution of forms of consciousness through the distribution of different forms of knowledge, essentially regulating what knowledge gets distributed at the level of policy and curriculum (Bernstein, 1996). The recontextualising rules are derived from the distributive rules and entail a transformation of knowledge into pedagogic communication. These rules recontextualise both *what* (subject content) and *how* pedagogic discourse is rendered. For Bernstein (1996), evaluative rules are essentially where the distributive effects of the pedagogic device are realised. He emphasises that the evaluative rules regulate the way in which students show that they can produce the required text and that the purpose of pedagogic practice entails the transmission of evaluative criteria. Appendix A provides a more detailed discussion of the pedagogic device.

The pedagogic device is a useful conceptual resource for describing the structuring effect that the Revised National Curriculum Statement (RNCS) has on the form that mathematics takes in pedagogic contexts in South

African schooling. Curriculum reform, following the demise of apartheid in the South African context, was a response to an educational crisis caused by apartheid policies where education resources were unequally distributed, to the benefit of a relatively small minority of the population. (Appendix B provides extensive commentary on the lineage of curriculum transformation from the apartheid years to the present political dispensation.) With the need to address the educational inequalities generated by apartheid, one of the specific aims of the Constitution (Act 108 of 1996) was to ‘heal the divisions of the past and establish a society based on democratic values, social justice and fundamental human rights’. It was the Constitution that provided the ‘basis for curriculum transformation and development in South Africa’ (DoE, 2002: 1), in that way attempting to transform the content of the distributive rule of the pedagogic device.

The past seventeen years, i.e. (1997-2013), have been characterised by major attempts by the State to radically transform the education terrain by implementing new education policies and educational practices in an attempt to transform the content of the recontextualising rule of the device. Since the inception of Curriculum 2005 (C2005) in 1997, the curriculum has undergone several reviews and we are currently experiencing a third wave of curriculum reform with the implementation of the Curriculum and Assessment Policy Statement (CAPS)¹. Following an investigation that culminated in the Chisholm Report², C2005 was redesigned to produce the RNCS which was completed in 2002. A third attempt at curriculum reform, CAPS was implemented for Grades 1- 3 and Grade 10 in 2012 and is currently in use for Grades 4-9 and Grade 11 this year.

My study focuses on the GET phase of the RNCS, specifically on the teaching of Grade 8 mathematics in the context of the implied changes to mathematics and to our conceptions of the learner of mathematics, and so is a focus on the way in which the content of the evaluative rule of the device is realised in a selection of schools. The empirical research context for this study is an archive of video records of the teaching of mathematics in 15 Grade 8 mathematics lessons across five schools. The five schools form part of the Dinaledi³ Mathematics,

¹ In the process of writing a new curriculum i.e. CAPS, the ‘writing brief centred on three important ideas: simplification, improvement, and clarification. So rather than generate a completely new curriculum, something the educational community could probably not withstand, the proposed plan was to use what was good from the existing RNCS and replace what appeared not to be working. The first thing was to let go of all the OBE terminology: Critical and Developmental Outcomes, Learning Outcomes and Assessment Standards were cut. They have re-appeared in a different form under the General Aims section of the CAPS documents and the Specific Aims sections [...]’ (NAPTOSA, 2011: 14).

² The Chisholm Report addressed failure present in curriculum (C2005), where it was found that its initial ideas were too imprecise and needed revision in an attempt to reclaim knowledge (Chisholm, Lubisi *et al.*, 2000).

³ Dinaledi schools are the National Department of Education’s flagship Science and Mathematics initiative (Dinaledi means “creating tomorrow’s stars today”).

Science and Technology Focus Schools (MST)⁴ and have been identified as having a high population of learners from disadvantaged backgrounds. Four of the schools draw student populations mainly from the ‘African’ townships of Khayelitsha and Phillippi, which are classified as being predominantly lower working class areas. The remaining school is populated by students from mostly working class areas in greater Cape Town. Video footage of the recorded lessons and the accompanying transcripts will be analysed to generate data. I will investigate the constitution of Grade 8 mathematics by exploring how the evaluative rule functions in the specific sites of my study.

1.3 A brief discussion of shifts in curriculum

At the level of the distributive rule of the pedagogic device, curriculum reform which culminated in the production of the RNCS sought not only to change what mathematics should be taught and to whom but also to construct new pedagogic identities for teachers and learners. For Bernstein (2000: 66), any educational reform can be regarded as ‘the outcome of the struggle to project and institutionalise particular identities’. Policy statements are rather explicit regarding the kind of teacher and learner envisaged. For example, the RNCS proposed that:

[...]Teachers and other educators are key contributors to the transformation of education in South Africa. This Revised National Curriculum Statement Grades R-9 (Schools) envisions teachers who are qualified, competent, dedicated and caring. They will be able to fulfil the various roles outlined in the Norms and Standards for Educators.

And

The promotion of values is important not only for the sake of personal development, but also ensure that a national South African identity is built on values very different from those that underpinned apartheid education. The kind of learner [...] is one who will be inspired by these values, and who will act in the interests of a society based on respect for democracy, equality, human dignity, life and social justice. (DoE, 2002: 3)

These extracts, taken from the Introduction to the *RNCS for Grades R-9 (Schools): Mathematics*, provide a description of what the persons doing mathematics (both teachers and students) should embody. Bernstein (2000: 65) might describe this as an attempt ‘to construct in teachers and students a particular moral disposition, motivation and aspiration, embedded in particular performances and practices.’

The notion that all South African citizens are considered as active participants at the level of society is translated at the level of the curriculum where students are no longer ‘passive’ recipients of knowledge. The definition of mathematics provided by the RNCS projects an image of mathematics as a ‘human activity [...] (which is) a product of investigation by different cultures – a purposeful activity in the context of social, political and

⁴ MST is the National Strategy for Mathematics, Science and Technology employed to improve performance and enrolments in these subjects, particularly of Grade 12 learners from historically disadvantaged backgrounds.

economic goals and constraints’ that enables creative and logical reasoning where mathematical knowledge is constructed by ‘observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves’ (DoE, 2002:4). Coupled to this, the way in which knowledge was conceived of had also undergone a shift. That is, mathematical knowledge could be ‘developed and contested over time through both language and symbols and by social interaction and is thus open to change’ (DoE, 2003a: 9). If one considers what this transformation of knowledge into pedagogic communication entails, bearing in mind Bernstein’s recontextualisation rule, there appears to be a reconfiguration of who the student is and this ultimately changes the way mathematics is presented in pedagogic contexts.

One effect of stressing the activity of mathematics, as well as its provisional nature, is that the explicit use of propositions, deductive argumentation and definitions has been de-emphasised and relegated to the higher reaches of schooling as well as resulting in a shift in emphasis on the content of mathematics to the activity of the knower of school mathematics. This begs the question of what gets produced as mathematics in pedagogic situations where there is a different conception of students, teachers, mathematics and mathematics education.

1.4 A brief discussion of State-proposed shifts in pedagogy

Pedagogy, as prescribed by the RNCS, echoes its ideological message of the active construction of knowledge by the student. Consider the list shown in Figure 1.1, which was distributed at a WCED Grade 8 Mathematics workshop for teachers in August 2000.

Process and Thinking Skills in Mathematics

Analysing	Creating a mathematical model	Connecting mathematics
Synthesising	Problem-solving	Designing explorations
Comparing	Verifying	Posing “what-if” questions
Contrasting	Justifying	Identifying assumptions
Collecting data	Disproving	Constructing proof
Making observations	Reasoning logically	Taking intellectual risks
Recognising patterns	Sequencing	Collaborating with peers
Forming conjectures	Predicting	Engaging in mathematical discourse
Finding counterexamples	Identifying	Constructing understanding
Generalising results	Classifying/ Communicating	

Figure 1.1: ‘Process and Thinking Skills in Mathematics’ (WCED, 2000: 23)

The purpose of the workshop was to provide an elucidation of the curriculum reform process in schools, which had commenced in 1998, and to equip teachers with an understanding of the principles and strategies of Outcomes-Based Education (OBE) as defined by the Department of Education (DoE) and the WCED. The “process and thinking skills” in Figure 1.1 have certainly been promoted and legitimised by the curriculum. These process and thinking skills, furthermore, suggest a curriculum shift in focus from knowledge statements regarding mathematics to one that focuses on the knower of mathematics. The idea of the active student as an observer of patterns is a central idea in the GET Band as expressed in the definition of mathematics offered by the RNCS:

Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena [...] (DoE, 2002: 4)

As a prelude to this study, it is useful to reflect on work done by Davis & Johnson (2007b) on the constitution of school mathematics in five working class schools⁵, three of which participated in my project. It was observed that teachers spent an inordinate amount of time to present worked examples to students as means for explicating solution procedures for particular classes of problems. Davis & Johnson (*ibid.*) compared the amount of time spent on teaching and learning through worked examples and time spent on exposition of ideas, principles and definitions. Their findings revealed that on average, three to four problems were covered per lesson, each problem requiring between nine and eleven minutes to complete and that approximately ten minutes were spent per worked example. Even though the pace of lessons was slow and a great deal of time was devoted to a sequence of topic-specific procedures, students’ results in grade-specific tests (Grade 10, 11 and 12) in 2007 suggested a poor grasp of subject matter.

They also noted that explicit use of knowledge of objects referenced by mathematical statements in explanations or in solution procedures was often absent. Besides the lack of attention to the existential nature of the mathematical object engaged with, definitions of objects and processes, general principles or the inter-relations between different objects and processes within and across topics were not discussed. The general principles of mathematics had to be synthesised by students from a worked example and the focus was on procedures only applicable to specific mathematical contexts. An under-specification of definitions was noted in all of the five schools in their study, mirroring the under-specification of definitions in the RNCS.

⁵ The five schools that participated in Davis & Johnson’s (2007b) study were ‘populated by so-called ‘African’ and ‘coloured’ learners from residential areas housing largely working class families in the Western Cape. Most of the learners attending the schools live in areas that are among the lowest 20% in terms of social-economic status as measured by the City of Cape Town in 2001. For now we refer to learners from areas of such low socio-economic status as working class because the majority of working adults (20+) living in such areas are employed as semi-skilled and unskilled workers and labourers according to the census data used by the City of Cape Town (2006, 2008).’ (Davis, 2010c: 378)

This particular observation draws my attention to difference between descriptions of mathematical objects and *natural kind* terms. The Kripkean view describes ‘natural kind terms’ as words for natural substances, species and phenomena (e.g., light, gold, tiger, water) which are not definitions (Kripke 1980: 127). Natural kinds are able to maintain their identity even if their properties are changed and are described as rigid designators.

Formal definitions of mathematical objects provide the fundamental truth regarding a concept from which all other facts about it are deduced i.e. a definition of a concept in mathematics has a distinctive logical status. The existence of mathematical objects is constituted in terms of a field of predicates announced in definitions, postulates and propositions and mathematical concepts have properties that are entirely different from descriptions of the things we encounter in everyday life; they are defined in terms of a list of properties which stand as the constitutive elements of the object. Mathematical objects are referred to as *definite descriptions* (Potter, 2004) and, unlike natural kinds, altering any of the predicates that participate in the definition of a mathematical object produces a different object or even no object at all by having produced a term with an empty extension. Given the de-emphasis on definitions and formal mathematical propositions, it appears that the distinction between definite descriptions and natural kinds has been dissolved in the curriculum.

In the pedagogy described in Davis & Johnson (2007b), there is a de-emphasis of explicit mention of formal definitions of mathematical objects and processes *and* there appears to be an absence of process and thinking skills prescribed by the RNCS in Figure 1.1. If students were therefore to gain access to the general principles of mathematics, they would need to synthesise these general principles from worked examples. There are, however, significant features in the empirical context of the five working class schools in my study that are merely suggestive that for working class students teaching has continued as before, irrespective of curriculum reform, and the meagre attention previously given to formal propositions and definitions of mathematical objects has been evacuated. I expect, therefore, that the pedagogic contexts in my study display similar features to those discussed in Davis & Johnson (*ibid.*).

My study examines the realisation of curriculum at the level of pedagogy in a general milieu where there is a distinct de-emphasis of formal definition of mathematical objects and processes in favour of more empirical means of justification. My focus in this study is to investigate what substitutes for the predicted absence of definitions, propositions and axioms in the mathematics constituted in the five schools.

Bernstein’s (1996) pedagogic device is helpful in organising schooling from the macro-level of policy and curriculum to the micro-level of pedagogic practice where I will focus on what gets constituted as Grade 8 mathematics in this general milieu of curriculum reform. My intention, however, is not to establish a causal relation between the curriculum context and the mathematics constituted in pedagogic situations. Such a task

would entail a much larger study. Bernstein (1996), states that pedagogy is necessarily evaluative. As such pedagogic evaluation reveals criteria for the recognition and realisation of mathematical objects or procedures in pedagogic contexts. In essence then, pedagogy is the carrier for recognition and realisation rules and hence the carrier of criteria. The nature of the criteria circulating in the pedagogic contexts of the schools in my study will be examined as a means for analysing the regulation of what comes to be constituted as mathematics and how mathematics is constituted in pedagogic contexts.

My research question can now be stated more fully:

What is constituted as Grade 8 mathematics in the pedagogic spaces of five working class schools in the Western Cape in the context of the GET curriculum that has de-emphasised the use of formal definition and propositions of mathematical objects and processes?

1.5 Overview of the dissertation

Chapter One contextualises the study by providing a background and brief discussion of how curriculum change has presented mathematics in a manner where the student is considered in a more direct way. The study is located in the GET phase of mathematics education and is interested in how a suspension of formal definition of mathematical objects and processes from the RNCS is dealt with in the teaching and learning of mathematics.

Chapter Two presents a review of relevant literature which explores potential reasons for the dismal mathematics performance of South African school students in relation to students in other countries. This chapter outlines the state of play in South African school mathematics by consulting studies that relate to student performance in regional and national tests, classroom based research as well as various accounts of the constitution of school mathematics as understood in pedagogy.

Chapter Three presents the general theoretical orientation structuring the research. The elaboration of school mathematics will be described and translated using Bernstein's (1996) theory of pedagogic discourse and the pedagogic device. Bernstein's pedagogic device will serve as a means for describing the pedagogic discourse from the macro-level of policy implementation in the curriculum to the micro-level pedagogy in the classroom – in particular, the notion of evaluation and the associated evaluative criteria.

Chapter Four constructs an analytic framework by drawing on Davis (2010a, 2010b, 2010c, 2011a , 2011b, 2011c, 2012, 2013a & 2013b) and will be used to enable a description and analysis of mathematical activity by focusing on the pedagogic *criteria* that indicate the particular *operations* in play and the collections of *objects* over which the operations range. In my description and analysis of the constitution of mathematics in the fifteen

Grade 8 mathematics lessons, I will draw on the relevant curriculum statements, the texts used for teaching and learning, and the lesson transcripts.

Chapters Five, Six, Seven, Eight and Nine offer an account of what is constituted as the mathematics content of curriculum topics in each of the five schools i.e. School P1, P2, P3, P6 and P7 respectively. The appendices contain the actual production and analysis of data for the fifteen lessons, which is too extensive to be included in the body of this project and may be referred to for clarity. Appendix E, F, G, H and I therefore provide an analysis of data for School P1, P2, P3, P6 and P7 respectively.

Chapter Ten and Chapter Eleven are the concluding chapters and present a summary of the findings and results across the research.

University of Cape Town

Chapter 2

Literature review

The research problem engaged with in this study focuses on the constitution of school mathematics in five working class schools in the Western Cape in a general milieu of curriculum reform in the GET phase of schooling where the use of formal definitions and propositions of mathematical objects and processes has been backgrounded. In order to position my study in relation to other work in the field of mathematics education in such contexts, I will examine various categories of literature that comment on the state of play in South African school mathematics. Studies that relate to student performance in mathematics in regional, national and international tests as well as research that attempts to explain the problem of poor performance in mathematics in pedagogic contexts in South African schools will be discussed. The latter studies describe the mathematics produced in pedagogic contexts in terms of firstly, the academic-everyday distinction and secondly, the procedural-conceptual distinction. Finally, I present two accounts of the constitution of mathematics which do not pay too much attention to the procedural-conceptual distinction, but rather explicitly ask questions about the constitution of mathematics in pedagogic situations. I reflect on studies conducted by Chevallard (1988, 1992) who deploys the theory of didactic transposition to provide an account of mathematics in schooling and then I present a range of work on the constitution of school mathematics done by researchers on local contexts that takes the computational activity of the student and teacher into account when examining mathematics teaching and learning.

2.1 Studies measuring student performance in mathematics

Curriculum reform immediately post-apartheid South Africa was envisaged as a means to transform the inherited apartheid education system, which consisted of seventeen departments into a single national department that provided equal educational opportunities to all citizens, regardless of race, class and gender. As outlined in Chapter One, the curriculum entailed a shift in the way in which mathematical knowledge, the learner and the teacher was viewed. This inevitably necessitated a change in emphasis on how mathematics ought to be taught and learned. The particular focus of my study relates to analysing the teaching and learning of Grade 8 mathematics in this general milieu of curriculum transformation, particularly in the five schools populated by students from lower socio-economic communities.

2.1.1 Regional, national and international testing

As a means of measuring the outcomes of curriculum reform in South Africa, first instituted in 1994, national initiatives in the form of systemic evaluation of learners were instituted by government, and the results were anything but favourable. The crisis in South African mathematics education has been documented extensively

and the most notable indicator is the performance in the national matriculation examinations for the years 2008, 2009 and 2010 which reflected a pass mark⁶ ranging between 46% - 47%⁷ for mathematics (Taylor, 2008). Bloch (2009: 58) describes education in South Africa as being a national disaster, stating that

[...] international tests suggest that South African schools are among the world's worst performers in maths and literacy. Worse still is the tragedy that our schools are reinforcing the social and economic marginalization of the poor and vulnerable. [...] South Africa routinely comes last on all international scores. [...]

SACMEQ (Southern and East African Consortium for Monitoring Educational Quality) Grade 6 mathematics test showed that a significant proportion of Grade 6 learners performed at Grade 3 level (Fleisch, 2008: 7). Nationally, the Department of Education also implemented official tests to quantify performance of learners in basic numeracy. The first of these systemic evaluations was conducted in 2001 for Grade 3 and then later in 2004 in Grade 6 revealing deficiencies in reading, writing and numeracy skills (DoE, 2003b). The average mathematics score was a mere 30% for the approximately 51 000 Grade 3 learners evaluated and 27% for 34 105 Grade 6 learners (Bloch, 2009: 62). In essence, then, the scores for numeracy from both the Systemic Evaluations in Grade 3 and Grade 6 show that the majority of children, particularly 'African' children are not mathematically competent.

These studies report on both the Foundation Phase (Grade 3) and the Intermediate Phase (Grade 6) of the schooling system, strongly suggesting that students entering the Senior Phase in Grade 8, the focus of my study, are bound to under-perform as well.

Another project conducted in 1999, the Monitoring Learning Achievement (MLA) study designed by UNESCO and UNICEF as a means for obtaining information regarding the effectiveness of basic education in terms of actual learning achievement, reported that from a sample of Grade 4 learners in a range of countries, South Africa fared the poorest, averaging 30% in numeracy tests (Fleisch, 2008: 10-11). According to the Third International Mathematics and Science Study (TIMSS) conducted in 2003 for Grade 8, South Africa also fared worst out of 50 countries surveyed for mathematics (Reddy, 2006; 2005).

Fleisch (2008: 8) describes the primary school mathematics and science performances of learners participating in academic performance tests such as MLA and SACMEQ in terms of a 'bimodal distribution'. He, amongst others, notes the huge disparities displayed in the learning achievement between privileged compared to poor

⁶ The Department of Education considers 30% as a pass rate for mathematics.

⁷ This pass mark is less than the requirement for entry into mathematics related higher education fields of study.

working class children (Taylor, 2008; van der Bergh & Louw, 2006, 2007; Reddy & Kanjee, 2007). The dismal mathematics performance in national, regional and international tests suggest that learners are unable to perform tasks at the appropriate grade levels or at the minimum expected standard levels (OECD⁸, 2008, Schollar, 2008). While the results of international assessment studies such as SACMEQ II and TIMSS 2003 as well as the national systemic evaluation conducted in South Africa draw attention to serious problems in mathematics education in schools located in lower socio-economic precincts, they do not report on what might be going on in classrooms.

The clear failure of students to meet the demands of regional, national and international tests implies that what is constituted as mathematics in schools is not adequate to realise what is expected internationally. The test results point to the issue of what gets constituted as mathematics, nationally and internationally and the performance of South African students suggests that while an elite sub-group of students may be performing well, the majority of students are failing. The test results not only suggest that different types of students are getting different forms of mathematics, but also provides a rationale for further emphasising that what gets realised as mathematics in different contexts seems to vary. Curriculum reform in South Africa, in its attempt to render mathematics in a more ‘meaningful’ way, has backgrounded the use of formal definition and propositions of mathematical objects and processes and hence a different form of mathematics may be constituted in the classroom.

Despite the fact that systemic testing reports on the outcomes and difficulties encountered in these contexts, these studies do not investigate pedagogy and the nature of mathematics constituted in pedagogic contexts. As a result, systemic testing provides little discussion on what is constituted as school mathematics, other than suggesting that the form of mathematics that the majority of students in South Africa are exposed to may be problematic. I now reflect on research on mathematics pedagogy to further explore this issue of what is constituted as mathematics.

2.2 Describing the problem and its cause

The empirical research that follows describes various accounts and arguments for why South African mathematics learners are lagging behind the rest of the world. In the first instance, this section presents a discussion of what is constituted as mathematics in pedagogic situations by reflecting on studies focused on the teaching and learning of mathematics in local schooling i.e. Hoadley (2005, 2007 & 2008); Parker & Adler (2005); Carnoy & Chisholm *et al.* (2008, 2011) and Reeves & Muller (2005). I also discuss school improvement studies (Reeves, 2005; Reeves and Muller, 2005) in Appendix C that examine the quality of mathematics

⁸ OECD – Organisation for Economic Co-operation and Development

teaching in schools and that attribute the differential performance in mathematics firstly, to different forms of mathematics being distributed at the level of the curriculum and policy, and ultimately teaching and secondly, to the social class background of learners.

2.2.1 Studies of mathematics teaching and learning

A number of studies (Chisholm, 1992; Taylor, 1999; Muller & Taylor, 2000) have attempted to provide greater clarity for the differential achievement in school mathematics along social class lines. One such study conducted by Hoadley (2007) describes the different kinds of knowledge made available to primary school learners from different social class backgrounds. Hoadley (2007: 704) presents a description of how the inequalities noted by Fleisch (2008: 8) as a 'bimodal distribution' are prevalent in a sample of primary school Grade 3 classrooms in a South African context. She claims that middle-class learners are presented with specialised knowledge of mathematics compared to working-class learners who are presented with local, everyday knowledge, meanings and practices.

Hoadley (*ibid.*), in her analysis of the reproduction of social class differences through pedagogy with respect to middle class and working class children, bases her study on the Bernsteinian (2000) proposition of orientation to meaning⁹. Bernstein (2000) distinguishes between context-dependent meanings and context-independent meanings, arguing that working class students have a *restricted* orientation to meaning in that they are predisposed to context-dependent meanings whereas middle class students have an *elaborated* orientation to meaning in that they have access to both context-dependent and context-independent meanings, privileging context-independent meanings in school contexts. Bernstein (*ibid.*) claims that working class children enter school with a *restricted* orientation to meaning compared to middle class children who enter with a more *elaborated* orientation to meaning. Middle class students, according to Bernstein, are therefore more likely to succeed at school.

Using Bernstein's (2000) proposition, Hoadley (2007) claims that the pedagogy in the working class context in her study fails to disrupt students' purported restricted orientation to meaning acquired in the home whereas pedagogy in the middle-class 'context' amplifies students' elaborated code acquired in the home. While this empirical study suggests that the poor performance of South African students may be as a result of different

⁹ Bernstein's proposition, orientation to meaning, was derived from the work of Luria (1976) and Holland (1981) in British schools.

orientations to meaning as well as the differential distribution of knowledge in different social contexts, it provides rather limited insight into the constitution of mathematics.

Jaffer (2011b) notes that the methodological resources deployed by Hoadley (2007) to analyse mathematics pedagogic practices provide very little insight into the content in so far as what is constituted as mathematics. Jaffer (2011b: 270) claims that in Hoadley's use of Bernstein's concept of classification with respect to mathematics and the everyday, she confounds so-called everyday knowledge with reference to 'everyday' objects. Jaffer (2011b: 670) further notes that when Hoadley (2007) believes that teachers are not teaching as they should, she sees criteria for the production of privileged text as implicit, rather than recognising the existence of criteria that teachers are actually using in their elaboration of mathematics.

In the context of my study, which focuses on the constitution of mathematics in a general milieu of curriculum reform, Hoadley's methodological resources remain inadequate as a means for describing mathematics constituted in pedagogic contexts. My study primarily aims to explore the ways in which mathematics is constituted at the micro-level of the classroom by focusing on the objects and operations brought into play by the regulative criteria of pedagogic discourse.

In a different study on teacher education reform in South Africa conducted by Parker & Adler (2005), like Hoadley (2007), they argue that in an attempt to provide access to mathematics for the socially and economically disadvantaged, there is a tendency to regard school mathematical knowledge like everyday knowledge resulting in learners not being given access to the specialised knowledge of mathematics.

Besides the studies conducted by Hoadley (2007, 2008) and Parker & Adler (2005), who explain students' differential achievement in mathematics along social class lines in terms of the distinction between everyday and academic knowledge, other research has also explored differences in students' mathematics performance along social class lines (Chisholm, 1992; Taylor, 1999; Muller & Taylor, 2000) within the field of educational sociology.

A large-scale, classroom-based study, focusing on mathematics teaching and learning was conducted by Carnoy & Chisholm *et al.* (2008, 2011)¹⁰ on a sample of forty Grade 6 Gauteng-based primary schools. Their research focused on investigating the impact of the teacher's pedagogic content knowledge on students' learning. The

¹⁰ Questionnaires were completed by students, teachers and the principal and students participated in two tests at different times of the year. Mathematics lessons were also video-recorded and analysed and teachers were presented with questions ranging from mathematics teaching to specific content and pedagogic content knowledge in their questionnaires.

results suggested low average levels of pupil and teacher mathematical knowledge, and an inequitable distribution of mathematical knowledge among those who taught students of lower and higher socio-economic background. In essence, then, their research claimed that

students who are disadvantaged academically in terms of family resources (including regularly using the language of instruction employed by the school) are also likely to be instructed by teachers with less capacity to impart mathematical understanding to students in the classroom. [...] (and) the lack of an adequate pool of teacher mathematics content and pedagogical content knowledge seems to be a major factor in influencing how much mathematics the students we observed are likely to learn. (Carnoy & Chisholm *et al.*, 2008: 69)

Carnoy & Chisholm *et al.* (*ibid.*) provided a description of mathematics pedagogy in terms of the procedural-conceptual distinction, a notion which I discuss in detail in Appendix D.

The empirical studies of mathematics pedagogy presented above provide commentary on the way in which knowledge is described and distributed in both middle class and working class contexts. These studies generally argue that the differential mathematical achievement of students is attributed to the fact that certain groups of students, particularly those from working class backgrounds, are presented with a form of mathematical knowledge that is not compatible with schooling.

2.2.2 School improvement studies

Many studies (Fleisch, 2008, Adler & Reed, 2002, Taylor, 2008, Taylor & Vinjevold, 1999, Reddy, 2006, Carnoy & Chisholm *et al.*, 2008, 2011) specify the nature of the problem prevalent in South African mathematics schooling in terms of low levels of teacher content knowledge, inadequate teacher training and a need to improve the quality of schooling, yet they do not seem to report on what exactly is constituted as mathematics and how such constitution comes to be realised at the micro-level of the classroom in working-class contexts. I reflect on two methodological resources deployed in school improvement studies, namely the notions of *opportunities-to-learn* (OTL) and *cognitive demand* in more detail in Appendix C.

2.2.3 Summary of the problem

While the findings presented in the studies on mathematics pedagogy report on the outcomes of particular teaching styles and process of mathematics learning, either in terms of the academic-everyday distinction or the procedural-conceptual distinction, they tell one very little about the constitution of school mathematics at the level of its content. The procedural-conceptual distinction is another widely-used theoretical distinction in the field of mathematics education and provides a means for describing mathematics teaching and learning. I elaborate on the latter distinction in Appendix D.

The procedural-principled distinction provides insight into how mathematics is taught and how mathematics ought to be constituted in a reformed curriculum yet it does not present what is actually taught or if the topic announced by texts and teaching has in fact been acquired. It describes conceptual knowledge as the privileged knowledge of what ought to be present in pedagogy, rather than what is present. This foundational distinction in mathematics education, however, is not rigorous enough in the context of my study since it provides limited insight into the constitution of mathematics – referencing a topic at the start of a lesson does not necessarily mean that the content thought to constitute the topic has been taught or learnt (Chitsike, 2011a, 2011b; Davis, 2011a, 2011b, 2011c, 2012, 2013a, 2013b; Jaffer 2011a, 2011b, 2012). A more productive and detailed way of describing what happens inside the mathematics classroom is necessary.

2.3 Studies on the constitution of mathematics

The literature presented in this section provides another account of the constitution of mathematics. It explicitly asks questions about the constitution of mathematics as it emerges in the pedagogic context rather than what mathematics ought to be. In this section I consider anthropological studies of mathematics pedagogy (Chevallard, 1988, 1992) as well studies that focus on the micro-level operational activity of teachers and students.

2.3.1 An anthropological stance

A body of work that considers the emergence of mathematics in didactical situations is the work of Chevallard (1988, 1992). Chevallard's (1992) anthropological theory of didactics locates both mathematical activity and the activity of studying mathematics within the set of human activities and social institutions. The theory of *didactic transposition*, originally developed by Yves Chevallard, is a means of precisely describing and explaining the phenomena of the transformation of knowledge from its production to its teaching. According to Chevallard (*ibid.*), in order to understand mathematics in schooling, one needs to consider what emerges as mathematics in pedagogic situations rather than pre-defining what mathematics is. In other words, he claims that what gets constituted as mathematics is not obvious in schooling because of the transformative effect that the mathematics produced by scholars undergoes when it enters a didactical system.

Chevallard (1988: 5) states:

The transition from knowledge regarded as a tool to be put to use, to knowledge as something to be taught and learnt, is precisely what I have termed didactic transposition of knowledge.

In essence, then, Chevallard in Adler & Huillet (2008) describes the modification of knowledge under instructional purposes as didactical transposition and he presents a model describing six moments of

mathematical organisation (MO)¹¹: the first encounter with the MO, exploration of the task and emergence of the technique, construction of the technological-theoretical block, institutionalization, work with the MO and evaluation.

Chevallard's account of the transformation of mathematical knowledge into school knowledge resonates with Bernstein's notion of recontextualisation which he describes as the delocation of knowledge from the field of production to the official recontextualising field (ORF) and the pedagogic recontextualising field (PRF) (Bernstein, 1996: 45).

Chevallard's anthropological approach is unique to other studies in the field of mathematics education, since it considers mathematics as it emerges in pedagogic situations. I now draw on other work that resonates with Chevallard's idea that research should consider mathematics that emerges in pedagogic situations. This body of work, however, is methodologically different from the anthropological studies conducted by Chevallard. Although both bodies of research consider the constitution of mathematics, Chevallard's theory of didactic transposition considers the six moments of MO whereas the second body of work considers the operational activity of teachers and students in pedagogic situations. Below, I present a brief description of these studies.

2.3.2 A focus on the operational activity at the micro-level of pedagogy

Closely related work by a number of local researchers on different aspects on the constitution of school mathematics in five working class schools in the Western Cape (Arendse, 2011; Basbozkurt, 2010a, 2010b; Chitsike, 2011a, 2011b; Davis, 2009, 2010a, 2010b, 2010c, 2011a, 2011b, 2011c, 2012, 2013a, 2013b; Davis & Johnson, 2007a, 2007b, 2008; Davis & Gripper, 2012a, 2012b; Gripper, 2011a, 2011b; Jaffer, 2009, 2010a, 2010b, 2011a, 2011b, 2012; Jaffer & Davis, 2012; Johnson & Davis, 2010; MacKay, 2009, 2010, 2011, 2012; Matobako, 2012, 2013; Roberts, 2009, 2010) present descriptions of the operational activity entailed at the micro-level of pedagogy. They generate descriptions and analyses of the teaching and learning of school mathematics by focusing on the objects and operations brought into play by the regulative criteria of pedagogic discourse. This approach provides a lens for researching what is entailed as mathematics and how such a constitution is realised.

The methodological resources employed, common to all of the research listed above, enable one to comment on the constitution of school mathematics as it emerges in the pedagogic situation in a systematic and productive

¹¹ Chevallard's (2002) notion of mathematical organisation consists of a practical block (or know-how) which is represented by the kind of task and the associated technique and a theoretical or knowledge block characterised by justification, explanation and production of techniques.

fashion when generating data and will be discussed at length in both Chapter Three and Chapter Four. The literature presented in this section attempts to map out why South African students might be so far behind many of their counterparts in other countries and why their accomplishments scarcely register on the TIMSS scale and other standardised tests.

A more in depth analysis of what is constituted as mathematics at the micro-level of the working-class classroom is required as a means for examining the problem of student underachievement at a more refined level. The literature surveyed for the purposes of this research allows me to locate the South African context of curriculum reform in relation to other literature in the field. This will allow me to consider the implications of a curriculum that places great emphasis on describing mathematics in terms of the activity of the learner rather than a description of mathematics in terms of formal proofs, axioms and definitions. In so doing I aim to address my research question: What is constituted as Grade 8 mathematics in the pedagogic spaces of five working class schools in the Western Cape in the context of the GET curriculum that has de-emphasised the use of formal definition and propositions of mathematical objects and processes?

Chapter Three presents the general theoretical framework that structures this study.

Chapter 3

Theoretical Framework

This chapter presents a description of the general theoretical framework that frames this study which aims to investigate the constitution of school mathematics in present curriculum conditions; specifically, in what is constituted as mathematics, and how this is effected in the teaching and learning of school mathematics in fifteen Grade 8 mathematics lessons in five working-class schools. In essence, this study aims to get to the core of what is constituted as mathematics and how this constitution is realised in a general milieu of curriculum reform which seems to suggest a shift in emphasis from the content of mathematics to the knower of mathematics.

An engagement with relevant theoretical resources will inform the analytic framework which will be used for the generation and analysis of data from the empirical setting. The development of the analytic framework is discussed in Chapter Four. Bernstein's (1996) theory of a pedagogic device serves as the overarching conceptual frame for structuring my research and functions in providing a description of pedagogic discourse from the macro-level of policy and curriculum to the micro-level of the classroom. As a means of exploring the constitution of mathematics in a curriculum context that has back grounded and to a certain extent, suspended, more formal presentations of mathematics in favour of more exploratory activity of the student, the work of Davis (2010a, 2010b, 2010c, 2011a, 2011b, 2011c, 2012, 2013a, 2013b) will be drawn on to describe the constitution of school mathematics in the five schools.

3.1 The Pedagogic Device

Bernstein's (1996) theory of pedagogic discourse provides a framework for examining how knowledge is transformed into pedagogic communication in the classroom. As discussed in Chapter One, Bernstein (1996: 42) proposed that knowledge is transformed into pedagogic communication through three hierarchically related rules; namely, the distributive, the recontextualising and the evaluative rules – constituting the pedagogic device.

In essence, the distributive rule decides who gets what knowledge, the recontextualising rule is responsible for transforming knowledge into pedagogic discourse and the evaluative rule transmits evaluative criteria for the production of legitimate text. The pedagogic device is a useful theoretical model for providing an analytical description of the way pedagogy functions in classrooms, tracing how knowledge is transformed from curriculum and textbooks into pedagogic communication in a general milieu of curriculum reform as outlined in Chapter One. According to Bernstein (1996: 42), it is at the level of the evaluative rule that the distributive effects of curriculum, texts for teaching and pedagogy are realised and this is primarily where my study is located. As mentioned earlier, Appendix A provides extensive commentary on the pedagogic device and its relevance to this study. Since what is constituted as mathematics in pedagogy is not immediately obvious from

transcript records and classroom observation, the pedagogic device is invaluable in providing a description of what is constituted as mathematics and how this is effected when reflecting on the both the distributive and recontextualising rules. The evaluative rule allows one to see what gets realised in pedagogy and this is the central area of scrutiny in my study. An account of what gets realised in mathematics pedagogy by focusing on what teachers and students do will also be related to what is found in the *mathematics encyclopaedia*¹². The mathematics encyclopaedia, which has the requirement of consistency and completeness of content, will provide a general picture of the content of the topics announced in classrooms.

3.2 Evaluation

Evaluative activity has a structuring effect on what is constituted as mathematics in classrooms where the recognition and realisation rules enable students and teachers to recognise what the content is and how it should be realised. What is constituted as mathematics does not only refer to the topics and content to be acquired, but also the various methods employed by students and teachers to successfully complete problems. In essence, then, the nature of the evaluative activity constitutes mathematics in a particular way.

Consider an example of the structuring effect of evaluation on the constitution of mathematics in the task¹³ shown in Figure 3.1. The task was generated by web-based software designed for the purpose of practising multiplication. Mathematics tasks, which are recontextualised in this manner, usually present a story which describes a singular event and immediately poses a question which forces the learner to inductively generalise. The inductive step is perhaps not immediately obvious, but the nature of the evaluative judgement encoded in the software ensures that proceeding successfully requires an inductive leap.

¹² Mac Lane (1986: 409) describes that the “development of Mathematics provides a tightly connected network of formal rules, concepts, and systems” and Davis coined the term ‘mathematics encyclopaedia’ to describe this complex system of mathematical knowledge.

¹³ <http://www.mathplayground.com/katiebegin.html>

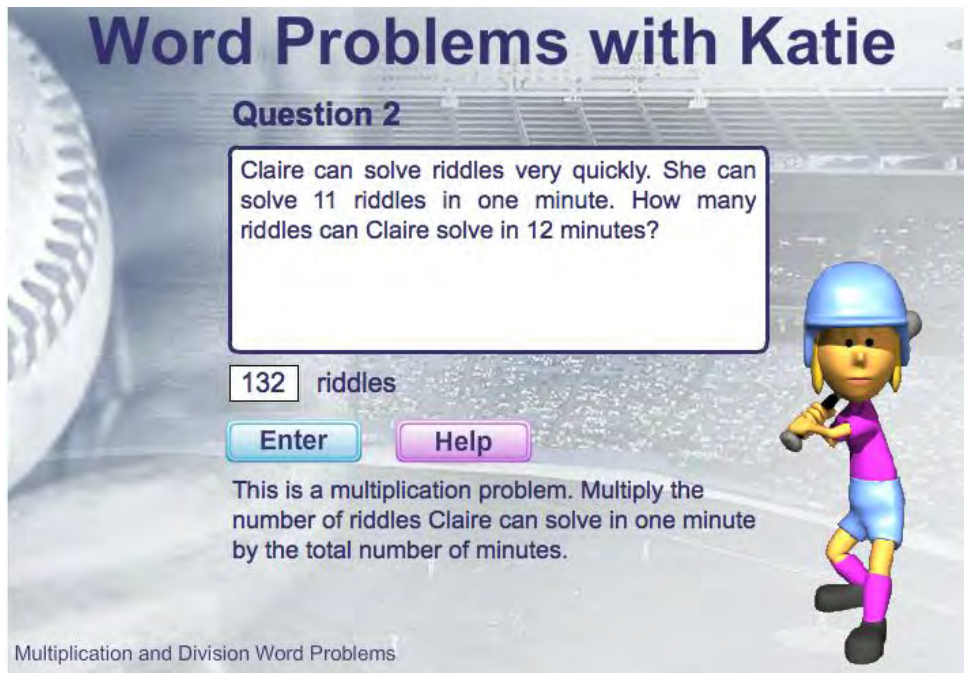


Figure 3.1: Web-based evaluative activity

The software will not accept as correct values that do not correspond to $a \times b$, as in 11×12 . In that way, the software recognises as legitimate only those responses that employ multiplication of the two supplied values. Implicit to the evaluative judgement encoded in the software is an inductive leap from a singular event to a universal existential feature: Claire always solves 11 riddles in a minute.

Table 3.1: A description of howmany riddles Claire can solve in 12 minutes

SINGULAR EVENT	Claire solves 11 riddles in one minute.
QUESTION	How many problems can Claire solve in 12 minutes?
INDUCTIVE LEAP	Claire always solves 11 riddles in a minute
SOLUTION	Claire can solve $11 \times 12 = 132$ riddles in 12 minutes

Claire can therefore solve 11 riddles in each of the individual minutes making up the 12 minutes, hence the calculation, 11×12 . The solution follows: Claire can solve 132 riddles in 12 minutes. The singular event describes the context and is only really a description in terms of this one instance, i.e., that particular minute. The only way of enabling the privileged solution, $11 \times 12 = 132$, is to force the learner to make an inductive leap, i.e., Claire is able to solve 11 riddles in each of the 12 minutes without being affected by time or context. The problem is actually impossible to answer without the student making the inductive assumption described in Table 3.1. The software will not accept any value other than 132 as a solution, forcing the student to accept an

inductive treatment of mathematics in order to solve the problem. So here we find mathematics constituted as an inductive activity as an outcome of evaluation.

3.3 Objects and operations and its association with the evaluative nature of pedagogy

It is through the process of evaluation that the criteria employed by students and teachers, engaged in mathematical activity, are illuminated. In pedagogic contexts, teachers confront students with knowledge they need to acquire. Typically, teachers present worked examples to illustrate procedures for solving classes of problems. Following Davis (2010b, 2010c, 2010d, 2011a & 2011c), the mathematical activity of teachers and students will be described and analysed in terms of the computational activity, i.e., the collections of objects and operations employed by teachers. Computational activity is synonymous with what Davis (2010b) refers to as operational activity in Figure 3.2. Davis (*ibid.*) provides an approach (Figure 3.2) for constructing descriptions of mathematical operations and objects and their inter-relations by examining the scriptural practices of students and teachers¹⁴.

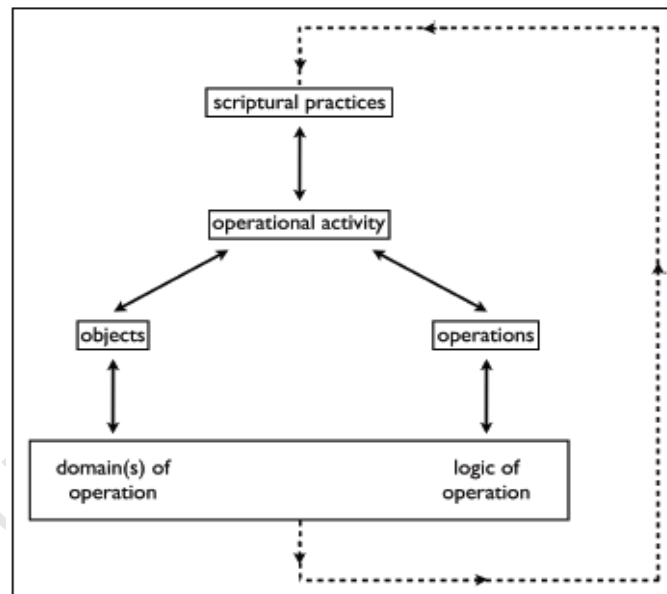


Figure 3.2: Successive re-descriptions of scriptural practices (Davis, 2010b: 102)¹⁵

The unit of analysis introduced by Davis (2003, 2005, 2010a: 5) is an *evaluative event*, and will serve as a means of segmenting the transcript and videos into sequences of pedagogic activity and is described as being “composed of a sequence of pedagogic activity, starting with the presentation of specific content in some initial

¹⁴ Davis (2010b: 102) describes ‘a series of successive re-descriptions of scriptural practices (indicated by the double-headed arrows) is thought of as producing a description of the relation between the domain(s) and logic of operation, so describing a generative structuring of scriptural practices (indicated by the dotted line).

¹⁵ Operational activity as described by Davis (2010b) is synonymous with the term computational activity.

form, and concluding with the presentation of the realisation of the content in final form”. These segments of pedagogic activity are classified with respect to both mathematical topic and type of activity that teachers and learners are engaged in. The evaluative event thus illuminates what is constituted at the level of content. This information is crucial in providing insight into my research question: what is constituted as mathematics and how? If one reflects on the web-based evaluative activity in Figure 3.1, it is evident that evaluation has a productive effect on what is realised as mathematics since the manner in which evaluation is encoded in this software program implicitly implicates an inductive generalisation. The evaluative event, which has a starting point and an end point, will provide access to both the computational activity that students and teachers engage in over a period of time and the regulation of such activity.

Pedagogy in the mathematics classroom involves teaching procedures and concepts as well as meanings of words, such as ‘derivative’, ‘transformation’, ‘multiple’, etc. and the use of sketches or figures as representations. So, one is constantly engaged in semiological activity in which signs are related. The importance of providing a semiotic perspective of pedagogy is useful and will allow for a description of the nature of mathematical meanings emerging in the context of this research where the constitution of what emerges as mathematics and how this is effected is of primary interest. In each of the evaluative events, signification is not unique and I draw on the Saussurian notion of the arbitrariness of *signifier-signified* relationship when confronted with different empirical settings in pedagogy. This allows one to consider meanings that emerge in the context (Saussure, 1983: 110-120). I am interested in what emerges in the context of mathematics pedagogy in the empirical site. For Saussure, all signs have arbitrarily ascribed meanings since the relationship between the signifier (sound, expression) and the signified (the object represented by the signifier) is not fixed. In essence, the representation of the object i.e. signifier does not define it and the relationship between signs are constantly changing where different signifier-signified couplets may be present in different contexts. The aim of this research is to establish what this relationship is in the particular context.

Chapter Four will offer a more in depth account of the evaluative event and how it will be operationalised.

Davis (2010d, 2011a) argues that mathematics is fundamentally constituted as the composition of functions at the level of its operations. This essential feature guarantees mathematics with great stability since functions have unique outputs for given inputs. Gallistel & King (2010: 43-44) describe the notion of function as follows:

A function is a deterministic mapping from elements of one set of distinct entities, called the *domain*, to elements from another set of distinct entities, called the *codomain*. [...] Functions of computational interest are almost always described by *algorithms* (step-by-step processes) that will determine the mapping defined by a function.

[...] An algorithm allows us to determine, given any element in the range¹⁶ (typically called the *input* or *argument* to the function), the corresponding member of the codomain (typically called the *output* or *value* of the function). (Italics in original.)

Davis (2010d: 100) further argues that any change to the essential requirement that operations are functions, result in operations being unstable. Davis (2011a: 98) states: ‘(a)n operation, $*$, is defined in general terms as a function of the form $*$: $D_1 \times D_2 \times \dots D_k \rightarrow C$: where the sets D_j are the domains of the operation, the set C is the codomain of the operation; the fixed non-negative integer k , which indicates the arity of the operation.’

The computational activity of teachers and students deployed in pedagogic situations may involve operations normally found in the mathematics encyclopedia or may entail what Davis (2010d, 2011a) refers to as *operation-like manipulations*¹⁷. A commonly used operation-like manipulation found in procedures for solutions of equations in school classrooms, can be described as ‘change sides, change signs’, is nowhere to be found in the mathematics encyclopaedia even though it is ubiquitous to the pedagogic treatment of mathematics in South African schools. A focus on the objects and operations in pedagogic encounters will enable the identification of operation-like manipulations and will enable one to gauge whether the mathematical activity in the classroom corresponds with the objects and operations in the mathematics encyclopaedia.

Operational resources found in the encyclopaedia allow us to select procedures based on mathematical definitions, properties and axioms. Since operations are functions, and the rules for any function are essentially limitless, the outcome of an operation can be arrived at in countless ways. Davis (2010d, 2011a) describes this unique property of operations in his methodology for describing pedagogic activity. Lawvere & Schanuel (1997) claim that a rule, i.e. the process where each element of the domain is associated with an element of its codomain, for a function is not exclusive with respect to the function. An example used by Lawvere & Schanuel (*ibid.*: 22-23) depicting how two different rules produce the same output follows in the extract where they discuss the rule f where one adds 1 to the input value followed by squaring the result, whereas for rule g , you ‘square the input value, double the input value, add the two results and then add 1’:

What the equation $(x + 1)^2 = x^2 + 2x + 1$ says precisely that $f = g$, not that the two rules are the same rule (which they obviously are not; in particular one of them takes more steps than the other).

¹⁶ Range in this instance does not refer to the mathematical notion referred to as range.

¹⁷ Operation-like manipulations are not found in the mathematics encyclopaedia, but commonly used in procedures to obtain solutions to problems in school classrooms e.g. ‘change sides and change signs’ when solving equations or ‘remove signs, subtract values and attach the sign of the bigger value’ when adding integers with different signs.

Having access to the basic axiomatic features relating to a particular topic allows one freedom to select any procedure which obeys the properties and axioms relating to that topic, rather than limiting one to a specific procedure imposed by the operational resources for that topic.

In mathematics, the encyclopaedic description of any topic is simplified, yet complete and certain, where the objects are homogeneous, the operations are functional and the interrelations between objects and operations can be explained in purely mathematical terms. The pedagogic treatment of mathematics has interesting features not necessarily prevalent in the mathematics encyclopaedia. When a topic is presented in school mathematics, the procedures used by the teacher give an indication of the criteria required to regulate the activity. The criteria are, generally, the rules for regulating mathematical activity in the recognition and realisation of content, and they govern the selection of operational resources that are used by teachers and learners. These criteria may or may not be found in the mathematical encyclopaedia, yet they function in producing a succession of transformations in the procedure and the final transformation usually produces the answer. An attention to the content found in the mathematics encyclopaedia as well as the recontextualised mathematics circulating in pedagogic contexts is of interest in this study. Describing the computational activity of teachers provides insight into what is constituted in the pedagogic context. A focus on the mathematics encyclopaedia in relation to school mathematics is an attempt to describe the mathematics in schooling and how this is achieved (mostly implicitly) to satisfy the axiomatic foundations of mathematics as noted by Davis & Gripper (2012a: 166):

[...] whatever emerges in school as mathematics is obliged to register some computational fidelity to mathematics in general. As a result, whatever does emerge in schooling as mathematics is implicitly designed to realise a reasonable degree of structure-preservation with respect to the axiomatic foundations of the content, even if only at a local level, and the fashioning of operations of the type we encountered here is one way in which teachers and their students attempt to achieve that goal.

This extract is helpful in explaining the need for consulting the mathematics encyclopaedia – primarily to describe what and how teachers implicitly substitute for the axioms that underpin school mathematics.

3.4 The role of the mathematics encyclopaedia

We need to draw a distinction between mathematics contents as described in, for want of a better term, the mathematics encyclopaedia, and mathematics contents as realised in pedagogic situations. In contrast to the completeness of the content as rendered in the mathematics encyclopaedia, the pedagogic treatment of mathematics often exhibits features not found in the mathematics encyclopaedia. Since criteria are regulative they function to produce the succession of transformations that make up procedures, with the final transformation in a sequence producing the desired outcome. However, the transformations often consist of alternative operations or manipulations to that found in the mathematics encyclopaedia.

When examining the implications of mathematics constituted in a milieu that has backgrounded the use of clear, explicit definitions of mathematical objects, as well as the use of more formal propositional statements of mathematical relations between mathematical objects, it may be useful to attempt to understand the role of definitions in this particular context. Formal definitions announce the existence or features of objects. In other words, objects and processes emerge from these definitions. However, in the context of curriculum reform where there is a de-emphasis in the use of explicit definition of objects and processes, this study will attempt to illuminate what and how mathematics is constituted by analysing the propositions, procedures and descriptions of mathematical content that emerge in the pedagogic context.

Topics, from the encyclopaedic perspective, are organised into fields (set of objects with two operations, namely addition and multiplication) and it is this field structure that allows the topic to be realised, since it allows one to perform all computations. The set, in turn, presents the objects which serve as arguments for an operation. Domains of objects refer to objects as elements of sets e.g. natural numbers, real numbers, integers, characters etc. and they constitute inputs or arguments of operations. It is useful to distinguish between objects in the mathematics encyclopaedia and objects in the pedagogic activity when describing what is constituted as mathematics and how this materialises. When the contents of the encyclopaedia are taken up for transmission and acquisition in pedagogy, the contents named in the curriculum e.g. multiple, derivative, equation etc. are associated with ideas and relations between ideas. Ideas would therefore necessitate a choice of object for association with a name taken from the mathematics encyclopaedia – and an attempt on the part of the teacher to pedagogise this.

3.5 Regulating the computational activity

The computational activity involved in mathematics pedagogy is usually related to a particular goal for realising content. In essence, then, procedures involved in computational activity consist of a series of transformations where the possible outputs (codomain) of one operation are the inputs (arguments) for the next operation and so on. An analysis of the computational activity requires that we not only focus on the nature of the objects operated on but also on the objects which are the inputs and outcomes of a particular operation. However, solely analysing the computational activity and its associated objects and operations does not provide one with a precise account of how the evaluative events may be regulated in pedagogy. It does not offer a detailed enough description of what regulative mechanisms may be governing the computational activity. In order to fully analyse the data generated from the evaluative events in lessons, additional theoretical resources that relate to the regulation of computational activity are required. The regulation of the computational activity is important since it not only describes the type of activity that students and teachers are engaged in, but will also provide insight into the criteria employed in each evaluative event.

The regulation of the computational activity will be elaborated on in my analytical framework. Firstly, I explore the regulative resources underlying pedagogic activity, which can be understood as a type of ground, as discussed by Davis & Johnson (2007a, 2008) and Davis (2010a, 2011b) in Table 3.2. Davis & Johnson (*ibid.*) describe ground as a means of describing both teacher and learner references to objects where each class of ground indicates a specific domain of objects being operated on.

Davis & Johnson (2007a, 2008) and Davis (2011b) describe *propositional ground* as the fundamental grounding upon which any proceduralising of mathematics rests but that the fundamental mathematical objects and operations indexed by mathematical expressions are often blurred by the pedagogic strategies at work. Davis & Johnson (2007a, 2008) and Davis (2010a, 2011b) describe the use of iconic resources in pedagogy as *iconic ground*, which involves regulation of the production of knowledge statements by referring to iconic similarity of expressions. They also describe *algorithmic ground* – use of a ‘standard form’ and rules, which involve the selection of operations from a collection of rules frequently used in computing certain types of procedures. *Empirical ground* is another category which entails the regulation of a mathematical activity by empirical investigation or measurement.

Table 3.2: Categories of ground (Davis, 2011b)

Ground	Central regulative resource	Objects of central concern
Iconic	Iconic similarities and differences of expressions	Graphical and/or symbolic expressions treated as images
Empirical	Empirical testing of expressions	Graphical and/or symbolic expressions treated as ‘measurable’
Propositional	Knowledge of mathematical objects and propositional relations	Mathematical objects indexed by the axioms, definitions and propositions signified by expressions
Algorithmic	Meta-rules governing an algorithm	Operations commonly used within a particular algorithm, and their sequencing

A series of analytical tools derived from the theoretical framework in this chapter will now serve to guide my interpretation of the data production and analysis from the research archive comprising of fifteen Grade 8 lessons from five schools. This follows in Chapter Four.

Chapter 4

A framework for the production and analysis of data

My study is a qualitative analysis of the constitution of school mathematics against a general background of curriculum reform in the GET phase of schooling where the use of formal definitions and formal statements of propositions and proofs have been de-emphasised. This chapter focuses on a discussion of the methodology for the production of data from the information archive consisting of fifteen video-recorded Grade 8 lessons from the five schools selected for this study. Firstly, I present an overview of the schools, followed by an account of the data collection process. I will use my analytical framework to elaborate school mathematics that emerges in the pedagogic contexts.

The process of developing an analytic framework will enable the theoretical framework to provide an accurate description of the empirical field. The development of analytic resources, which usually entail a system of categories and sub-categories, will function to activate the theory to do the work of data production and analysis from the research archive comprising of fifteen Grade 8 lessons from five schools.

4.1 Research design

4.1.1 The schools

The five schools forming part of a research and development program at a South African University are situated in Cape Town and have been identified as having a high population of learners from disadvantaged backgrounds. Four of these schools draw student populations mainly from the ‘African’ townships of Khayelitsha and Phillippi, which are classified as being predominantly lower working class areas. The remaining school draws its student population from mostly working class areas in greater Cape Town.

The schools will be referred to as: School P1 (well-resourced ex-Model C school in the central metropole of Cape Town situated in a former ‘white’ area populated by ‘coloured’ and ‘African’ learners), School P2 and School P3, (ex-Department of Education and Training (DET) schools situated in former ‘African’ townships), School P6 (ex-DET school situated in a ‘white’ area in the central metropole of Cape Town) and School P7 (well-resourced ex-House of Representatives (HoR) school situated in a former ‘coloured’ area on the Cape Flats with the learner population that is almost exclusively ‘coloured’).

4.1.2 Data collection

In February 2009, mathematics lessons were observed and video-recorded in grades 8, 9 and 10 in each of the five schools by a number of researchers¹⁸ from a South African university. As already mentioned, these five schools participated in a research and development program at a South African University; Schools P1, P2 and P3 having participated in the first phase of the program in 2007 and Schools P4 and P5 being replaced by schools P6 and P7 in Phase II.

The information archive created in Phase II of the program consists of observation notes, video records and transcripts - a total of 43 lessons were recorded across the five schools. Three consecutive single period lessons per grade were observed and video-recorded, except for two schools where only two lessons were video-recorded. The data collection process involved the use of two cameras, one focusing on the activity of the teacher and the other focusing on the activity of students. Transcription of the video records followed and, where necessary, isiXhosa and Afrikaans teacher and learner speech were translated into English.

For the purposes of my research, video records of lessons, observation notes and lesson transcripts were compiled from three consecutive Grade 8 mathematics lessons in each of the five schools, i.e. 15 lessons, which have been analysed using the analytic framework that follows in this chapter. Following this, Chapters Five, Six, Seven, Eight and Nine will present a discussion of the data generated by my analysis of the transcripts and video records of Schools P1, P2, P3, P6 and P7 respectively.

4.2 Analytical framework and data production

The analysis of any given lesson will proceed in the following manner:

- (i) Lessons will be segmented into sequences of evaluative events and sub-events.
- (ii) Evaluative events will be described and analysed in terms of the computational activity of the teacher by examining the particular operations in play and the collections of objects over which the operations range in each procedure i.e. the domains and co-domains.
- (iii) Selections related to the announced topic from curriculum documents and the textbook or other curriculum resources used by the teacher will be discussed.¹⁹
- (iv) Aspects of the mathematics encyclopaedia relevant to the mathematical topic announced in the pedagogic context will be described.²⁰

¹⁸ I did not personally observe and video-record these fifteen lessons.

¹⁹ Drawing on curriculum documents and textbook extracts that relate to the announced lesson topic will allow me to relate what emerges in the pedagogic context to what is found in these documents as I explore the constitution of mathematics.

²⁰ The mathematics encyclopaedia provides a description of the mathematical body of knowledge in its entirety in terms of the fundamental mathematical propositions, definitions and propositions. So relating the computational activity entailed in

- (v) The manner in which each evaluative event is regulated²¹ will be examined and categorised using the schema of analysis developed in Chapter Four as a means for re-describing the computational activity outlined in Step (ii).

At the primary level of data production, i.e. Step (ii), Appendices E, F, G, H and I provide a precise account of what is entailed, operationally, by offering descriptions and analyses of how the mathematical content is recognised and constituted in the 15 lessons in the archive. In addition, the above-mentioned appendices include descriptions of aspects of the mathematics encyclopaedia relevant to the mathematical topic announced in the pedagogic context. The regulation of evaluative events constitutes the secondary level of data production as described by Step (v).

I will now focus on providing a more detailed account of each of these steps in my analytic framework.

4.2.1 Evaluative events

The transcripts from each of the five schools will be segmented into evaluative events, which serve as the unit of analysis. The notion of an evaluative event was developed by Davis (2003) through his engagement with the work of Bernstein, Hegel and Lacan. An evaluative event traces the introduction of an idea or procedure in a pedagogic context over time - registering criteria that are employed to regulate activity, specifically with respect to computational activity.

In pedagogic contexts, teachers confront students with knowledge they need to acquire over time. An evaluative event is composed of a sequence of pedagogic activity, starting with a presentation of specific content in some initial form, and concluding with the presentation of the realisation of the content in a (provisionally) final form. Such finality might only be temporary, as in cases where content requires elaboration over several lessons (or even over several grades). The idea of the evaluative event is used to partition records of pedagogic situations into sections that are homogeneous with respect to the mathematical topic and the particular type of activity that participants in the pedagogic situation are engaged in (Davis 2011a). Consider the transcript extract from school P1 (Lesson 1) which shows the shift from Evaluative Event 1 (EE1) to Evaluative Event 2.1 (EE2.1) where the topic changes from Prime factorisation of natural numbers to Multiples of natural numbers:

EE 1 Teacher: So, but in the exam we won't specify, we won't say you must use this method or that. It will simply say, write it as a product of its prime factors and then you choose the method that you find easiest.

EE 2.1 Teacher: Okay, then in the lab now, we started with multiples and we said that the multiples are the answers that that you get when you do what?

the procedures carried out by teachers and students for an announced topic to relevant selections from the mathematics encyclopaedia is helpful in establishing what is constituted as mathematics and how.

²¹The regulation of each evaluative event will reveal the criteria which may be prevalent in textbooks or the curriculum.

Evaluative events are at times partitioned into sub-events. Sub-events are identified as instances when teachers digress from the topic as a result of misunderstanding, an interruption in the pedagogic encounter or dialogue entered into to assist learners in acquiring content related to the evaluative event (Davis, 2011a). In addition, sub-events may be used to distinguish a change in the pedagogic activity of the teacher and/or students. The evaluative events are further classified in terms of the pedagogic activity that ensued during a particular period of time and can be categorised as follows:

- Expository – entailed an exposition of new content or revision of content by providing explanations, definitions or by doing a series of worked examples.
- Exercises – entailed learners completing a series of problems individually or in a group
- Marking – entailed learners’ work being evaluated either by the teacher, themselves or them doing solutions on the blackboard in the classroom.
- Investigative – entailed students exploring mathematics content generally by grappling with a practical activity.

A Grade 8 lesson at School P1, for example, can be described as consisting of three evaluative events, a summary of which is listed in Table 4.1.

Table 4.1: Evaluative events spanning a Grade 8 lesson at School P1

Evaluative Event	Type	Length
0 1 Prime factorisation of natural numbers [00:00 – 01:26]	Expository	01:26
2.1 Multiples of natural numbers [01:26 – 02:50]	Expository	01:24
2.2 Addition of fractions with same denominators [02:50 – 04:50]	Expository	02:00
2.3 Addition of fractions with different denominators [04:50 – 07:35]	Expository	02:45
3.1 Practice calculating lowest common multiples [07:35 – 14:08]	Expository	06:33
3.2 Calculating lowest common multiples [14:08 – 19:46]	Exercise	05:38

Evaluative Event 2 consists of three sub-events and Evaluative Event 3 consists of two sub-events.

4.2.2 A description of the computational activity

Once the transcript has been segmented into evaluative events, the next step entails examining the scriptural practices of teachers and students following Davis (2011b). As discussed earlier, evaluative events present a particular pedagogic sequence that introduces and elaborates a particular idea. Davis (2011b) examines the

scriptural practices of learners and teachers where scriptural practices are the ‘mathematical statements and their transformations’ which allow one to mark out the interconnection between two fundamental features of computational activity (Davis 2010b: 102), namely, the operations employed and the collections of objects operated over.

[W]e start by fixing on what it is that teachers and students do, which is then redescribed in terms of operations or operation-like manipulations. Since operations and objects are compossible, the construction of a description of an operation, or of an operation-like manipulation, also produces the construction of an associated description of the objects operated upon. From the objects we can generate descriptions of the domains over which mathematical activity operates. The selection and organizations of the particular operations and operation-like manipulations in play enable the production of descriptions of the operational logic at work (*ibid.*).

When attempting to examine what is constituted as mathematics in schooling and how such mathematics comes to be constituted, the elements under consideration are the transformations. Transformations entail one or more operation or operation-like manipulations and associated collections of objects.

Consider an extract from a transcript of a Grade 8 lesson where we encounter an instance of the constitution of the notion of a multiple. This extract is helpful in providing a description of how mathematical objects emerge, are referred to, or described in the context of pedagogy. I then provide a description of how the objects and operations, that constitute the computational activity, are recognised as well as an account of how the curriculum and texts for teaching present the notion of a multiple.

- Teacher: Okay, then in the lab now, we started with multiples and we said that the multiples are the answers that you get ... When you do what?
- Learner: Times.
- Teacher: When you?
- Learner: Times.
- Teacher: Times what?
- Learners: Times ... By the um .. By the um ...[inaudible].
- Learner: By the prime number.
- Teacher: By the prime number? [Speaks with rising intonation and frowns, indicating her disagreement.]
- Learner: [Many learners call out at once.] No. No.
- Teacher: No. Okay. Let's say I ask you for multiples of five. [Teacher cleans a section of chalkboard as she speaks.] Remember we did .. factors. Hey? So now we're doing .. multiples.
- Learner: Natural numbers. [It appears that the learner is reading from the textbook. See Figure 4.3.]
- Teacher: So I'm asking you now for the multiples of five.
- Learner: [Many learners call out at once.] Five. Five. Five, ten. Five. [Teacher gestures with her hands, indicating that she wants learners to count off multiples of five. – See Figure 4.1.]
- Teacher: It's five, ten ..
- Learner and Teacher: Ten, fifteen, twenty.

Teacher: [Interjecting.] So we said .. How do we get to those answers? We multiply it by?

Learner: Multiply. Multiply by five.

Teacher: We multiply it by?

Learner: Five.

Learner: By one.

Teacher: By one.

Learners: By two. By three.

Teacher: By two. By three. .. The answers are the multiples. So the multiples of five will be? ... Five. [Teacher writes /5/, followed by /5, 10, 15, 20, 25/ as learners call out the multiples of five.] (School P1, Grade 8, March 2009, lines 12-34.)

Consider Figure 4.1 where the teacher gestures with her hands, signalling that she wants learners to count off multiples of five. This, perhaps, unintentional approach also gives insight into the notion of a multiple is constituted.



Figure 4.1: How to calculate multiples of five: 'five', 'ten' and 'fifteen'

I now describe the computational activity that unfolds when examining the transcript for Evaluative Event 2 in more precise terms by focusing on the procedures used by the teacher for computing multiples. The teacher suggests two methods for calculating multiples. The first method relies on the use of the multiplication times table and the second method primarily involves the operation of multiplication. The procedure for the two methods can be restated in the following manner:

Method 1: Read off or recite multiplication times table for the natural number a .

Method 2: Given a number, a (implicitly a natural number), multiply that number by 1, then 2, then 3, then 4, and so forth, until you obtain the number of multiples that are required. The central operation of the procedure is multiplication over the natural numbers, resulting in the production of a finite sequence of n numbers, like: 5, 10, 15, 20, ..., $5n$ for some $n \in \mathbb{N}$.

In other words, what we have is a function, f , from the natural numbers (domain) to the natural numbers (codomain). That is, $f(n) \rightarrow an$, where $a, n \in \mathbb{N}$. The particular rule (simple multiplication), which effects the mapping from \mathbb{N} to \mathbb{N} and so generates the required sequence of multiples, has a particular number, a , as one argument, and $n \in \mathbb{N}$ as the other. That is, $x(a, n) \rightarrow an$, where $a, n \in \mathbb{N}$. As a means of summarising the two methods just presented, I now present an account of the objects that are arguments (input or domain) of operations and the associated values (output or codomain) entailed in the constitution of the notion of a multiple in Table 4.2:

Table 4.2: Describing the objects involved in the computational activity in terms of the domain and codomain for the two methods for computing multiples (School P1 Lesson 1 EE2)

Lesson 1	EE2.2[01:26-02:50]	Multiples	Input (domain)	Output (codomain)	Operation
Method 1			$a \in \mathbb{N}$	The multiplication table for a .	Multiplication
Method 2			$a \in \mathbb{N}$ and \mathbb{N}	Sequence: (a_n) \mathbb{N}	Multiplication

4.2.3 Curriculum and textbooks

Both the curriculum and texts for teaching provide invaluable insight into the constitution of mathematics in pedagogic contexts and I am interested in how teachers use these resources to inform what is constituted as mathematics and how. I present extracts from both the RNCS and the textbook used in this particular pedagogic encounter with multiples.

Multiples and factors are included in the curriculum but only in terms of being able to recognise, classify and represent them with no reference being made to defining explicitly what they are:

OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them:

- integers;
- decimals, fractions and percentages; [...]
- multiples and factors; [...] (DoE, 2002: 69)

The curriculum does not state that learners are required to define what a multiple is. The learners are merely expected to ‘recognise, classify and represent’ multiples.

The textbook used by the teacher, *Preparing for High School Maths*, presents the definition of a multiple either as ‘a number (that) can be found in the ‘multiplication table’ of that number’ or ‘the multiples of a number are obtained by multiplying it by the natural numbers’ as presented in Figure 4.2. and Figure 4.3. (Bull & Hepworth,

2008: 20). Both definitions for multiple provided by the textbook are defined in a computational manner and both are supplemented with an example.

2 NUMBER THEORY

DEFINITIONS :

A **prime number** has only two factors – itself and one.

A **composite number** has more than two factors.

Factors are the whole numbers which divide exactly into the number.

Common factors are those factors which are shared between two or more numbers.

Multiples of a number can be found in the ‘multiplication table’ of that number.

Example: The first 6 multiples of 10 are 10, 20, 30, 40, 50, 60.

Figure 4.2: Definitions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

SKILL 5 MULTIPLES

The multiples of a number are obtained by multiplying it by the natural numbers.

Example:
Find the first five multiples of the number 8.

Solution:
The first five multiples of 8 are:
8, 16, 24, 32, 40

1st multiple is $1 \times 8 = 8$

2nd multiple is $2 \times 8 = 16$

3rd multiple is $3 \times 8 = 24$

4th multiple is $4 \times 8 = 32$

5th multiple is $5 \times 8 = 40$

Figure 4.3: Multiples in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

When relating the two methods for finding a multiple prescribed by the teacher with the ‘definitions’ provided by the textbook in Figure 4.2 and Figure 4.3, it appears that the textbook may be her primary resource since the methods are so similar.

4.2.4 The mathematics encyclopaedia

The mathematics encyclopaedia is a useful analytic resource, since, what is constituted as mathematics in the pedagogic context often stands in place of the field axioms. In this particular instance, I describe how the teacher's constitution of the notion of multiple relates to the notion of multiple in the encyclopaedia.

Typically, a mathematical definition of multiple is described as a cluster of inter-related ideas: factors, co-factors, divisors, divisibility, products and, usually, integers. Outside of dictionaries, appropriate definitions are found in texts on number theory, an example of which follows.

In the case in which the quotient resulting from the division of a by b is an integer, denoting it by q , we have $a = bq$, i.e. a is equal to the product of b by an integer. We will then say that a is divisible by b or that b divides a . Here a is said to be a *multiple* of b and b is said to be a *divisor* of the number . The fact that b divides a is written as: $\frac{a}{b}$. (Vinogradov, 1954: 1-2; italics in original.)

The definition maps out the set of inter-related existential features of a number when it can be described as a multiple. In terms of the definition, the recognition of one number as a multiple of another entails an activation of a web of ideas, as indicated earlier, that are much richer than the idea of multiples as the answers we get when we multiply by one, by two, by three, and so forth.

Within an evaluative event, there is some way in which a *name*, e.g. 'factor', indexes some kind of existence and marks out a particular mathematical entity. Based on Saussure's (1983) proposition regarding the arbitrary nature of the signifier-signified relation, the name associated with the emergent idea that the student pieces together from the context might not have its origin in the encyclopaedia.

It is necessary for me to reflect on how mathematics is rendered in these two contexts, i.e., in the mathematics encyclopaedia and in pedagogy (including the curriculum, texts for teaching and pedagogy) when attempting to analyse what is constituted as mathematics in pedagogic contexts and how it emerges. Relating the pedagogic context to what is found in the mathematics encyclopaedia will provide a means of helping me determine what the mathematics that emerges in pedagogic contexts stands in place of and if, at all, there has been any substitution of mathematical content.

In this instance, the notion of a multiple is constituted in a manner very different to what is found in the mathematics encyclopaedia. No explicit formal definition is provided by the RNCS, textbook or teacher and students are encouraged to 'define' a multiple either by using the multiplication times table as a resource or by using multiplication as the central operation.

Names provided in the encyclopaedia make no reference to the agent of mathematics, i.e. the student, and there is a direct approach to the mathematical object. An analysis of evaluative events, in terms of its computational activity, will no doubt reveal interesting features when relating the encyclopaedic version of mathematics to the pedagogic version of mathematics.

I now shift to the next level of analysis where I discuss the regulation of each evaluative event. This will provide even more clarity when attempting to provide a description and analyses of what emerges in pedagogic contexts.

4.2.5 Regulation of the computational activity

In addition to analysing each evaluative event in terms of the computational activity of the teacher, I am also interested in developing resources that enable me to describe the mechanisms that regulate computational activity. In developing resources for describing this regulation, I draw on the theoretical resources discussed in Chapter Three. I am specifically interested in how each evaluative event is regulated as this would provide insight into what and how mathematics is constituted. The regulation would describe the type of activity engaged in and provide insight into criteria employed in each evaluative event.

I have identified four categories for classifying the regulation of an evaluative event or lesson in the research archive. They resonate very closely with Davis & Johnson's (2007a, 2008) and Davis' (2010a, 2011b) categories of ground²² in the production of the description and analysis of mathematics teaching as outlined in Chapter Three in Table 3.1. These categories may be one of the following:

- 1) a set of *interconnected propositional statements*²³ of mathematical relations between mathematical objects;

²² Davis & Johnson (2007a, 2008) and Davis, 2011b describe *propositional ground* as the fundamental grounding upon which any proceduralising of mathematics rests but that the fundamental mathematical objects and operations indexed by mathematical expressions are often blurred by the pedagogic strategies at work. Davis & Johnson (2007a, 2008) and Davis (2010a, 2011b) describe the use of iconic resources in pedagogy as *iconic ground*, which involves regulation of the production of knowledge statements by referring to iconic similarity of expressions. They also describe *algorithmic ground* – use of a 'standard form' and rules, which involve the selection of operations from collection of rules frequently used in computing certain types of procedures. *Empirical ground* is another category which entails the regulation of a mathematical activity by empirical investigation or measurement.

²³ The mathematics encyclopaedia is populated with operational resources that provide access to a selection of procedures based on mathematical definitions, properties and axioms. Since operations are essentially functions, and the rules for any function are necessarily limitless, the effects of an operation can be arrived at in a variety of ways. Access to the interconnected propositional statements of mathematical relations between mathematical objects allows one to choose any procedure that obeys to the axioms and properties relating to that topic.

- 2) a *computational resource* for producing a required solution which may entail a proposition related to particular ordering of operations to solve problems where the selection and sequence of operations must be correct; e.g., choosing an appropriate procedure, calculation, algorithm or rule;
- 3) a *reliance on what the solution may look like*, e.g., symbolic representation or a specific spatial distribution of characters;
- 4) *general descriptions of mathematical concepts* rather than precise, formal mathematical definitions where there is no explicit engagement with the core axioms; e.g. an empirical encounter with mathematics where students rely on inductive generalisation in an attempt to find out the features of mathematics.

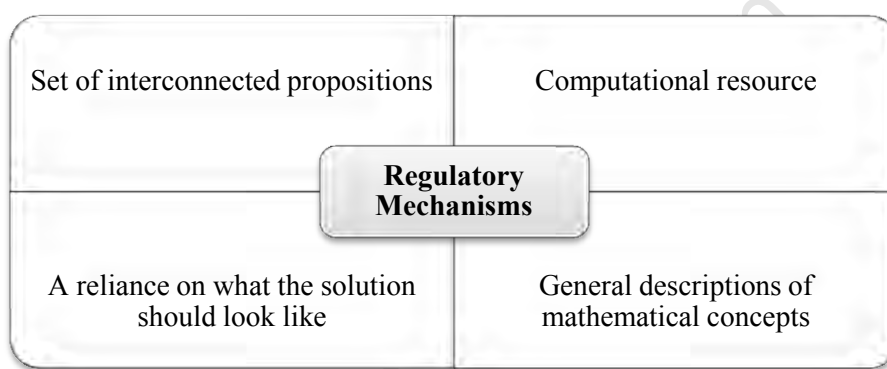


Figure 4.4: Forms of regulation

Ultimately, each evaluative event will be classified in terms of these forms of regulation so that I may analyse the computational consequences for what is constituted as mathematics and how.

The categories outlined in Figure 4.4 outline the forms of regulation governing an evaluative event or procedure. Once an account of the computational activity has been generated by examining the mathematical objects and operations, Davis (2011b) describes the utility of further classifying evaluative events using the categories of ground as discussed in Chapter Three. Davis (*ibid.*) highlights three key features that are outcomes of analysing the computational activity in pedagogic encounters. Firstly, the operations that emerge; secondly, the collections of objects over which these operations range and thirdly, the criteria regulating the selection and sequencing of these operations. I will operationalise these three features in this study and the forms of regulation prevalent in each evaluative event will reveal the criteria that regulate the student and teacher mathematical activity.

Davis, Adler & Parker (2007: 37) argue that “any evaluative act, implicitly or explicitly, has to appeal to some or other authorising ground in order to justify the selection of criteria” and so the criteria provide a description of how the mathematical activity in pedagogic encounters may be regulated.

The transcript extract on multiples presented earlier is a record of how mathematical objects emerge, are referred to, or described, in the context of pedagogy that does not provide clear definite descriptions of objects and processes.

An interesting feature of the constitution of mathematics is the nature of the objects operated over. The distinction between *definite descriptions* of mathematical objects (definitions) and descriptions of *natural kinds* is useful. Here the work of Kripke (1980) and Potter (2004) are helpful for describing mathematical objects and processes. The difference that exists between objects that are encountered empirically (natural kinds) and mathematical objects, whose existence is constituted in terms of a field of predicates announced in definitions, postulates and propositions, will illuminate the implications of a curriculum that employs definitions that are more referential than predicative, and essentially what this entails for the constitution of mathematics. The Kripkean (1980: 127) view describes ‘natural kind terms’ as words for natural substances, species and phenomena (e.g., light, gold, tiger) which are not definitions and are rigid. In other words, natural kind terms defy all attempts at providing them with a priori definitions:

A priori, all we can say is that it is an empirical matter whether the characteristics originally associated with the kind apply to its members universally, or even ever, and whether they are in fact jointly sufficient for membership in the kind. The joint sufficiency is extremely unlikely to be *necessary*, but it may be *true*. In fact, any animal looking just like a tiger is a tiger – as far as I know – though it is (metaphysically) possible that there should have been animals that resembled tigers but were not tigers. The universal applicability, on the other hand, may well be necessary, if true. (Kripke, 1980: 139; italics in the original.)

When attempting to render a natural kind in definitional form, the definition remains incomplete since objects belonging to a natural kind form a group of objects having a list of features that are not constructed through definition, but which are, instead, inductively accumulated and which are also not always necessary.

The teacher attempted to elicit answers from the learners to the question of what a multiple might be, and then prompted them on how to calculate multiples rather than providing them with an explicit, definite description of the notion of a multiple: “we said that the multiples are the answers that you get ... When you [...] times. (line 12-13)” When the learners failed to offer a suitable answer to the teacher’s question she resorted to using a specific example and used gesture (see Figure 4.1) to indicate that she wanted learners to count off multiples of five: “No. Okay. Let’s say I ask you for multiples of five. (line 21)” It seems that the teacher’s finger gestures prompted learners to assume that multiples of five are composed of a sequence of counting numbers, each of which is multiplied by five: 5×1 , 5×2 , ... and so forth. What the teacher wanted as an answer was 1×5 , 2×5 , ...: “By one. [...] By two. By three. (line 32-33)” and so forth.

Given that the teacher indicated counting numbers through gesture (Figure 4.1), it would not be unreasonable for learners to think of a sequence of counting numbers, each of which is multiplied by five as the concept of a multiple. Multiplication tables served as central regulative resources rather than the definition of a multiple and the relations between multiple, factor or divisor. Multiplication tables would therefore be considered as a *computational resource* in terms of my analytic framework to read the data. Table 4.3 summarises the procedure used, the content realised through the procedure, content substituted by the procedure as well as the form of regulation for the constitution of a multiple.

Table 4.3: The constitution of a multiple

Table 4.5: The constitution of a multiple

Announced topic	Procedures used	Form of regulation	Realised content	Substitutes for
Multiples	Multiplication tables, Multiplication	Computational resource	Multiplication tables	Definition of a multiple and relations between multiple, factor, divisor

4.3 An overview

The analytical framework described in this chapter provides a structure for the analysis and production of data for the study. The data production will enable me to generate statements regarding the constitution of mathematics prevalent in teachers' pedagogic practice in the five schools selected for this study. Chapter Five, Chapter Six, Chapter Seven, Chapter Eight and Chapter Nine that follow will detail the analyses and data production for lessons presented in School P1, School P2, School P3, School P6 and School P7 respectively. This chapter concludes with a discussion about the reliability and validity of my study.

4.4 Reliability and validity of the study

The fundamental properties of mathematics as described by Mac Lane (1986: 410) in his elaboration of the precision and completeness of rules, axioms, definitions and proofs of mathematics provide my study with the necessary reliability in this regard.

My research evidence is based on five working class schools in underprivileged contexts and I have provided extracts from texts, transcripts and the curriculum. I am therefore only able to draw conclusions about the constitution of mathematics in a milieu of curriculum reform that has under-specified mathematical objects and processes with reference to firstly, what the content is and secondly, the propositions relating to the content based only on these particular cases. Dowling and Brown (1998: 83) would describe the presentation of my argument and resulting interpretation as an 'elaborated description' (that) '[...] both develops and makes visible the operationalisation of the theoretical problem, elaborating its validity.' This study aims to investigate what is

constituted as mathematics and how in this particular context by employing methodological resources that will map out the computational activity entailed in pedagogy in a precise manner, rather than drawing any general conclusions.

4.5 Ethical considerations

This research project is a collaborative venture between a South African University and the five schools that participated in this study. One purpose of the observations was to generate a general description of the constitution of mathematics as a means for designing intervention strategies aimed at improving the quality of mathematics teaching at these schools. Written permission and ethics clearance was secured by the research and development project manager from the concerned parties at the five schools that participated in this study.

What follows is an account of the constitution of mathematics in each of the five schools in the research archive. Chapter Five, Chapter Six, Chapter Seven, Chapter Eight and Chapter Nine presents an account of the constitution of mathematics for School P1, School P2, School P3, School P6 and School P7 respectively.

Chapter 5

A discussion of a sequence of Grade 8 lessons at School P1

Having situated the study empirically and methodologically in the preceding chapters, this and the following four chapters focus on the production and analysis of data for each of the five schools. As discussed in Chapter Four, three video-recorded Grade 8 lessons per school were considered, resulting in an information archive consisting of fifteen video-recorded lessons. Each lesson was subjected to the procedures outlined in the analytic framework discussed in Chapter Four for the purposes of producing and analysing data. The aim of this and the next four chapters is to ascertain—through descriptions of the computational activity of teachers and students discussed in detail in the Appendices—what it is that comes to be constituted as the mathematics content of curriculum topics in the particular pedagogic situations recorded in the archive.

Previous research on teaching and learning in the schools that I refer to in this project characterises the mathematics lessons in the five schools in the following way: (1) the worked example serves as the general mechanism for the elaboration of mathematics; (2) worked examples are used to exemplify procedures for the solution of particular classes of problems; and (3) the explicit discussion of mathematical definitions and propositions is mostly absent from lessons (Davis & Johnson, 2007b, 2008; Mackay, 2009, 2010 & Jaffer, 2010a, 2010b). As I discussed in Chapter Three, I take the position that school mathematics is ultimately grounded in the axioms, definitions and propositions of mathematics even if teachers and students do not explicitly refer to the axioms and definitions, and even if teachers and students use a series of auxiliary computational resources to do their mathematical work (Davis, 2012). What the computational content is that is constituted to substitute for the explicit use of expected definitions and propositions in the computational activity of teachers and students in each instance is of central importance to my project.

Appendix E focused on the production and analysis of data for this Chapter from the records of lessons for School P1. Appendices F, G, H and I provide an analysis of data for Schools P2, P3, P6 and P7 respectively. Due to space constraints, the Appendices contain detailed analysis for each school. An analysis of the evaluative events will allow me to bring together information for a particular lesson in an attempt to extract data. For each evaluative event, I provided a discussion of the computational activity of the teacher and students and this was described in terms of what operations were employed in tasks and the domains and codomains of objects entailed in the operation.

In the Appendices for each of the five schools, the analysis of any given lesson proceeded following the protocol laid out in Chapter Four. This purpose of this chapter is to provide a summary of what is constituted as

mathematics for the teacher observed at School P1. I now present a brief description of the school setting and an overview of the three lessons followed by summary of the central points of concern to my research problem.

5.1 A synopsis of the lessons for School P1

School P1 is a former Model-C school situated in a middle class suburb but draws students from predominantly black, working class townships. The school building is well-maintained, the classrooms are spacious and the general classroom environment is conducive to teaching. All three lessons, which were not consecutive, were presented by the same teacher. Her teaching style reflected a combination of ‘chalk and talk’ methods as well as group activities. The Grade 8 class she taught comprised only of male students. The first two lessons followed the usual presentations generally adopted by teachers; that is, an introduction to the subject matter followed by worked examples and a selection of exercises that were to be completed by the students. The lesson format displayed a great deal of questioning from both the teacher and the students and students participated freely. Students generally responded to questions by raising their hands. Lesson 3 was approached differently, where the class engaged in a group work activity as a means of consolidating content that had already been covered in the previous two lessons. Students and teachers used the text, *Preparing for High School Maths*.

The main topic areas dealt with in the three lessons were *prime factorisation*, *multiples*, *lowest common multiples (LCM)* and *highest common factors (HCF)*. The first lesson started with a recap of two methods used for prime factorisation, the ladder method and the factor tree method. The recap was followed by a discussion of multiples and ultimately a procedure for determining the lowest common multiple. Lesson 2 focused on multiples, finding the LCM and the HCF of two numbers. Lesson 3 was presented as a practical activity, where groups of students were presented with two word problems as a means for further elaborating the ideas of the highest common factor and lowest common multiple. Consider the evaluative events for School P1 in Tables 5.1, 5.2 and 5.3.

5.2 A summary of the evaluative events for Lessons 1, 2 and 3

Tables 5.1 to 5.3 summarises my initial analysis of the lessons where I constructed the evaluative events for each lesson using the procedure laid out in Chapter Four:

Table 5.1: Segmentation of Lesson 1 into evaluative events

Evaluative Event	Type	Length
1 Prime factorisation of natural numbers [00:00 – 01:26]	Expository	01:26
2.1 Multiples of natural numbers [01:26 – 02:50]	Expository	01:24
2.2 Addition of fractions with same denominators [02:50 – 04:50]	Expository	02:00
2.3 Addition of fractions with different denominators [04:50 – 07:35]	Expository	02:45
3.1 How to calculate lowest common multiples [07:35 – 14:08]	Expository	06:33
3.2 Calculating lowest common multiples [14:08 – 19:46]	Exercise	05:38
		19:46

Table 5.2: Segmentation of Lesson 2 into evaluative events

Evaluative Event	Type	Length
1.1 Filling in missing multiples and number patterns [00:00 – 02:52]	Exercise	02:52
1.2 How to check if a number is a multiple of another [02:52 – 04:43]	Exercise	01:51
2.1 Lowest common multiple and highest common factor [04:43 – 08:44]	Expository	04:01
2.2 Highest common factor of two numbers [08:44 – 20:29]	Exercise	11:45
2.3 Emphasising the ladder method for finding highest common factor between two numbers [20:29 – 42:41]	Expository	23:12
		42:41

Table 5.3: Segmentation of Lesson 3 into evaluative events

Evaluative Event	Type	Length
1.1 Practical problem solving: Making a bicycle tassel [00:00 – 23:50]	Exercise	23:50
1.2 Using ‘mathematical knowledge’ – HCF to solve problem [23:50 – 35:20]	Exercise	11:30
2 Practical problem solving: Train arrival times [35:20 – 52:30]	Exercise	17:10
		52:30

After segmenting the three lessons into evaluative events, I described the computational activity of the teacher and students in order to establish what was constituted as the topic contents in the lessons and to identify what content did the work of the explicit use of apposite topic-related definitions and propositions. A detailed exposé

of the computational activity for School P1 can be found in Appendix E. Appendix E also provided a discussion of the regulation of topics for School P1 by presenting a summary of procedures used, the content realised through the procedures, the regulatory mechanisms as well as the content substituted by the procedures for each of the main topic areas. I now present an account of the constitution of mathematics for the teacher at School P1.

5.3 Summary of the constitution of mathematics in School P1

The main topics under consideration across the three lessons included prime factorisation, multiples, lowest common multiples and highest common factor.

These topics were explicated by way of providing students with the necessary criteria for performing computational procedures. The procedures, as described earlier in Appendix E, served as the main vehicles for explicating the content.

- **Prime factorisation**

For the topic, prime factorisation, the proposition stating that the smallest divisor of a natural number is prime is not taught. This particular proposition is implicit and my analysis in Appendix E shows that there are two ways of dealing with the absence of this proposition. In the first instance, the ‘factor tree method’ does not require one to know what a prime number is. When factorising a composite natural number using the ‘factor tree method’, factors are composite or prime and one keeps on applying the same rule to composite numbers – ultimately resulting in prime factors only. The central operation employed here is multiplication over the natural numbers. The second way that the teacher at School P1 copes with the absence of this proposition is by employing the ‘ladder method’. This method does require knowledge of the set of primes, but the central operation employed is division.

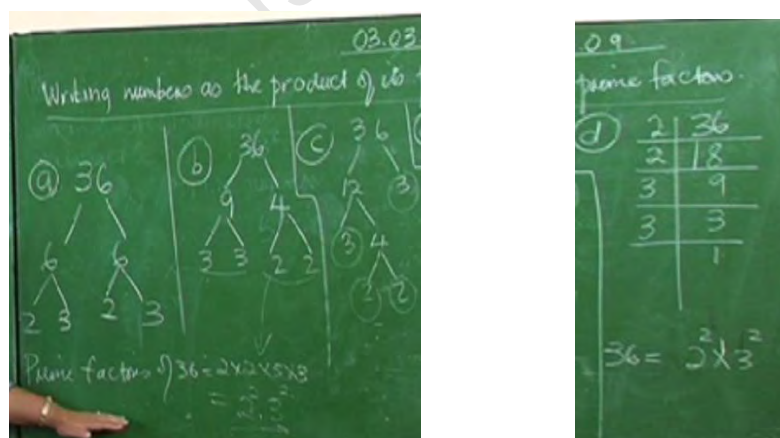


Figure 5.1: An exemplification of the use of the ‘factor tree’ method and the ‘ladder method’

Both the ‘factor tree method’ and the ‘ladder method’ displayed in Figure 5.1 are computational resources which are employed by both the textbook and the teacher and they substitute for the proposition that the smallest

proper divisor is prime.

- **Multiples**

In the evaluative events dealing with multiples, the definition of a multiple is not in sight. Rather multiplication tables and number patterns are the primary means of regulation for achieving the necessary outcome and stand in place of the definition of a multiple and relations between multiple, factor and divisor. The definition of a multiple and relations between multiple, factor and divisor emerges as result of a calculation procedure rather than its formal definition being made available when the topic is announced. This evaluative event provides a description of how mathematical objects surface, are referred to, or are described, in the context of pedagogy that does not offer clear definite descriptions of objects and processes.

- **Lowest common multiple**

The LCM is introduced to students using examples on addition of fractions having the same and different denominators. Thereafter, the solution procedure which entailed listing the multiples of the given values, circling the LCM and summarising the conclusion in a systematic manner is emphasised and certainly regulates finding the required solution. The layout of the solution in terms of how the answer should be written out is also an important means of regulation in this evaluative event since the teacher stresses that all problems of this nature should be done in this specific way.

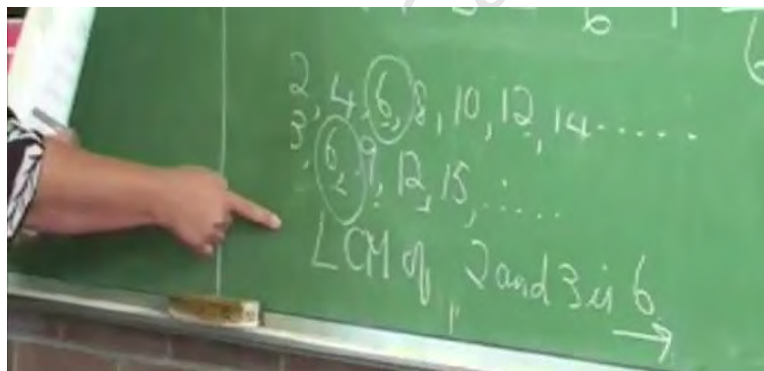


Figure 5.2: The spatial distribution of symbols in finding the lowest common multiple of 2 and 3

So the teacher's computational procedure as shown in Figure 5.2 substitutes for the definition of LCM and the relations between factor, multiple and divisor.

- **Highest common factor**

The teacher presents two methods for calculating the HCF. Method 1, the 'factor tree method' is presented as well as Method 2, the 'ladder method' already covered when prime factorisation was taught. She advocates the use of the latter method for calculating the HCF since there is less likelihood of making silly errors. Both the computational procedure prescribed by the teacher in both methods as well as the use of the ladder template to effect prime factorisation is emphasised to a great extent and certainly stands in for the formal definition of HCF and the relations between factor, multiple and divisor.

5.4 Regulation of the computational activity

I re-visit the synopsis outlined in Appendix E in Table 5.4 as a point of reference for summarising the constitution of mathematics as well as the regulation of the computational activity.

Table 5.4: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P1

Announced topic	Procedures used	Regulation of computational activity	Realised content	Substitutes for
Prime factorisation	Factor tree method	Computational resource	Multiplication of whole numbers	Definition of prime numbers Smallest proper divisor is prime
Prime factorisation	Ladder method	Computational resource Reliance on what solution looks like	Division of whole numbers	Relations between divisors and factors
Multiples	Multiplication tables, counting	Computational resource	Multiplication tables	Definition of a multiple and relations between multiple, factor, divisor
Lowest common multiple (LCM)	Multiplication tables, counting	Computational resource Reliance on what solution looks like	Multiplication tables	Definition of LCM Relations between factor, multiple, divisor
Highest common factor (HCF)	Factor tree method	Computational resource Reliance on what solution looks like	Multiplication of whole numbers	Definition of HCF Relations between factor, multiple, divisor

Table 5.4 is meant to show how for each of the announced topics an appropriate procedure or computational resource is always employed and that the realised content seldom mirrors the announced topic. This table also provides an account of how the content, which is meant to be realised, is regulated as well as what substitutes for the explicit use of definitions and propositions.

In conclusion, the ‘definitions’ of concepts (e.g. multiples, primes, etc.) that were provided by the teacher and textbooks were framed in terms of calculations, rather than being guided by the mathematical definitions of the objects as specified in the mathematical encyclopaedia. It is also worth noting that the curriculum does not contain definitions and does not specify that students should know definitions. In general, there is an attempt to define concepts predominantly in terms of calculations and any notion of what the mathematical object might be emerges from examples and remains tied up with the activity of the student. It appears that the computational resources were the main means of regulation in each of the topics in Table 5.4.

Chapter Six follows and presents an overview of the constitution of mathematics at School P2.

Chapter 6

A discussion of a sequence of Grade 8 lessons at School P2

School P2 is an ex-DET school located in an ‘African’ township on the Cape Flats and the student population of the school consists of children from families who live in the area that could be classified as a predominantly lower working class area. The Grade 8 class comprised of approximately 49 students and they were seated in groups in a rather poorly resourced classroom. The general classroom atmosphere was one where the teacher often spoke while students were having their own conversations, and students provided very little input to the lesson – mostly just observing how things were done initially.

The language of instruction was both English and isiXhosa and the teacher often switched languages during the course of the lesson. Students responded mostly in isiXhosa and were also required to do exercises after the topic was presented. Students generally accepted everything the teacher presented, which was often read directly from the textbook, and there did not seem to be any uncertainty when she provided an incorrect solution in Lesson 3.

6.1 A synopsis of the lessons for School P2

The three consecutive lessons covered at School P2 culminated in the topic, *Transformation Geometry*. The evaluative events across these lessons in Tables 6.1, 6.2 and 6.3 provide an accurate description of the content areas focused on and the nature of the pedagogic activity. The content that emerged is described in detail in Appendix F where a precise account of the computational activity together with an overview of the curriculum and textbook prescriptions in relation to mathematics encyclopaedia is provided. The focus of Lesson 1 was on *plotting points* in a Cartesian plane, the end result being a picture of a boat when joining the points in alphabetical order. In Lesson 2, homework was checked and learners were required to plot eight points on a grid – eight learners came up to the board to plot these points and the teacher used the remainder of the lesson to check notebooks. The topic for Lesson 3 was Transformation Geometry, and *translations* were covered. The textbooks in use were *Oxford Successful Maths* and *Preparing for High School Maths*. Consider the evaluative events spanning Lessons 1, 2 and 3 in the tables that follow.

6.2 A summary of the evaluative events for Lessons 1, 2 and 3

Table 6.1: Segmentation of Lesson 1 into evaluative events

Evaluative Event	Type	Length
01 Plotting co-ordinates [00:00 – 18:26]	Expository	18:26
02 Plotting co-ordinates (Homework) [18:28 – 21:52]	Exercise	03:24
		21:50

Table 6.2: Segmentation of Lesson 2 into evaluative events

Evaluative Event	Type	Length
01 Plotting and joining co-ordinates [00:00 – 21:21]	Exercise	21:21
The entire lesson is spent on plotting points and drawing picture.		21:21

Table 6.3: Segmentation of Lesson 3 into evaluative events

Evaluative Event	Type	Length
01 Transformation Geometry - translations [00:00 – 08:49]	Expository	08:49
02 Translating an object three units right [08:49 – 19:12]	Expository	10:23
		19:12

6.3 Summary of the constitution of mathematics at School P2

The main topics under consideration across the three lessons, as presented later in Table 6.4, include plotting co-ordinates and translations as a sub-topic of Transformation Geometry.

These topics were presented by providing students with the necessary criteria for performing computational procedures. Both *computational resources* for plotting a co-ordinate pair as well as *general descriptions of mathematical concepts* i.e. transformations and translations., as described in Appendix F, served as the main vehicles for explaining the content. Table 6.4 also revealed inconsistencies between the announced topics and the realised content for both topics. What follows is a discussion of the constitution of mathematics at School P2.

There were various attempts at providing ‘definitions’ in these three lessons but they remained general and framed in terms of procedures rather than formal mathematical properties. The criteria that students picked up in this pedagogic context remain framed in terms of what had to be done and what the solution should look like rather than an understanding of the existential properties of the object.

- **Plotting co-ordinates**

The procedure for plotting points by firstly selecting x and then y to represent the co-ordinate pair is emphasised by both the teacher and the textbook. The criteria transmitted in the curriculum, texts for teaching and pedagogy all focused on producing an outcome – hence the emphasis on the computational resource as a means of regulation. The final picture i.e. the boat in L1EE1 and the shape in L2EE1 was another marker indicating that the required solution had been achieved. The procedure and the pictorial representations were possible substitutes for the formal definition of a co-ordinate pair and presented instances where students needed to rely far more on generalising from specific examples to arrive at generalised universal ideas that would previously have been made explicit with definitions in traditional curricula.

- **Translations**

The under-specification of the formal definition of translation and its replacement with a very general description of this concept by both the teacher (see Figure 6.1) and the textbook, presented an opportunity for teachers to refer to mathematical objects as if they were normal objects i.e. natural kinds²⁴.

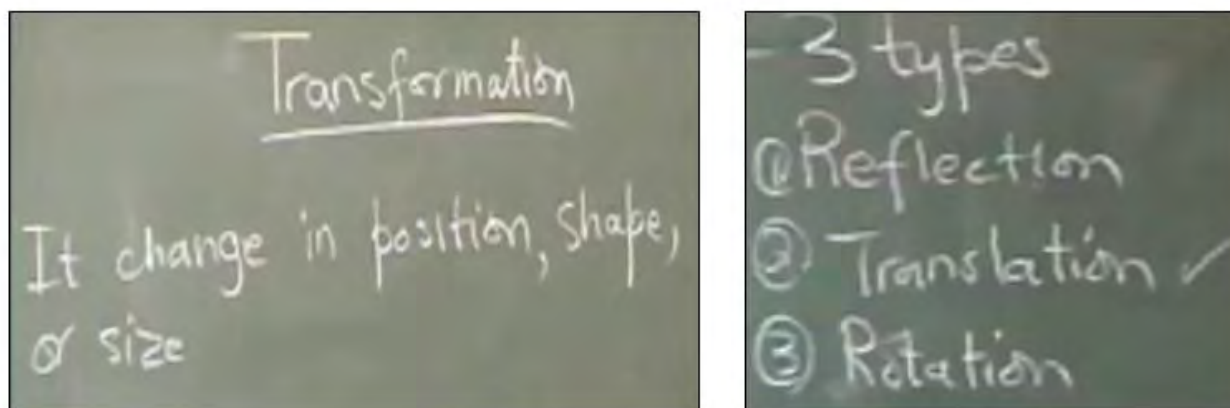


Figure 6.1: A ‘definition’ of transformation (S P2 L3 EE1)

²⁴ The distinction between natural kinds and definite descriptions is discussed in Chapter Four (4.2.5).

As mentioned in Chapter Four, a general description of a mathematical object is very different to a formal definition of a mathematical object, the latter being backgrounded to a large extent in the GET phase of schooling. The curriculum and textbooks appear to have blurred the distinction between descriptions of empirically encountered objects (natural kinds) and mathematical objects (whose existence depend on a set of predicates). The definition of a concept in mathematics has a distinctive logical status: it is the fundamental fact about the object from which all other facts about it must, ultimately, be deduced.

I now consider how, in the absence of formal mathematical propositions in teaching, textbooks and the curriculum, teachers are compensating for this absence.

6.4 Regulation of the computational activity

Consider the summary of the procedures used, the content realised through the procedures, the means of regulation as well as the content substituted by the procedures for each of the announced topics in Table 6.4.

Table 6.4: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P2

Announced topic	Procedures used	Means of regulation	Realised content	Substitutes for
Plotting co-ordinates	Selection of first number – x Selection of second number – y Count x units from the origin as indicated by the value of x in the co-ordinate pair $(a;b)$. Count y units from the origin as indicated by the y -value of the co-ordinate pair $(a;b)$. Trace the lines from $x = a$ and $y = b$. The point $(a; b)$ is where the two lines meet.	Computational resource A reliance on what solution may look like	Order of plotting points Pictorial representation	Mathematical definition of a co-ordinate
Translations	Slide object up or down or left or right by a certain number of units without changing the orientation of the object	General description of mathematical concepts i.e. linguistic description Computational procedure for translating objects	Sliding a figure to a new position without turning it Counting	Mathematical definition of a translation

The under-specification of formal mathematical propositions in the curriculum, texts and pedagogy at School P2 leaves students with little option but to generalise about mathematical objects, rather than training them in the first instance to work with mathematical objects. Names enter into a pedagogic context as a signifier (expression, sound, word) and the object of study here focuses on how these names get correlated with content (meaning) (Saussure, 1922: 98). In effect, pedagogy instructs the student to regulate mathematical signifiers by means of an activity to produce particular mathematical objects aligned with that particular name. When students are regulated in this type of indirect manner it prompts responses that are framed in terms of what needs to be done rather than existential questions regarding the nature of the object. Understanding the object in terms of its existential properties is not emphasised since the content associated with names in the mathematics encyclopaedia are mostly generalisations from activities and the associated computational resources i.e. choosing the appropriate procedure for plotting points.

I now present an account of the constitution of mathematics for School P3 in Chapter Seven.

Chapter 7

A discussion of a sequence of Grade 8 lessons at School P3

School P3 is an ex-DET school located in an 'African' township on the Cape Flats and the student population of the school consists of children from families who reside in the area that could be classified as a predominantly lower working class area. The classroom was not well-resourced and was only equipped with the bare necessities. The layout of the classroom allowed students to interact with each other since they sat in groups. However, teaching proceeded in a manner where students only responded to questions posed by the teacher. The teacher at School P3 was rather authoritarian, threatening to beat and kick students if they failed to complete homework. Any student who had not done homework had to stand for the entire lesson.

IsiXhosa and English were used interchangeably by both the teacher and the students during lessons; however, board work was done in English. Of the three lessons observed, only Lessons 2 and 3 were consecutive and focused on the same topic.

The purpose of this chapter is to summarise what is constituted as mathematics for this teacher as well as illustrate what stands in place of the definitions and axioms found in the mathematics encyclopaedia.

7.1 A synopsis of the lessons for School P3

The main topic areas covered during these three lessons were *types of numbers* and *addition of fractions*. In Lesson 1, *prime numbers*, *composite numbers*, *factors*, *highest common factor* and *lowest common factor* were presented. Both Lesson 2 and 3 focused on *addition of fractions* with the same and different denominators and as already mentioned, Lesson 2 and Lesson 3 were consecutive. The textbook in use at School P3 was *Preparing for High School Maths* from which students completed a range of exercises. The series of evaluative events displayed in Tables G.1, G.2 and G.3 were used to generate data for how it is that criteria emerged and subsequently to mark out the objects that get constituted against a background of curriculum reform that has veered away from the use of explicit definition of mathematical objects and processes. Appendix G focused on the production and analysis of data for this chapter from the records of lessons for School P3. In the Appendix, each evaluative event was described and analysed in terms of the computational activity of the teacher by paying close attention to the operations at play as well as the collections of objects over which the operations ranged for each procedure. Consider the evaluative events for School P3 in Table 7.1, 7.2 and 7.3.

7.2 A summary of the evaluative events for Lessons 1, 2 and 3

Table 7.1: Segmentation of Lesson 1 into evaluative events

Evaluative Event	Type	Length
01 Definition of a prime number [00:00 – 03:46]	Expository	03:46
02 Composite numbers, factors and prime numbers [03:46 – 08:02]	Expository	04:16
03 Highest common factor and lowest common factor [08:03 – 20:20]	Expository	12:16
		20:18

Table 7.2: Segmentation of Lesson 2 into evaluative events

Evaluative Event	Type	Length
01 Types of fractions – equivalent, proper fractions and improper fractions [00:00 – 04:04]	Expository	04:04
02 Addition of fractions [04:04 – 24:19]	Expository	20:25
03 Algorithm for addition of fractions [25:58 – 30:03]	Expository	05:05
		29:34

Table 7.3: Segmentation of Lesson 3 into evaluative events

Evaluative Event	Type	Length
01 Addition of fractions with same and different denominators [00:00 – 41:41]	Exercise	41:41
Entire lesson is spent checking homework on the board		41:41

I now present an account of the constitution of mathematics for the teacher at School P3.

7.3. Summary of the constitution of mathematics at School P3

The key topic areas dealt with in Lesson 1 included a discussion on types of numbers, i.e. primes, composite numbers, factors, and the highest common factor. Lesson 2 and Lesson 3, which were consecutive, focused on recapping types of fractions and the greater part of these two lessons, was spent on applying the teacher's algorithm for addition of fractions with equal and unequal denominators. These topics were presented mainly by using *general descriptions of mathematical concepts* as well as emphasising the use of the algorithm for adding fractions as a *computational resource*.

- **Prime numbers, composite numbers, factors**

Initially, in Lesson 1, there was an attempt by the teacher to provide definitions of the announced topics, namely prime numbers and composite numbers, to students. Consider the following transcript extract presenting prime numbers and composite numbers:

Learner:	A prime number have two factors, a composite has more than two factors.
Teacher:	That's the difference between those two, a prime number got only two factors and a composite number gets more than two factors. (S P3 L1EE2 lines 76 – 78)

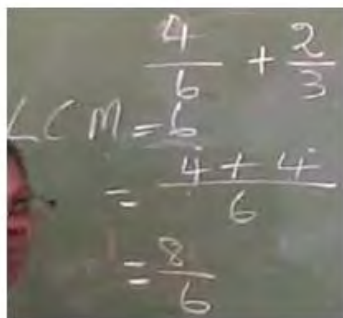
For every definition supplied, an example always followed. However, these 'definitions' were *general descriptions of mathematical concepts* and were substitutes for the formal definitions of prime numbers, composite numbers and factors found in the mathematics encyclopaedia.

- **Highest common factor**

When finding the HCF of two numbers, 18 and 24, an algorithm was provided for this procedure where students were required to list factors of both numbers, identify the common factor and then classify factors as being the highest or the lowest. Both the teacher and the textbook provided a procedure for calculating the HCF which served as a *computational resource* that stood in place of the formal definition of the highest common factor.

- **Addition of fractions**

In Lessons 2 and 3 the algorithm for adding fractions with unequal denominators avoided an encounter with rational numbers. Consider Figure 7.1 which presents the algorithm for adding $\frac{4}{6} + \frac{2}{3}$:



The image shows a chalkboard with the following handwritten work for adding $\frac{4}{6} + \frac{2}{3}$:

$$\begin{array}{r} \frac{4}{6} + \frac{2}{3} \\ \text{LCM} = 6 \\ \hline \frac{4 + 4}{6} \\ \hline = \frac{8}{6} \end{array}$$

Figure 7.1: The algorithm for adding $\frac{4}{6} + \frac{2}{3}$:

Rational numbers, i.e. the objects entering the pedagogic space ($\frac{4}{6}$ and $\frac{2}{3}$) were not recognised as such; rather they were perceived as whole numbers²⁵ which were operated on. Consider the procedure for adding fractions with unequal denominators:

1. Find the LCM of the denominators of the fractions.
2. Divide each denominator of fraction into the lowest common denominator.
3. The quotient of each calculation is then multiplied by the numerator of the fraction.
4. These two answers are added and constitute the numerator of the answer.
5. The denominator of the answer is the lowest common multiple of both fractions.
6. Simplify the final answer fully.

Students, in their presentation of board work copied the form and layout provided by the teacher for achieving a solution so there was also a *reliance on what the solution looked like*. The topic, addition of fractions, no longer regulated the outcome.

7.4. Regulation of the computational activity

After describing the computational activity for each evaluative event for each of the three lessons, the data produced provided clarity both in identifying what was constituted as mathematics as well as what content stood in place of formal definitions and propositions. Appendix G provided an in depth account of the computational activity for School P3 as well as a discussion of how various topics were regulated by providing a summary of the procedures used, the content realised through the procedures, the means of regulation as well as the content substituted by the procedures for each of the key topic areas. Now consider Table 7.4 which describes how the realised content is regulated and it provides an account of what substitutes for the explicit use of formal definitions and propositions.

²⁵ See the discussion describing the computational activity for Lesson 2 (Evaluative Event 2) in Appendix G.

Table 7.4: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P3

Announced topic	Procedures used	Regulation of computational activity	Realised content	Substitutes for
Prime numbers	Textbook definition and example * i.e. 'Prime numbers have only two factors, themselves and 1.' (Bull & Hepworth, 2008: 23)	General description of a mathematical concept	Prime numbers	Formal definition of a prime number Smallest proper divisor is prime
Composite numbers	Textbook definition and example * i.e. 'Composite numbers are the whole numbers which have more than two factors.' (Bull & Hepworth, 2008: 24)	General description of a mathematical concept	Composite numbers	Formal definition of a composite number
Factors	Textbook definition and example * i.e. 'A factor is a number which divides into another number exactly with no remainder.' (Bull & Hepworth, 2008: 22)	General description of a mathematical concept	Factors	Formal definition of a factor
Highest common factor	List factors Identify common factor Select the highest common factor	Computational resource	Multiplication of whole numbers	Definition of HCF Relations between factor, multiple, divisor
Equivalent fractions, mixed numbers and improper fractions	Multiplication, addition	Computational resource	Multiplication and addition of whole numbers	Definition of equivalent fractions, mixed numbers and improper fractions

* The textbook definition and example is not a procedure as specified by the column title. As discussed earlier in this chapter, the teacher is providing a 'definition'. These definitions, however, are recontextualised versions of the encyclopaedic definitions.

Table 7.4 continued: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P3

Announced topic	Procedures used	Regulation of computational activity	Realised content	Substitutes for
Addition of fractions with the same denominator	<p>If the denominators of the fractions are the same, add the numerators of the fractions and retain the denominator.</p> <p>The sum of the numerators constitutes the numerator of the final answer.</p> <p>The original denominator remains the denominator of the final answer.</p>	<p>Computational resource</p> <p>Reliance on what solution looks like</p>	Multiplication, division and addition of whole numbers	<p>Fractions may be represented as ordered pairs of integers $(a; b)$, $b \neq 0$, for which an equivalence relation has been specified (an equality relation of fractions), namely, it is considered that $(a, b) = (c, d)$ if $ad = bc$. The operations of addition, subtractions, multiplications and division are defined in this set of fractions by the following rules:</p> $(a, b) \pm (c, d) = (ad \pm bc, bd),$ $(a, b) \cdot (c, d) = (ac, bd)$ $(a, b) : (c, d) = (ad, bc)$ <p>(thus, division is defined only if $c \neq 0$).</p> <p>Operatory properties over rational number addition</p>

Table 7.4 continued: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P3

Announced topic	Procedures used	Regulation of computational activity	Realised content	Substitutes for
Addition of fractions with different denominators	<ol style="list-style-type: none"> Find the lowest common multiple of the denominators of the fractions. Divide each denominator of fraction into the lowest common denominator. The quotient of each calculation is then multiplied by the numerator of the fraction. The two answers obtained in Step 3 are added and constitute the numerator of the answer. The denominator of the answer is the lowest common multiple of both fractions. Simplify the final answer fully. 	Computational resource Reliance on what solution looks like	Multiplication, division and addition of whole numbers	See above

Furthermore, Table 7.4 provides an overview of how procedures or computational resources are often utilised for each of the named topics and that the realised content rarely reflects the announced topic. For addition of fractions, it seems that the *computational procedure* replaces the operator properties associated with rational number addition. The set Q of rational numbers is closed under addition, subtraction and multiplication.

General descriptions of mathematical concepts are another dominant means of regulation at School P3 and these are simply recontextualised versions of the definitions found in the encyclopaedia. They are different to formal mathematical definitions, propositions and axioms.

I now present an account of the constitution of mathematics at School P6 in Chapter Eight.

Chapter 8

A discussion of a sequence of Grade 8 lessons at School P6

School P6 is an ex-DET school situated in a ‘white’ area in the central metropole of Cape Town. The particular Grade 8 class in this study consisted of approximately 45 students of mixed gender. The three consecutive lessons that were observed and video-recorded took place in a poorly resourced classroom which formed part of a double room cordoned off by a room divider that did not close properly. As a result, the lesson in progress in the adjacent venue was just as audible as the lesson in progress in the Grade 8 mathematics classroom. Desks were spaced quite closely leaving little room for movement between desks.

The language of instruction was both English and isiXhosa and the teacher often interchanged languages during the course of the lesson. Although the teacher presented all three lessons in a traditional ‘chalk and talk’ manner, students responded with ease either by calling out answers or raising their hands. Students mostly responded in isiXhosa and were also given opportunities to do exercises in their notebooks to evaluate their progress.

The intention of this chapter is to provide a summary of what is constituted as mathematics for this teacher at School P6 as well as clarify what stands in place of the definitions and axioms present in the encyclopaedia. What follows is a brief account of the school site and an outline of the three lessons followed by summary of the central issues of concern to my research problem.

8.1 A synopsis of the lessons for School P6

Three consecutive lessons were analysed by chunking them into evaluative events as displayed in Table 8.1, 8.2 and 8.3. The evaluative events provided a clear description of the content segments covered in each of the three lessons and they also offered an overview of the nature of the pedagogy i.e. expository, evaluative or an exercise. In order to further describe the constitution of mathematics at School P6, each evaluative event was defined and analysed using the procedure laid out in Chapter Four. Appendix H provides a detailed account of the computational activity for each evaluative event by marking out the particular operations in play and the collections of objects over which these operations ranged in each procedure in order to establish what is constituted as the topic contents. Furthermore, Appendix H offered selections from curriculum documents, textbooks, other curriculum resources and the mathematics encyclopaedia related to the announced topic. Finally, each evaluative event was examined in terms of how it was regulated using the schema of analysis developed in Chapter Four as a means for re-describing the computational activity.

The broad topic areas examined in and across these three lessons focused on *operations on integers* i.e. addition, subtraction (not dealt with explicitly), multiplication and division. The *rules* for performing integer arithmetic

were emphasised and presented as a means of introduction to all three lessons; these were the criteria that provided a basis for successfully computing problems involving integers.

8.2 A summary of the evaluative events for Lessons 1, 2 and 3

Tables 8.1 to 8.3 summarises my initial analysis of the lessons where I constructed the evaluative events for each lesson using the procedure laid out in Chapter Four:

Table 8.1: Segmentation of Lesson 1 into evaluative events

Evaluative Event	Type	Length
01 Recap of rules for addition and subtraction of integers [00:00 – 03:46]	Expository	03:46
02 Operations (addition and subtraction) on integers [03:46 – 21:03]	Exercise	12:17
03 Recap of rules for multiplication and division of integers [21:03– 40:36]	Expository	19:53
		40:36

Table 8.2: Segmentation of Lesson 2 into evaluative events

Evaluative Event	Type	Length
01 Recap of the rules for multiplying and dividing integers [00:00 – 02:12]	Expository	02:12
02 Integer arithmetic (all four operations) [02:49 – 36:09]	Exercise	04:04
		36:09

Table 8.3: Segmentation of Lesson 3 into evaluative events

Evaluative Event (Period 1)	Type	Length
01 Recap of the rules for multiplying and dividing integers [00:00 – 08:13]	Expository	08:13
02 Order of operations [08:30 – 45:38]	Expository	01:24
		36:52
Evaluative Event (Period 2)	Type	Length
03 Description of the commutative property and exercise[45:38 – 47:20]	Expository	01:42
04 Description of distributive property [47:20 – 51:10]	Expository	03:50
05 Distributive rule over division [51:10 – 59:15]	Expository	08:05
06 A description of the utility of the associative, commutative and distributive properties [59:15 – 66:45]	Expository	07:30
		21:07

What follows is a discussion on the constitution of mathematics at School P6.

8.3 Summary of the constitution of mathematics in School P6

As already mentioned, the main topic under consideration across the three lessons was *integer arithmetic*. A range of rules were presented for dealing with all four operations, including the computational resource of BODMAS. The *commutative property* as well as the *distributive property* was also presented as a tool for dealing with integer arithmetic more efficiently. I will now reflect on how the absence of the propositions identified (later in Table 8.4) is accounted for by examining how teachers are compensating for their absence.

- **Integer addition when signs are the same and when signs are different**

An analysis of the computational activity discussed in Appendix H shows that the announced topic at the commencement of Lesson 1 is integers. However, a procedure which entails altering the nature of the integers by separating integers into signs and whole numbers was presented by the teacher. For integer addition, the integers with the same sign were added together and the common sign was re-attached to the final whole number answer. Consider the transcript extract that follows

Teacher: [...] the rules of adding and subtracting integers. So let's quickly reflect what are those rules. When adding one take the rule, when adding integers of the same signs what do you do, kanene? {again} Yes, bhuti {boy}, add like normal and put the sign over. Huh? Same sign, you add them you put the

common sign for example... let's start with the normal four plus five. (See Figure H.1) So you add them. Which is?

Learners: Nine.

Teacher: Positive?

Learners: Nine.

Teacher: Minus five and minus four? (See Figure H.1) Yes, bhuti {boy}? Negative nine. Remember we add the common sign which is negative, [...] (S P6 L1 EE1 lines 1-8)

When the signs of the integers were not the same, a different rule applied. Consider the following rule presented by the teacher:

Teacher: [...] and then for different signs ... and then izandla ziphakanyisiwe {put up your hands}, yes? You subtract the smaller digit from the bigger digit, okay? We subtract the smaller digit from the bigger digit and put the sign of the bigger digit. (See Figure 8.2) [...] (S P6 L1 EE1 line 8-10)

For this particular procedure, integer addition was once again the announced topic. Integers were separated into whole numbers and signs. The rule presented by the teacher instructed students to subtract the smaller whole number from the bigger whole number and re-attach the sign of the bigger whole number to the final whole number answer. The selection of the correct series of computations was reliant on the signs of the integers to be added or subtracted. The *computational procedure* therefore replaced the operational properties associated with integer addition and subtraction. Addition over the integers has some very convenient operatory properties, namely, associativity, commutativity, closure, identity and nullity which were not referenced during the explication of this topic.

- **Multiplication and division of integers**

Multiplication and division of integers was announced, but as with addition and subtraction of integers, the rules remained a central means of regulation for producing the required answers. Consider the following transcript extract which described how this topic was dealt with:

Teacher: [...] let's talk about imultiplication {multiplication} as well as division because the rules are more or less the same. Multiplication and division of integers. [...] (S P6 L1 EE3 lines 130-131)

Teacher: So ngamanye amazwi {in other words} it's a rule, ithi irule yethu {our rule says}, when you multiply a negative multiplied by negative we get a positive. When you multiply a positive by a positive you get a?

Learners: Negative.

Learners: Positive. (S P6 L1 EE3 lines 149-151)

Teacher: So when signs are the same, that is very important. Negative negative or positive positive, you get a?

Learners: Positive answer.

Teacher: But when the signs are different when you are multiplying you get a?

Learners: Negative. (S P6 L1 EE3 lines 204-207)

The topic, multiplication and division of integers and their corresponding operatory properties did not necessarily regulate finding the solutions to any of the problems presented in the respective lessons. The emphasis was rather on selecting the appropriate rule relating to integer multiplication and integer division for

generating the correct answer. Students were constantly reminded of these rules. Consider a recap of the rules as presented by the teacher in Figure 8.1:

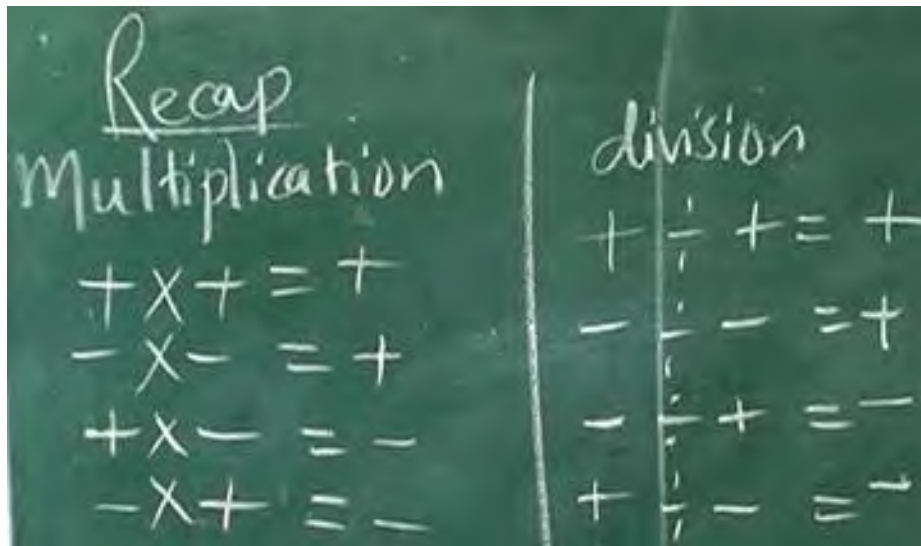


Figure 8.1: The rules for multiplication and division of integers (S P6 L2 EE2)

- **Order of operations, commutativity and distributivity**

Students were introduced to use yet another computational resource, namely, BODMAS to perform integer arithmetic. In addition to presenting BODMAS, the teacher also summarised three main rules for processing more than one operation in a problem. Appendix H provides a detailed account of the computational activity involved in presenting BODMAS in Lesson 3. The criteria that students picked up from these rules related to counting the number of operations in a given problem, identifying these operations and then applying the correct procedure in terms of which operation needed to be performed first.

The commutative property as well as the distributive property were also dealt with extensively in Lesson 3 as an extension of integer arithmetic. For both properties, the focus appeared to be on performing the correct operations on the integers, rather than recognising their value at the operational level. Only specific examples were presented for how one applied both the commutative and distributive properties rather than students being presented with the general form of the property i.e. $a + b = b + a$ or $a \cdot b = b \cdot a$ and $a \cdot (b + c) = a \cdot b + a \cdot c$. Like BODMAS, both properties were presented as computational procedures given that the teacher expressed the following: '[...] you can simplify calculations by using the commutative, associative and distributive properties of operation' (S P6 L3 EE2 lines 442).

Integer arithmetic at School P6 required an elaborate set of context dependent rules that students had to remember and apply depending on the respective operation. The manner in which students approached a problem was always reliant on the rules that needed to be followed in order to successfully produce a solution

rather than focusing on the existential nature of the announced object. In this instance, the convenient operatory properties pertaining to integer arithmetic were not referenced at all. The sequence of procedures and rules were prioritised and the computational activity shows that the objects that were operated on in this context were not integers as announced by the teacher but whole numbers. Integers were transformed into whole numbers by disassociating signs from numbers and applying a series of context specific rules where one operated on the signs and on the whole numbers separately. On closer analysis of the series of evaluative events across the three lessons, it appeared as though integers were described and treated as if they were *natural kinds* (objects encountered empirically), rather than presenting them in terms of their formal definition and associated operatory properties i.e. its *definite description*. Sundering and concatenation of integers as described by Jaffer (2009)²⁶ altered the fundamental features of what is clearly a mathematical object.

Presenting integers as though they are natural kinds renders their definition incomplete and students are encumbered with a range of different rules to use in specific instances. What follows is an overview of how the above-mentioned topics are regulated.

8.4 Regulation of the computational activity

The purpose of Table 8.4 is to demonstrate how for each of the named topics a suitable computational resource was always made use of and that the announced topic often differed from the realised content. This table is also helpful in identifying how the realised content is regulated and summarises what stands in place of the explicit use of definitions and propositions. The central means of regulation for all of the announced topics relied on the use of an appropriate computational resource. Consider this synopsis in Table 8.4:

²⁶ Davis (2010) is the source of the concept of sundering and concatenation of integers. ‘*Sundering* is understood in general as a pseudo-operation that separates a mathematical expression into a series of two or more expressions. The component expressions into which an expression is sundered are not unique and the production of the component expressions is based on the decision of the individual performing the sundering’ [...] *Concatenation* is understood in general as a pseudo-operation which links two or more expressions into a composite expression. The order of the concatenated elements is decided by the individual performing the concatenation (Jaffer: 2009: 50) in Davis (2010a). (italics in original)

Table 8.4: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P6

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Integer addition when signs are the same	Add whole numbers and attach common sign to answer	Computational resource	Whole number arithmetic	Operatory properties of integer addition
	Poker chip model involving pairing and cancelling	Computational resource	Counting	Additive inverse of integers $(\forall x \in \mathbb{Z}, \exists -x \in \mathbb{Z} x + (-x) = 0)$
Integer addition when signs are different	Subtract whole numbers and attach sign of the bigger whole number to answer	Computational resource	Whole number arithmetic	Operatory properties of integer addition
Integer multiplication	Separate signs and numbers and multiply separately	Computational resource	Whole number arithmetic	Operatory properties of integer multiplication
Integer division	Separate signs and numbers and divide separately	Computational resource	Whole number arithmetic	Operatory properties of integer division
BODMAS	Proceed in this order: Brackets, Of (\times), Division, Multiplication, Addition, Subtraction	Computational resource	Whole number arithmetic	Operatory properties of integer arithmetic
Commutative property	Addition Multiplication	Computational resource	Commutative property	$a + b = b + a$ $a \cdot b = b \cdot a$
Distributive property	Multiplication Addition Division Subtraction	Computational resource	Distributive property	$a \cdot (b + c) = a \cdot b + a \cdot c$

An analysis of Table 8.4 reveals a discrepancy between the announced topics and the realised content for all but two of the named topics. For both the commutative and the distributive properties, $a + b = b + a$, $a \cdot b = b \cdot a$ and $a \cdot (b + c) = a \cdot b + a \cdot c$ is exactly what the teacher is referring to as well. So this is not really a case of substitution but merely a symbolic representation of the commutative property of addition. In the absence of teaching students the fundamental axioms and propositions regarding integers, schooling has to make up for propositions in an alternative manner by presenting students with a collection of procedures and rules for producing the same result.

I now present an overview of the constitution of mathematics for the final school, School P7 in the research archive, in the next chapter.

University of Cape Town

Chapter 9

A discussion of a sequence of Grade 8 lessons at School P7

School P7 is a well-equipped ex-HoR school located in a former 'coloured' area on the Cape Flats and comprised of a learner population that was almost exclusively 'coloured'. The classroom was in a general state of disrepair with a broken wall and graffiti on the walls and desks. The classroom organisation and layout was conventional, with the teacher being the primary source of knowledge and where the students learnt by listening to the teacher and observing what he did. The teacher spoke for most of the lesson and checked homework problems on the board. Students were often instructed by the teacher to stop writing down notes and pay attention to specific instructions for doing a problem. The teaching style was such that the teacher was the primary source of knowledge and individual students were only allowed to answer questions posed by the teacher by raising their hands or the entire class chorused answers.

As in the previous four chapters, this chapter will present what comes to be constituted as mathematics content for the curriculum topics in the pedagogic situations in one classroom at School P7. Appendix I provides an extensive account of the computational activity of the teacher and his students for the series of three lessons. The computational activity was described in terms of what operations were employed in tasks and the domains and codomains of objects entailed in the operations. This chapter will focus on providing a synopsis of what is constituted as mathematics for the teacher at School P7 in order to investigate what substitutes for the axioms and definitions found in the encyclopaedia. What follows is an overview of the topics covered during these three lessons followed by the main issue of concern in my research problem.

9.1 A synopsis of the lessons for School P7

Three consecutive lessons were analysed by chunking them into evaluative events as displayed in Table 9.1, 9.2 and 9.3. As described previously, the evaluative events provided a clear description of the content segments covered in each of the three lessons and they were categorised in terms of the nature of the pedagogy. In order to further describe how the objects were constituted in the pedagogic context, broad topic areas were extracted from the series of evaluative events.

Both Lesson 1 and Lesson 2 related to types of numbers, factorisation and exponential expressions. Lesson 3 presented an introduction to bar graphs and their associated features and focused specifically on the layout and spacing of the axes, the bars and the grid for drawing the graph. Lesson 1 commenced with the teacher checking homework on the board relating to factors and generating a method for finding the highest common factor from this list of factors. Lesson 2 focused on checking a worksheet on the Number System and covered squares, cubes, square roots, cube roots, odd numbers, even numbers, composite numbers and prime numbers. This particular lesson could be characterised as one that consolidated knowledge of the Number system, rather than being introductory. Exponential expressions were also discussed when squares and cubes were covered in Lesson 2. The broad topic areas examined in and across these three lessons included *factors*, *types of numbers*, *exponential expressions* and *bar graphs*. Consider the evaluative events for the three consecutive lessons at School P7 in Tables 9.1, 9.2 and 9.3:

9.2 A summary of the evaluative events for Lessons 1, 2 and 3

Tables 9.1 to 9.3 summarises my initial analysis of the lessons where I constructed the evaluative events for each lesson using the procedure laid out in Chapter Four:

Table 9.1: Segmentation of Lesson 1 into evaluative events

Evaluative Event	Type	Length
1.1 Factors [00:00 – 02:10]	Exercise	01:26
1.2 Generating highest common factors [02:19 – 05:44]	Exercise	01:24
02 Calculating cube roots and cubes [05:46 – 13:09]	Expository	02:00
03 Recap: Types of numbers [13:10 – 14:38]	Expository	02:45
04 Exponents [14:41 – 28:44]	Expository	06:33
05 Classification of numbers - primes, odd, composite, squares, cubes [35:50 – 39:15]	Exercise	05:38
		39:15

Table 9.2: Segmentation of Lesson 2 into evaluative events

Evaluative Event	Type	Length
01 Powers [00:00 – 02:32]	Expository	02:32
02 Square roots and cube roots [02:32 – 02:55]	Expository	00:23
03 Finding the first two-digit number, the digits of which has a sum of 12 and finding the first three digit number, the digits of which has a sum of 14 [02:55 – 09:20]	Expository	06:25
04 Recap of the meaning of 5^2 [09:30 – 10:50]	Expository	01:20
05 Listing odd numbers, even numbers greater than 30, prime numbers and composite numbers greater than 30 [11:30 – 14:15]	Exercise	02:25
		38:51

Table 9.3: Segmentation of Lesson 3 into evaluative events

Evaluative Event	Type	Length
01 Drawing bar graphs [00:00 – 31:02]	Expository	31:02
		31:02

I now present an account of the constitution of mathematics for the teacher at School P7.

9.3 Summary of the constitution of mathematics in School P7

For the majority of the topics presented at School P7, the teacher referenced a computational procedure for explicating the topic rather than drawing on formal mathematical definitions. The procedures, as described in Appendix I, served as the main vehicles for producing the content. I now consider how the absence of propositions is accounted for in teaching by examining how teachers are compensating for this absence.

- **Factors**

The notion of a factor is the result of a calculation where the teacher's procedure for generating factors of a list of even numbers relies on listing factors in order from smallest to largest *and* the use of the everyday colloquial expression 'goes into' which is meant to signal divisibility. This process suggests a sequence of natural numbers: $1, 2, 3, \dots, n, \dots$ with individual numbers tested from left to right, until n is reached. The definition of a factor and its related properties do not regulate the procedure as described from the onset; rather the *computational procedure* appears to be the primary object and stands in place of the following proposition: $\forall m \in \{1, 2, \dots, m\}$, if $m|n$, then m is a factor of n . So the concept of a factor is constituted as a procedure for finding factors.

- **Highest common factor**

A list of factors of even numbers generated in ascending order was referenced by the teacher as part of the procedure for determining the highest common factor. Consider the following transcript extract where he explains the procedure for calculating the highest common factor of 30, 48 and 72:

- Teacher: [...] Now you look at all the factors of thirty, forty eight and seventy two. Who can tell me, what's the highest [02:00] common factor of those three numbers? What do you see? Here are thirty factors, here are forty eight factors and there we have seventy two factors. Which is the highest one? Now you need to examine thirty's factors. Let's look at ten. Is ten a factor of forty eight?
- Learners: No.
- Teacher: No, ok. Which one do you think fits in by all three? [chair noise makes word unclear]. Yes my girl?

When indicating to students how one identifies the common factor of 30, 48 and 72 he says that the common factor should 'fit in by all three'. This description is unclear and inadequate and hardly provides students with explicit instructions for how to proceed. As already mentioned, the list of factors of even numbers written on the board served as a computational resource for calculating the highest common factor and substituted for any formal definition of the highest common factor.

- **Cube roots**

The list of perfect cubes from one to nine which the class counted out aloud was used as a resource for finding the cube roots of large numbers. The instruction given by the teacher for finding cube roots of larger numbers

entailed observing the last digit of the number to be cube rooted and estimating its cube root, based on the list of perfect cubes. The implicit nature of this procedure is regulated by the teacher's knowledge of the answer. Consider the transcript extract for finding the cube root of a value that ended with a 7 as well as the method for finding what the answer would end with:

Teacher: [...] ... Ok. The next one, you have to try and get a starting point. What starting point can we use here?
Learner: Seven.
Teacher: Hmm? Wait, it's going to end in a, ... your answer is going to end in a?
Learners: Three

The transcript extract suggests that $\sqrt[3]{27} = 3$ is used as a resource for the above calculation and as a result, finding the cube or cube root of a number necessitates adhering to a computational activity rather than referencing its formal definition found in the mathematics encyclopaedia. However, the description presented in the transcript extract does not capture the implicit aspects of the teacher's procedure.

- **Exponents**

Counting off on fingers and using the calculator to check answers are the central means of regulation for computing problems that involve exponential expressions. As a result, the topic of exponents emerges as the outcome of an activity that usually involved counting rather than the general structure of an exponential expression, as one that comprised of a base and an exponent, being made explicit to students. The teacher also preferred to use specific examples when presenting this topic. Counting off on one's fingers as one multiplies seems to stand in place of the general idea that any number raised to a power n could be calculated by multiplying the number by itself n times.

- **Bar graphs**

The means of regulation in this evaluative event relies on the computational procedure provided by the teacher and on the symbolic illustration of the bar graph, mainly because he says that 'a graph is a picture of a set of numbers' (S P7 L3 EE1 line 7-8). Producing a replica of the teacher's bar graph stands in place of any formal definition of what exactly a bar graph might be or why it is an appropriate representation of the data.

9.4 Regulation of the computational activity

What follows is an overview of how the above-mentioned topics are regulated. Table 9.4 presents a summary of the procedures used, the content realised through the procedures, the means of regulation for each of the announced topics as well as what stands in place of the content substituted for by the procedures:

Table 9.4: Summary of procedures, realised content, substituted content and regulatory mechanisms for School P7

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Factors	Colloquial expression: 'goes into' to signal divisibility'	List of even factors on the board - Computational resource	Division of natural numbers	$\forall m \in \{1, 2, \dots, m\}$, if $m n$, then m is a factor of n .
Highest common factor	Textbook procedure List factors in numerical order, find common factor followed by highest common factor	List of even factors on the board - Computational resource	Division of natural numbers	Definition of highest common factor
Cube roots	Smaller numbers: List of cubes from 1 to 9	List of cubes from 1 to 9 - Computational resource	Counting Estimation	Procedure for finding cube root
	Bigger numbers: List of cubes and observe the last digit of value to be $\sqrt[3]{\quad}$	Using the last digit of the perfect cube and the last digit of its cube root – Computational resource		
Exponents	Counting on fingers to denote the exponent Calculator Specific examples	General description of a mathematical concept	Counting Multiplication	Definition of an exponential expression: any number raised to a power n could be calculated by multiplying the number by itself n times.
Bar graph	Measuring Drawing – vertical and horizontal axis with a scale and title	Procedure - Computational resource A reliance on what solution may look like	Replica of teacher's bar graph	Definition of a bar graph Definition of independent and dependent variable

This table is meant to show how for each of the announced topics an appropriate procedure or computational resource is always employed and that the realised content is often very different to the announced topic. This table also provides an account of how the content, which is meant to be realised, is regulated as well as what substitutes for the explicit use of definitions and propositions

Following this chapter I present two concluding chapters that will provide a summary of all the findings across the five schools in the research archive.

Chapter 10

Summary of presentation of results

This chapter presents an overview of the results of the data production and analysis for the fifteen video-recorded Grade 8 lessons selected for this study discussed in Chapter Five, Six, Seven, Eight and Nine for School P1, P2, P3, P6 and P7 respectively. Using the analytical framework outlined in Chapter Four, each lesson was analysed in terms of the computational activity entailed as well as the means of regulation for each evaluative event.

10.1 Describing the use of time

Lessons across the five schools followed a similar structure where solutions to homework or classwork questions were checked by the teacher by either calling learners to the board to complete the solutions, which the teacher either accepted or re-did with the class. Alternatively, the teacher worked through a selection of mathematical problems during the lesson. Davis and Johnson²⁷ (2007b) observed how lesson time was spent in five working class schools²⁸ by describing it in terms of five categories. In describing my data, I adopt three of their five categories i.e. exposition of mathematical principles, exposition by worked examples and working through exercises and include an additional category, ‘exposition by providing general descriptions of mathematical principles’, which I described in Chapter Four. However, in contrast to Davis and Johnson (2007b), instead of calculating the amount of time spent on each category, I used the categories to describe the nature of the evaluative event, my unit of analysis. The categories I used are described below:

Exposition of mathematical principles – this entails the exposition of the mathematical ideas, principles, propositions and definitions that ground the procedures being rehearsed.

Exposition by worked examples – this involves teachers doing examples which learners have not seen before (i.e. they have not been set as homework or classwork questions).

Working through exercises – learners are involved in working through an exercise, alone or in groups. Often, the teacher walks around the class making commentary on learners’ work or marking their work.

Exposition by general descriptions of mathematical principles – this entails general descriptions of mathematical concepts rather than precise, formal mathematical definitions where there is no explicit engagement with the core axioms.

²⁷ I discuss the context and findings of Davis & Johnson’s (2007b) study in detail in Chapter One. In short, their study noted that the most common way of elaborating procedures in the five schools participating in their study was through carrying out worked examples and a minimal amount of time spent on exposition of ideas, principles and definitions.

²⁸ As outlined in Chapter One, three of these schools participated in my study.

Table 10.1 shows the number of evaluative events categorised in terms of the above categories for the five schools.

Table 10.1: Categorising the evaluative events for the fifteen lessons

School	Lesson	Number of evaluative events	Total evaluative events	Exposition of mathematical principles	Exposition by worked examples	Working through exercises	Exposition by general descriptions
P1	1	6	14	0	5	1	0
	2	5		0	2	3	0
	3	3		0	0	3	0
P2	1	2	5	0	1	1	0
	2	1		0	0	1	0
	3	2		0	1	0	1
P3	1	3	7	0	1	0	2
	2	3		0	3	0	0
	3	1		0	0	1	0
P6	1	3	11	0	2	1	0
	2	2		0	1	1	0
	3	6		0	6	0	0
P7	1	6	12	0	1	3	2
	2	5		0	4	1	0
	3	1		0	1	0	0
		Total number of evaluative events	49	0%	57%	33%	10%

Figure 10.1 displays the categorisation of the evaluative events in the fifteen lessons for the five schools as a whole while Figure 10.2 displays this same feature per school.

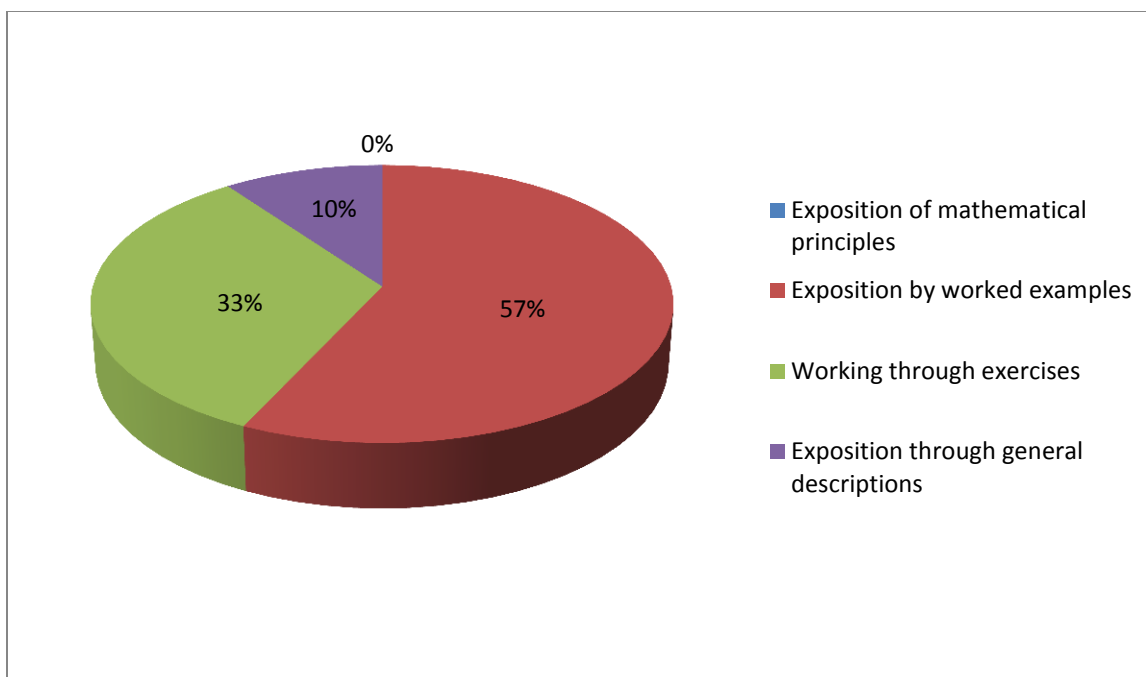


Figure 10.1: Categorising the evaluative events for the research archive

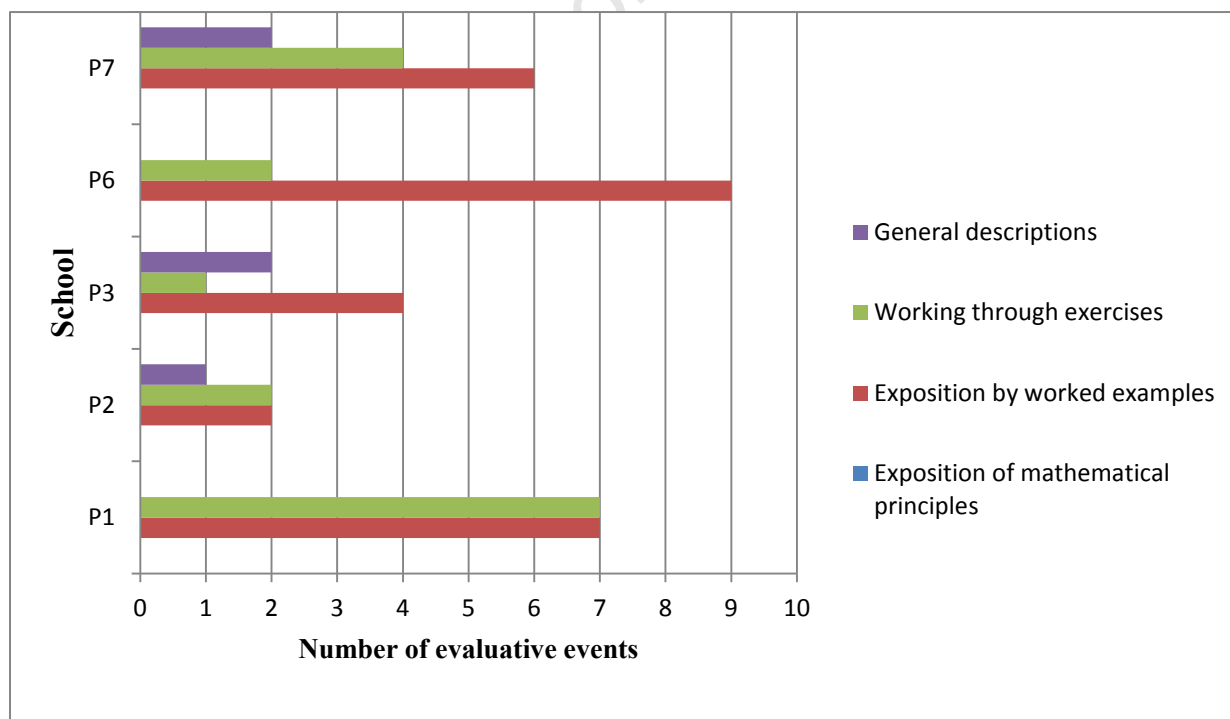


Figure 10.2: Bar graph showing a categorisation of the evaluative events per school

From Table 10.1 we see that for the fifteen lessons under investigation, content is mainly explicated by carrying out procedures, either as exposition of worked examples or by working through exercises. These two categories describe 90% of the evaluative events. This is consistent with Davis & Johnson's (2007b) conclusions from a summary of nineteen lessons across five schools that 'most time is spent on the elaboration of mathematics through worked examples' (Davis & Johnson, *ibid.*: 124). None of the evaluative events across the 15 lessons focused on exposition of mathematical ideas and propositions which suggests that the propositional ground underpinning the realised content is either absent from the evaluative criteria generated in these schools or taken as implicit, which has implications for the realisation of the content. Approximately 10% of the evaluative events could be categorised as general descriptions. This means that in 10% of the evaluative events, rather than providing students with formal definitions of mathematical concepts and the associated operatory properties, very rudimentary descriptions of these concepts were meant to suffice.

The analysis of the computational activity for each of the five schools in this research archive illustrates what the computational content is that is constituted to substitute for the explicit use of definitions and axioms present in the encyclopaedia.

What follows is a summary of what is constituted as mathematics at each of the five schools by examining both the realisation and the regulation of the content. The analysis for School P1, P2, P3, P6 and P7 can be found in Chapters Five, Six, Seven, Eight and Nine respectively followed by an extended analysis of the constitution of mathematics in Appendix E, F, G, H and I for each of the schools.

10.2 Realisation of the content across the five schools

I now present an overview of the announced topics in relation to the realised content for each of the main topic areas at the five schools in Table 10.2 followed by a summary of what is constituted as mathematics for each school:

Table 10.2: Summary of the constitution of mathematics for each school

School	Announced topic	Realised content
P1	Prime factorisation (Factor tree method)	Multiplication of whole numbers
	Prime factorisation (Ladder method)	Division of whole numbers
	Multiples	Multiplication tables
	Lowest common multiple (LCM)	Multiplication tables
	Highest common factor (HCF)	Multiplication of whole numbers
P2	Plotting co-ordinates	Order of plotting points, Counting Pictorial representation
	Translations	Sliding a figure to a new position without turning it Counting
P3	Prime numbers	Classification of whole numbers
	Composite numbers	Classification of whole numbers
	Factors	Classification of whole numbers
	Highest common factor	Multiplication of whole numbers
	Equivalent fractions, mixed numbers and improper fractions	Multiplication and addition of whole numbers Fractions are treated as pairs of whole numbers
	Addition of fractions with the same denominator	Multiplication, division and addition of whole numbers
	Addition of fractions with different denominators	Multiplication, division and addition of whole numbers
P6	Integer addition when signs are the same	Addition of whole numbers
	Integer addition when signs are different	Subtraction of whole numbers Counting using the Poker Chip model
	Integer multiplication	Multiplication of whole numbers
	Integer division	Division of whole numbers
	BODMAS	Arithmetic of whole numbers
	Commutative property	Arithmetic of whole numbers
	Distributive property	Arithmetic of whole numbers
P7	Factors	Division of whole numbers
	Highest common factor	Division of whole numbers
	Cube roots	Counting, Estimation
	Exponential expressions	Counting, Multiplication of whole numbers
	Bar graph	Replica of teacher's bar graph, Counting

At School P1 the four main topic areas identified were prime factorisation, multiples, HCF and LCM during three lessons which were not consecutive. The data for School P1 shows that the announced topics in all of the evaluative events are not aligned with the realised content as shown in the summary above and that the realised content is constituted as multiplication and division of whole numbers.

At School P2, plotting co-ordinates and transformations were the two central announced topics that were dealt with during three consecutive lessons. The content constituted in this school was mainly counting.

At School P3, the seven key topics announced included prime numbers, composite numbers, factors, highest common factor, types of fractions and addition of fractions with the same and different denominators. The topics of primes, composites and factors were constituted by providing students with a textbook definition of these mathematical entities which substituted for formal mathematical definitions and as a result the realised content was whole number classification. Multiplication, division and addition of whole numbers were constituted as the realised content for the remainder of the topics at School P3.

At School P6, integer arithmetic could be categorised into seven sub-topics which included integer addition, integer subtraction, integer multiplication, integer division, BODMAS, the commutative and the distributive property. The content constituted at this school for all the topics was whole number arithmetic.

At School P7 the five topic areas that were announced were factors, the HCF, cube roots, exponential expressions and bar graphs for a series of three consecutive lessons. The content constituted in this school was mainly multiplication and division of whole numbers and counting.

For the 25 announced topics across the five schools, 17 of them were realised as basic arithmetic, 5 were constituted as counting and 3 of the topics were constituted as whole number classification as presented in Figure 10.3. A detailed analysis of how each topic was constituted can be found in Appendix J.

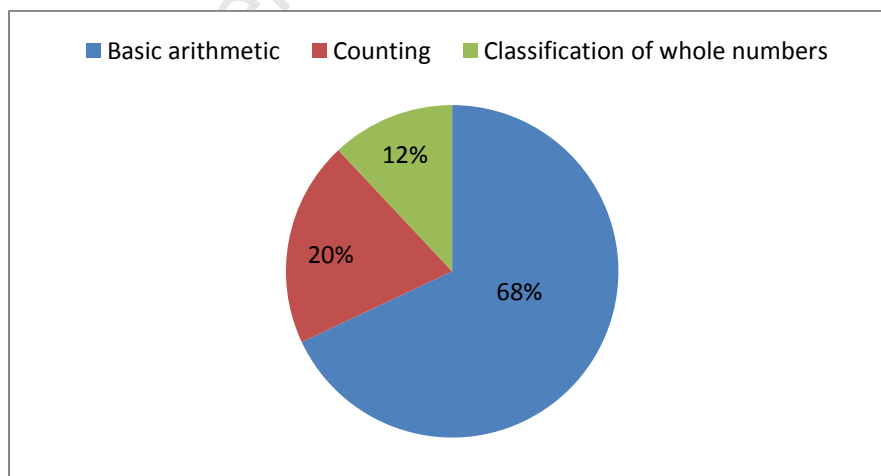


Figure 10.3: Realisation of the content across the five schools

10.3 Regulation of the content across the five schools

Besides providing a description of both the procedures used as well as the content realised through the procedures for each of the main topic areas, I also described how the computational activity was regulated for each of the evaluative events.

The four main categories, developed from the analytic framework in Chapter Four, for classifying how the computational activity for each evaluative event was regulated were: (i) a set of interconnected propositions; (ii) computational resources; (iii) a reliance on what the solution looked like and (iv) general descriptions of mathematical concepts.

What follows in Table 10.3 is an overview of how the content was regulated across the five schools by classifying each evaluative event in terms of the above categories. It should be noted that an evaluative event can be classified in terms of more than one category. Appendix K provides a comprehensive analysis of how each evaluative event was classified.

Table 10.3 shows that 4% of the evaluative events examined were regulated by a set of interconnected propositions, 16% by general descriptions followed by 18% being regulated by a reliance on what the solution looked like. Of the 49 evaluative events, 83% were regulated by computational resources. I now present a summary of the constitution of mathematics across the five schools.

Table 10.3: Summary of the regulation of the content for each of the five schools

School	Number of evaluative events	Interconnected propositions	Computational resources	Reliance on what solution looks like	General Descriptions
P1	14	0	14	3	0
P2	5	0	4	2	1
P3	7	0	5	3	3
P6	11	2	9	0	0
P7	12	0	9	1	4
Total	49	2	41	9	8

10.4 Summary of the constitution of mathematics across the five schools

The data collected from the fifteen lessons under analysis, shows that the content is mainly elaborated through the use of computational resources i.e. 83% of the evaluative events across the five schools. The data show that teachers generally employ computational resources when introducing new topics or when marking homework; the primary purpose being to demonstrate the application of routine procedures. The reliance on computational resources for producing the required solution includes choosing an appropriate procedure, calculation, algorithm or sets of instructions to compute.

The dependence on the use of computational resources suggests an emphasis on the activity of the knower. Furthermore, the data show that only two, i.e. 4%, of the evaluative events across the five schools were regulated by a set of interconnected propositional statements of mathematical relations between mathematical objects.

The description of the constitution of mathematics emerging from the analysis of the lessons across the five schools mirrors the shifts in curricula discussed in Chapter One. Recall that curriculum reforms since 1997 indicated a shift in emphasis from knowledge statements regarding the content of mathematics to the activity of the knower of school mathematics and de-emphasised the explicit use of propositions, deductive argumentation and definitions.

Table 10.4 presents a general overview of the main topic areas at the five schools as well as the realised content for each of these topic areas. Table 10.4 shows that the content realised in the evaluative events differed from the mathematics encyclopaedic content associated with the announced topic.

Table 10.4: Summary of the constitution of mathematics across the five schools

Announced topic	Realised content
Types of numbers	Classification of whole numbers
Fractions	Arithmetic of whole numbers
Integers	Arithmetic of whole numbers
Transformation geometry	Counting
Exponents	Counting
Bar graph	Counting and drawing

In the general context of a curriculum that to large extent de-emphasised the use of explicit definitions and

propositions, this study set out to examine the content that filled that space. Table 10.3 shows a de-emphasis of interconnected mathematical propositions. Table 10.4 displays a summary of the content that is substituted for the main topic areas for the fifteen lessons in my study as primarily being whole number arithmetic, classification of whole numbers and counting. As described in the analysis, the realisation of the content as whole number arithmetic is particularly evident in the procedure for adding fractions with different denominators as well as for integer arithmetic, where both the teachers and the students employ computational procedures that suspend knowledge of rational number arithmetic as well as integer arithmetic respectively. For fraction addition the computational procedure employed involves a change in the domain being operated over from rational numbers to whole numbers. A similar phenomenon of domain shifting was observed for integer arithmetic which entailed an existential shift from the domain of integers to the whole numbers.

A significant finding in my study shows that the content constituted for the announced topics is primarily basic arithmetic and counting. The data show that at Grade 8 level, students are taught contents that are announced as Grade 8 topics but are in fact realised as Intermediate Phase or even the Foundation Phase content. The level of content made available at these schools perhaps accounts for the poor performance of schools populated by learners from working class backgrounds. Thus, my study potentially offers an alternate explanation to the literature discussed in Chapter Two for the poor performance of schools populated by working class students. It must be re-iterated that this study merely provides an account of findings in these five schools and is by no means positing any causal claims regarding the curriculum, pedagogy and learner performance. At this juncture, it is not clear what picture would emerge in other social class contexts as I focused on working class schools only. This would be part of a much larger study where this work could be extended even further. What follows is a concluding chapter where I reflect on the key outcomes of my analysis in the context of the Revised National Curriculum Statement.

Chapter 11

Conclusion

This study set out to investigate the constitution of mathematics in pedagogic situations of schooling, specifically in the teaching and learning of mathematics in a selection of fifteen Grade 8 mathematics lessons at five schools in the Western Cape Province of South Africa. This study is situated within the broad framework of the sociology of education, specifically drawing on Bernstein's (1996) theory of pedagogic discourse and the pedagogic device.

The specific problematic that my study engages with is the constitution of school mathematics in the pedagogic situation of schooling in the GET phase of the context of the RNCS where we find that the use of explicit definitions of mathematical objects as well as the explicit study of mathematical propositions has been de-emphasised. In the context of the implied changes to the mathematics curriculum, my focus is on the way in which the content of the evaluative rule of the pedagogic device is realised in this particular selection of schools.

My theoretical framework draws on the work of Bernstein (1996) and Davis (2010a, 2010b, 2010c, 2011a, 2011b, 2011c, 2012). These theoretical resources were drawn on to describe and analyse the mathematical activity in the five schools as well as serving as a means for generating analytical resources for describing the constitution of mathematics.

In my analysis I presented a description of the computational activity in fifteen lessons by examining the operations employed and the collections of objects operated over in each procedure for each of evaluative events identified. I used these descriptions of the computational activity to discuss how the mathematical content was constituted in relation to selections associated with the announced topic from curriculum documents, textbooks, other curriculum resources as well as the mathematics encyclopaedia.

In this concluding chapter I summarise the key findings of my analysis with reference to the theoretical and analytic framework, followed by a discussion of the limitations and potential of this study.

I revisit the pedagogic device which served as the overarching structural framework for this research.

11.1 Reflecting on the pedagogic device

Bernstein's pedagogic device provided a means for describing the pedagogic discourse from the macro-level of policy implementation in the curriculum to the micro-level pedagogy in the classroom. This research focused on the realisation of curriculum at the level of pedagogy in a general milieu where there is a distinct de-emphasis of

formal definition of mathematical objects and processes in favour of more empirical means of justification. As a result, the notion of evaluation and the associated evaluative criteria were key in providing access to analysing the regulation of what comes to be constituted as mathematics and how in pedagogic contexts.

Although, mathematics is meant to be defined as definite descriptions, the data show that any attempt to describe objects in a precise, formal manner is largely absent. So what we have is an absence of definite descriptions. Consequently students have to synthesise general principles of mathematics primarily through computational procedures, general descriptions of mathematical concepts and a reliance on what the solution should look like. If one reflects on the general picture of policy and curriculum in South Africa, it shows that certain features are present but certain features are also absent. Although the curriculum proposes that, '(m)athematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena [...]' (DoE, 2002: 4), the data show that the active student aspect is largely absent. If one considers curriculum reform in this particular context, changes have been introduced in a way that does not really consider the student, except in a rhetorical way where there is no real activity. As already mentioned, formal mathematical definitions are also largely absent. It appears that the nature of pedagogy does not display any of the features of the 'active' student and at the same time does not involve formal definitions.

In addition to noting that computational procedures are the dominant resource for elaborating mathematics in these five schools, the findings of this research also show that mathematics is constituted in a manner that is different from the announced topic. As already mentioned in Chapter Ten, integers and fractions are constituted as whole number arithmetic, transformation geometry, exponential expressions and bar graphs are constituted as counting and types of numbers are constituted as whole number classification. An important outcome of this study reveals that Grade 8 topics are realised in terms of very elementary content, i.e. basic arithmetic and counting, and this may provide a possible explanation for the poor performance of South African students in regional, national and international testing discussed in detail in Chapter Two. As noted in Chapter Ten, I am not suggesting any causal relation between the curriculum, pedagogy and learner performance.

There are significant features in the empirical context of the five working class schools in my study that may suggest that for working class students teaching has continued as before, regardless of curriculum reform, and the inadequate attention previously given to formal propositions and definitions of mathematical objects is no longer referenced.

11.2 Limitations and potential of my study

My study has not at any stage considered the academic qualifications or teaching experience of the teachers at the five schools and hence I have no records of their knowledge of mathematical content. The focus of this study was on the teaching and learning of mathematics in the context of educational policy, curriculum and the available teaching resources in lesson segments. It would have been interesting to research the relation between the teacher and mathematical knowledge and how this shapes what is constituted as mathematics and how in the classroom.

Another area not researched in this study is the relation between teachers' choice of mathematical topics and the curriculum. Teachers in state-controlled schools are generally obliged to follow a set curriculum that officially specifies what the subject knowledge should be for mathematics. The Department of Education issues a planner in the form of pace-setters²⁹ that help teachers cover the prescribed topics for the year. As a result, teachers do not have that much control over which topics should be taught. Furthermore, the RNCS also dictates the way that this content should be learnt and assessed. This certainly influences the way in which teachers constitute mathematics in the context of pedagogy.

Due to the size of this research project, I was not able to carry out triangulation of data which would perhaps have provided a more detailed and balanced overview of the constitution of mathematics in these five schools. No students or teachers were interviewed and this may have provided more insight into how teachers interpreted the effects of curriculum reform related to the topic areas covered and whether they were in fact adhering to the curriculum prescriptions that emphasised learner activity or whether they were simply continuing as they had in the past. The mathematical competence of students was also not taken into account during this study. Interviews of this nature could be carried out in further research.

This study commenced by asking the question: What is constituted as Grade 8 mathematics in the pedagogic spaces of five working class schools in the Western Cape in the context of a GET curriculum that de-emphasised the use of formal definition and propositions of mathematical objects and processes? In Chapter One I presented a discussion on how curriculum reform in South Africa had presented mathematics in manner that considered the student in a more direct way. In Chapter Two I discussed an account of the state of play in South African school mathematics by reviewing literature that examined reasons for the poor mathematics performance of South African school students in relation to students in other countries.

²⁹ Pace-setters, issued by the WCED, provide a term by term outline of topics to be covered as well as the time that each topic requires to teach.

My study focused on the realisation of curriculum at the level of pedagogy and provided an account of the computational activity in fifteen Grade 8 mathematics lessons at five working class schools in the Western Cape in order to present a description of what and how mathematics was constituted in a general milieu of curriculum reform. The data in this particular empirical setting show that if students are to gain access to the general principles of mathematics, they would need to synthesise these general principles from computational procedures, general descriptions of mathematical concepts and a reliance on what solution should look like rather than a set of interconnected propositions. The data also show that Grade 8 topics are realised in terms of mostly elementary content i.e. basic arithmetic and counting.

I am not, however, able to relate these findings to other contexts or social class settings and am unable to make claims about whether the constitution of mathematics in the context of the RNCS differs from the constitution of mathematics in other curriculum contexts. It would be interesting to examine the constitution of mathematics in a selection of schools from different social class backgrounds using my analytical framework.

The methodology employed in my research generates productive potential for re-describing what teachers and students are engaged in the context of mathematics pedagogy and there is potential for extending these methodological tools even further. Furthermore, this research has the potential to inform or re-structure teaching, texts as well as curriculum.

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Appendix A: The Pedagogic device

A.1 The distributive rule

The distributive rule focuses on who gets what knowledge and it 'attempt(s) to regulate those who have access' (Bernstein, 1996: 45). At the level of policy, all students are expected to get the same content; however, Bernstein (*ibid.*) highlighted how knowledge is differentially distributed to different people when discussing the distributive rule of the pedagogic device. He states that the pedagogic device functions to

[...] provide a symbolic ruler for consciousness [...] it is a condition for the production, reproduction and transformation of culture [...] The device creates in its realizations an arena of struggle between different groups for the appropriation of the device, because whoever appropriates the device has the power to regulate consciousness. (Bernstein, 1996: 50-52)

In the context of curriculum reform in South Africa, the White Paper on Education (1995) spelt out the distributive imperative quite clearly in its pursuit of equal access to education for all in a successful modern economy:

An integrated approach implies a view of learning which rejects a rigid division between "academic" and "applied", "theory" and "practice", "knowledge" and "skills", "head" and "hand". Such divisions have characterised the organisation of curricula and the distribution of educational opportunity in many countries of the world, including South Africa. They have grown out of, and helped to reproduce, very old occupational and social class distinctions. In South Africa such distinctions in curriculum and career choice have also been closely associated in the past with the ethnic structure of economic opportunity and power. (DoE, 1995: Chapter 2)

Besides rejecting the previous 'authoritarian curriculum' in favour of a more integrated curriculum, access for all has also translated into more active ways of learning as described in another quote from the White Paper on Education (1995) below:

The Constitution guarantees equal access to basic education for all. The satisfaction of this guarantee must be the basis of policy. It goes well beyond the provision of schooling. It must provide an increasing range of learning possibilities, offering learners greater flexibility in choosing what, where, when, how and at what pace they learn. (DoE, 1995: Chapter 4)

At this point it might be useful to reflect on the features of mathematics as prescribed in the RNCS:

Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. (DoE, 2003a: 9)

The unique features of learning and teaching Mathematics include: investigating patterns and relationships: describing, conjecturing, inferring, deducing, reflecting, generalising, predicting, refuting, explaining, specialising, defining, modelling, justifying and representing. (DoE, 2002: 5)

Bernstein (1996) argues that it is from within the field of production of knowledge that a selection is made and re-located in the recontextualising field. The extracts provided above suggest that the curriculum should not only state the mathematical content but also give learners the opportunity to explore and actively construct their own mathematics. This study will examine the implications of how a de-emphasis of more definite descriptions of mathematics in the curriculum have influenced the form that evaluative activities take in pedagogy.

A.2 The recontextualising rule

As elaborated in Chapter One, the RNCS for GET is a revised, streamlined version of C2005 focusing on ‘learner-centred pedagogy’ where instruction ‘emphasise(s) individualized learning based on active engagement and empirical problem solving’ (Nykiel-Herbert, 2004: 251).

Bernstein (1996: 48) distinguishes between the

official recontextualising field (ORF) created and dominated by the state ... and a pedagogic recontextualising field (PRF) [...] consist(ing) of pedagogues in schools and colleges, and departments of education, specialized journals [...]

when describing how knowledge which is selected for teaching and learning is recontextualised. Specialised knowledge produced in institutions such as universities and research institutes undergo a transformation into pedagogic knowledge in the context of schooling. The curriculum extracts provided thus far suggest that, in addition to stipulating officially what the subject knowledge should be for mathematics, the RNCS also dictates the way in which it should be learnt and assessed. The curriculum is what Bernstein (1996: 45) would describe as the first transformation of knowledge; where official pedagogic knowledge is converted into what counts as school knowledge. According to Bernstein (*ibid.*), the conversion of knowledge from the field of production takes place within the official recontextualising field (ORF) and the pedagogic recontextualising field (PRF). The recontextualising field primarily consists of government departments of education, curriculum bodies, education journals, etc.

In another extract from the RNCS, the presence of a relationship between the content of mathematics and the activity of the learner is also evident where the learner is expected to display empirical recognition by

identifying and analysing regularities and change in patterns and relationships that enable (them) to make predictions and solve problems. (DoE, 2002: 9)

Pedagogy prescribed by the RNCS resounds with its ideological message of active construction of knowledge by the student where the general features of mathematics are made available for observation from which they are able to draw conclusions. Consider the extract in Figure A.1, the Contents page of a Grade 9 textbook used in 2006, illustrating the emphasis on the active student ‘finding patterns, finding rules’. The general idea suggested is that being able to convince oneself and others of a rule suffices as a form of mathematical argumentation as opposed to stating formal definitions upfront. It also displays how strongly the PRF is aligned with the principles of the ORF.

Part 2	Fun with Patterns, Functions and Algebra . . .	46
Unit 7	Rules within patterns . . .	47
Activity 1	Patterns play hide-and-seek	47
Activity 2	Finding rules for diagrammatic patterns	48
Activity 3	Finding patterns for rules	49
Activity 4	The 1 000th hexagonal number . . .	49
Reflection	Number patterns in beadwork	52

Figure A.1: Contents page of textbook (Antunes, Schalekamp & Setati, 2006: 49)

The recontextualising rule is evident in the way that the PRF uses textbooks, for example, to realise the prescriptions of the ORF. It appears that the RNCS has recontextualised the notion of a necessarily active learner by emphasising the behavioural characteristics of students. The need to actively construct new knowledge and make use of materials in order to demonstrate activity is implicated in pattern, amongst other resources, becoming a major pedagogic resource in establishing mathematical propositions.

A.3 The evaluative rule

The second transformation of knowledge occurs when school knowledge is transformed into acquired knowledge in the classroom. Bernstein (1996: 50) emphasises that it is the evaluative rule that condenses the meaning of the pedagogic device, stating that: ‘the essence of the (teaching) relation is to evaluate the competence of the acquirer’ (Bernstein, 1990: 66).

For Bernstein (1996), if the acquirer has achieved the criteria made available, teaching and learning has been successful. He (*ibid.*) would argue that access to both the recognition and realisation rules were granted, i.e., criteria were made available for the recognition of what should be produced as content and criteria for how to realise the content.

The problem that my study focuses on is located at this level of the pedagogic device. Both the distributive rule and the recontextualising rule are realised at the level of evaluation where evaluation is an attempt to realise what has been prescribed at the level of policy and the curriculum.

In a study conducted by Davis & Johnson (2007b: 121) on the constitution of school mathematics, already discussed in Chapter One, on the same five working-class schools selected for my study they claim that in

addition to the pace of teaching and learning being slow, students at these schools also ‘fail to learn the content adequately.’

We noted that while teaching in the five schools took place exclusively through exposition, little explicit attention was given to the exposition of mathematical ideas, principles and definitions that served to ground the procedures that were being rehearsed. Mostly, teachers briefly referred to definitions but without discussing or explicating the mathematical reasons for the production of definitions. We also noted that worked examples were used in three ways: (1) in expositions by teachers demonstrating the application of standard procedures through worked examples; (2) by students working through exercises to practice and demonstrate their acquisition of procedures; and (3) the routine marking of worked examples by teachers in which privileged solutions are demonstrated. (Davis & Johnson, 2007b: 123)

Davis & Johnson’s (*ibid*: 126) research revealed that on average, 10 minutes were spent per worked example and that approximately 3 to 5 problems were discussed per lesson. Their study claims that although students generally worked at a slow pace at these schools and had sufficient time to acquire the criteria, in this instance by way of the worked example as a primary resource for teaching, their grade-specific baseline test results measuring mathematical competence on content learnt in previous grades were weak. This may highlight the issue that the acquisition of the evaluative criteria by students does not necessarily imply success; Davis & Johnson (*ibid*: 121) claim that learners ‘fail to learn the content’. In essence, then, if it is the evaluative rule, which according to Bernstein (1996), determines whether the relay or transmission of legitimate text has been achieved or not then the distributive imperative of education for all has not been met.

Bernstein (1996: 47) furthermore describes pedagogic discourse as being an embedded discourse with the instructional discourse (ID) embedded in the regulative discourse (RD). The RNCS has not only fundamentally altered the nature of the knowledge (ID) by de-emphasising more formal presentations of mathematics, but has also shifted the way in which the reproduction of that knowledge is regulated in the classroom (RD) by focusing on demonstrating activity via the active student. In essence, then, the content of knowledge to be acquired has undergone change as well as the manner that students and teachers need to demonstrate that it has been acquired.

Bernstein’s notion that the process of evaluation, and hence the evaluative rule, encapsulates the meaning of the pedagogic device is a key feature that has influenced my selection of additional conceptual resources.

Appendix B: Tracing the lineage of curriculum reform in South Africa

The RNCS is distinguished from the content-driven, teacher-centred curriculum synonymous with apartheid, as an outcomes-based, learner-centred one where access to mathematics is defined as ‘human right in itself’ coupled to a portrayal of mathematics as ‘a human activity’ (DoE, 2002: 4). What follows is a journey tracing the educational transformation in South Africa where I reflect on how and why the apartheid school curriculum was reconstructed around an outcomes-based philosophy³⁰.

B.1 The apartheid curriculum

The apartheid education policy (1948-1994) was, especially, influenced by the philosophy of Fundamental Pedagogics³¹ that promoted an authoritarian, racist curriculum. Apartheid education was used as a means to enhance the divisions in society and reinforce inequalities that already existed in the various race groups in South Africa. The Apartheid Education System consisted of racially segregated departments of education, and so all government-funded schools received resources that were differentially allocated with race being the deciding factor. Besides the very definite racial distinctions, the apartheid curriculum reflected rigid subject boundaries and was strongly aligned with the New Math movement of the 1960s which stressed the importance of sets, an emphasis on knowing formal definitions and proofs and teaching mathematics by drawing on mathematical theory. The fundamental ideas of the New Math movement in secondary education were based on the vast work of Bourbaki³².

B.2 The Human Research Sciences Council (HSRC)

The education system under apartheid was in dire need of radical restructuring and transformation –differential allocation of funds to different race groups exacerbated inequalities. Resistance to apartheid education was inevitable and protests, strikes and demonstrations were common place in 1970s and 1980s.

Research conducted by the South African Institute for Educational Research, an institute of the state-aided Human Sciences Research Council (HSRC) in South Africa investigated curricula in liberal countries and

³⁰ An outcomes-based approach emphasises the outcomes to be achieved rather than the content to be mastered.

³¹ Fundamental Pedagogics originated in the Netherlands as a humanistic philosophy of education and was linked to the Christian National Education framework of the apartheid era (Vithal & Volmink, 2005: 5-6).

³² The Bourbaki were a group of mostly French mathematicians, who began meeting in the 1930s and aimed to write a thorough (formalised) and unified account of all mathematics, which could be used by mathematicians in the future.

suggested what a curriculum in a liberal democratic society should look like (Galant, 1997). A common thread emerging from their research was the need for the government to

pursue a policy of equal educational provision for all South Africans with respect to educational opportunities and standards, irrespective of race, colour, religion or gender. It also suggested that a single department of education for all groups should replace the system of fragmented provincial departments. (Burger, 2010)

These suggestions were presented as the De Lange Report to the Minister of Education on 31 July 1981.

The state responded to the educational crisis in South Africa by launching an investigation which the HSRC published in 1981: 'Provision of Education in the RSA.' While the main findings of the report were dismissed by the government as being too radical, many of the proposals resounded with both the legislation of the late apartheid state and the reform legislation of the post 1994 democratic government (Stevens, 2006). From this project emerged the production of constructivist ideas in mathematics education. Kraak (2002) suggests that the aim of the HSRC report was to divert the focus on official education based on the traditional 'academic' arts and sciences curriculum towards a more 'appropriate' skills-based vocational curriculum, particularly for black, working class learners.

B.3 Mathematics education projects in South Africa

The late 1980s and early 1990s were characterised by massive NGO activity in education in South Africa - mathematics education projects were established in an attempt to improve school mathematics and were mostly initiated by Mathematics and Education departments at universities. Amongst others, the following projects were committed to research and developmental work in school mathematics:

- MEP was launched at University of Cape Town
- RUMEP (Rhodes University)
- People's Education³³
- RUMEUS (Stellenbosch University)
- REMESA (University of the Western Cape)
- MALATI (Stellenbosch University)
- CASME (University of Kwazulu-Natal)
- National Education Crisis (Coordinating) Committee (NECC) Mathematics Commission³⁴

³³ People's Education was presented as an alternative to apartheid education and portrayed equal access for all, critical thinking, learner-centredness, teachers as curriculum developers, community participation and group work (Chisholm *et al.*, 2000).

The projects can be broadly described as constituting what Bernstein (1996) refers to as the Pedagogic Recontextualising Field (PRF). Bernstein (1996: 48; italics in original) distinguishes between the

[...] *official recontextualising field* (ORF) created and dominated by the state ... and a *pedagogic recontextualising field* (PRF) ... consist(ing) of pedagogues in schools and colleges, and departments of education, specialized journals [...]

when describing how knowledge which is selected for teaching and learning is recontextualised. Research projects, initiated by the PRF, were essentially focused on curriculum innovation in an attempt to address the educational crisis in South Africa, stating that ‘... this curriculum is in various ways inadequate, inaccessible and inappropriate ...’ (Levy, 1992: 6). Their aim, primarily, was to examine three key deficiencies in the South African mathematics curriculum, namely:

- What knowledge is taught.
- How knowledge is taught.
- Control of what is taught and how it is taught. (Levy, 1992: 7)

The form of mathematics that emerged from the PRF during this era was characterised by a shift in emphasis from rote learning and algorithms synonymous with a content-based curriculum to situations where learners were confronted with real-life situations and were able to appreciate the value and utility of mathematics in the pursuit of developing ‘an independent mathematical thinker, looking for a solution. [...] The change is to the child-centred approach’ (Levy, 1992: 228-9).

B.4 The interim curriculum

In 1994, South Africa embarked on a process of ‘educational reform that aimed to change the authoritarian, racist curriculum of ‘fundamental pedagogics’ into one that embraced – politically and philosophically – democratic, progressive, constructivist principles of teaching and learning’ (Nykiel-Herbert, 2004: 251). An interim curriculum emerged from studies on global reform, findings of the HSRC, NGOs and was primarily inspired by constructivist ideas. Initially, this revised curriculum was implemented by both the Natal and Western Cape Education Departments and its purpose is described in the following quotation:

After the change in government in 1994, an interim curriculum was introduced for all grades in primary and secondary schools in South Africa and which was essentially an edited version of the 1983 national curriculum so that all learners in SA were offered the same grade 12 examination, irrespective of which education department their school was in. (Engelbrecht & Harding, 2008: 57)

³⁴ NECC was formed in 1985 in response to school boycotts and unrest in the mid-1980s. Their slogan, ‘People’s education for peoples’ power’, symbolised a body that was established to challenge the apartheid system of education. (NEPI, 1993)

The 'Draft New Syllabus for Mathematics: Standards 2-4' certainly displayed a shift in emphasis, where the student was more involved as an 'active' mathematical thinker. An extract from the syllabus describes this trend in curriculum reform quite clearly:

In implementing the Mathematics Curriculum, due attention should be given, not only to the provision of mathematical knowledge, skills and concepts in the planned curriculum, but also to the mathematical processes by means of which pupils are actively and productively involved in learning, e.g. comparing, classifying, describing, representing, pattern searching, inferring analyzing, proving and problem-solving.

(DEC, 1991: 1)

The 'Draft New Syllabus for Mathematics: Standards 2-4' was introduced for senior primary mathematics with implementation dates in 1993 (Std 2), 1994 (Std 3) and 1995 (Std 4), 'where in addition to providing flexible methods, which are discussed and compared in class, it recommends a problem-solving approach to mathematical thinking, the development of algorithmic thinking (rather than mastery of only the standard algorithms), and the calculator as an essential tool' (DEC, 1991 in Adler, 1992: 26).

The Association for Mathematics Education of South Africa (AMESA), a professional organisation for mathematics educators in South Africa was founded in 1993 and consisted of nine previously distinct mathematics organisations. AMESA originated from MASA (Mathematics Association of South Africa), a mathematics association for 'white' mathematics educators and alongside state curriculum reforms, professional mathematics associations played a central role in curriculum reform. Galant (2007: 51) describes the role of MASA in structuring curricula and hence, official policy documents, as follows:

They reflect upon the state of mathematics education in South Africa in the 1970s and make recommendations for mathematics curriculum "innovations" that include the introduction of differentiated school mathematics syllabuses, the restructuring of teacher training courses and bureaucratic procedures for implementing curriculum changes. The research reports discuss the nature of mathematics, the selection and organisation of mathematical contents in school mathematics syllabuses, issues in the teaching and learning of school mathematics and describe teachers and students in particular ways.

AMESA, however, provided a more liberatory agenda with respect to 'formulating policy statements on matters regarding Mathematics Education and promoting such perspectives among its members, policy-making bodies and organs in civil society involved in education' (AMESA, 2011).

Consider the following extract from material produced by AMESA for teachers as a guide for teaching junior secondary geometry in response to the implementation of the Interim Core Syllabus for Mathematics for Standards 5-7 in 1995/6/7:

Most teachers would recognize that the existing junior secondary geometry syllabus [...] emphasis(es) the logico-deductive nature of formal Euclidean Geometry. In practice this means that definitions have to be learnt and proofs have to follow a logical sequence, reflecting an algebraic style of reasoning. Such an approach leaves little space for students to explore their own geometrical intuitions or to find ways of convincing themselves of the logic of certain formal proofs. The new geometry syllabus has been designed to open up this space and represents a shift towards the development of spatial awareness and ability. [...] The new syllabus encourages an informal

perspective and intuitive approach to junior secondary geometry. In particular it suggests that students' experiences in the world be used to introduce them to geometrical thought and language [...] that [...] such experiences have to be reflected upon, reinterpreted and then redescribed in mathematical terms. (AMESA, 1995a: 3)

This extract suggests a de-emphasis of a particular presentation of mathematics i.e. formal propositions of mathematical objects and processes in favour of a more intuitive, experiential approach and the teacher guide prescribes a range of activities where students are presented with real-life problems that mathematise these activities. A similar trend emerged from the teacher's guide for teaching junior secondary algebra, where an activity required students to formulate a function rule for a sequence of numbers:

Teachers should reflect on the processes they used themselves in solving the problem. In particular we should note the informal way in which many teachers solve [...]: we as teachers should not force formal 'algebra' (here meaning a formula using letter symbols and variables) on students when there is no need for it. On the other hand we should recognize such informal approaches as 'algebra'. One of the important features of algebra is the idea of *generality*. (AMESA, 1995b: 10; italics in original)

B.5 Curriculum 2005: A post-apartheid curriculum – 1997 and beyond

In 1997, C2005, the first curriculum of the new political dispensation, introduced progressive and constructivist ideas as a backlash against traditional, authoritarian curricula. The curriculum was viewed as the means for realising the imperatives of a democratic society and transforming the racially-based school education system – hence the decision to structure the curriculum around outcomes-based education (OBE).

When C2005 was introduced in 1997, it was premised on three critical elements: the introduction of eight new learning areas suffused with the values of democracy, non-racialism and non-sexism; outcomes-based education (OBE) and the provision of a foundation in general education up to and including the 9th Grade. (Chisholm, 2003: 268)

C2005 was developed as a result of a confluence of ideas – anti-apartheid political ideas, the importance of student participation and progressivist ideas from the United Kingdom and Australia (Spady, 1994) where the focus was on holistic education and competences rather than deficiencies. Traditional school subjects were replaced with learning areas and phase organisers (broad themes which guided learning across subjects) were designed to maintain coherence and ensure cross-curricular work. The form of school mathematics privileged by the changes in the way knowledge was organised in the curriculum was characterised by a shift towards a more learner-centred acquisition of knowledge and skills through active learning.

B.6 The Chisholm Report

During the second year of implementation of C2005, in response to an outcry from academics and the public, the National Department of Education commissioned research through the President's Education Initiative (PEI) to investigate the implementation of the recent curriculum reform policies. The authors of the PEI, in *Getting Learning Right*, reported that the new curriculum exhibited a low level of content across all subjects with

integration across subject boundaries. Presentation of subjects was unsystematic and the coherence provided by the phase organisers³⁵ was responsible for weak lateral demarcation between school and everyday knowledge and between different school subjects (Taylor & Vinjevoold, 1999). The purpose of this research was to provide scientific grounding for future planning and delivery of educator development and support programmes.

By early 2000, the inherent flaws in Curriculum 2005 were becoming obvious, with specific complaints about children's inability to read, write and count at appropriate grade levels, their lack of general knowledge and the shift away from explicit teaching and learning to facilitation and group work. Teachers did not know what to teach. Academics, and the media, took up call for a review of the curriculum. (MoE, 2009: 12)

The investigation and findings of the PEI Report in 2001 resulted in a Ministerial Committee being tasked with reviewing C2005. Weak conceptual coherence, especially in subjects such as mathematics, where skills, concepts and content were under-specified for each grade level, was noted as a design feature that required attention. The Report of the Review Committee (Chisholm, Lubisi, *et al.*, 2000) illuminated a number of other issues that also required attention in the curriculum:

- a skewed curriculum structure and design
- lack of alignment between curriculum and assessment policy
- inadequate orientation, training and development of teachers
- learning support materials that are variable in quality, often unavailable and not sufficiently used in classrooms
- policy overload and limited transfer of learning into classrooms
- shortages of personnel and resources to implement and support C2005
- inadequate recognition of curriculum as the core business of education departments

Following the Chisholm report, C2005 was reworked into the RNCS which was completed in 2002. The RNCS was implemented in Grades 1, 2 and 3 in 2004, Grades 4, 5 and 6 in 2005, into Grade 7 and 10 in 2006, Grades 8 and 11 in 2007 and Grades 9 and 12 in 2008.

³⁵ 'Curriculum 2005 was an attempt at radical change to this curriculum form by reorganizing the curriculum. Much of the change came in the way in which knowledge was organized. Thus the new curriculum: was competence based, and organized knowledge in integrated learning areas; ... This integration was achieved through: themes in 'phase organizers' and 'programme organizers', which crossed the divides between school subjects.' (Hoadley & Jansen, 2009: 173)

B.7 The revised C2005 becomes the revised national curriculum statement (RNCS)

The implementation of the RNCS followed and was a revised streamlined version of C2005 – officially introduced in 1997 – an Outcomes-Based Education (OBE).

It explicitly attempted to shift the curriculum agenda from a local, primarily skills-based and context-dependent body of knowledge inappropriate for a schooling system, towards a more coherent, explicit and systematic body of knowledge more suitable for a national curriculum in the twenty first century and more able to take its place amongst other regional and international curricula. It specifically set out to develop a high knowledge, high skills curriculum, resulting in a fundamental but necessary departure from *Curriculum 2005*. (MoE, 2009: 13)

For mathematics, there appears to be a shift away from

characterizing mathematics as a discipline of facts, procedures, formulae and proofs that can be transmitted by articulate teachers to diligent learners, towards one defined and learned based on constructive activity of the learners. (Fosnot & Dolk, 2003: 11)

It should be noted that the above extract is, in fact, Fosnot & Dolk's (2003) description of curriculum reform in the United States when the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) were implemented. The theory of learning prescribed by the reform-oriented Standards-based curriculum certainly echoed the general theme of progressive education endorsed by the RNCS, which also advocated student-centred discovery learning:

The outcomes encourage a learner-centred and activity-based approach to education. (DoE, 2002:1)

Ensor and Galant (2005: 284-285) describe the rationale for curriculum reform in South Africa as follows:

In education specifically, the apartheid era had bequeathed a school system which was divided on racial lines, and carried with it an inequitable distribution of educational goods – material as well as symbolic. [...] A key vehicle the government chose in order to deliver on its educational and social reconstruction aims was the National Qualifications Framework (NQF), a bold attempt to integrate education and training. Integration and relevance became the watchwords of the new educational discourse, underpinning the NQF and Curriculum 2005 (C2005), the new curriculum that was devised for schools. [...] (T)he intention of government was to loosen three sets of boundaries – between the academic and the everyday, between education and training, and between the different component contents of the academic curriculum, at both tertiary and school level. Furthermore, there was a strong push to loosen social relations in the classroom and promote greater learner participation. Eroding these boundaries was intended to loosen a further set of relations – those between different social groups.

The extracts presented not only suggest a change in the content of the curriculum but also a change in the image of the learner where access to mathematics is defined as a human right aligned with the definition of mathematics as 'a human activity' and 'a product of investigation of different cultures – a purposeful activity in the context of social, political and economic goals and constraints' (DoE, 2002: 4). It follows then that there would be a different conception of what constitutes mathematics as well as a different conception of both the

learner and teacher of mathematics. It is within the context of this shift in curriculum that I wish to explore what mathematics is taught and how it is learned in Grade 8 mathematics classrooms.

The implications of a more informal presentation of mathematics as prescribed by curriculum planners, will be explored and traced from the macro-level of policy implementation through texts for teaching and ultimately to the micro-level of the classroom.

B.8 Global and local implementation of the curriculum shift

Having tracked the history of curriculum reform in South Africa, I now examine the global influences on the curriculum. Curriculum reform in the United States provides a useful platform from which to discuss how and why this shift in the South African curriculum has been implemented. Contemporary reform curricula challenge students to make sense of new mathematical ideas through exploration, investigations of real-world situations and extended projects. This approach is juxtaposed with a so-called traditional mathematics syllabus that emphasised the use of formal definitions and procedures to obtain correct answers.

The National Science Foundation (NSF) in the United States, a government-based organisation that funds, amongst other research, work in mathematics education with the primary goal of improving mathematics teaching and learning for the last 50 years offered strong support for curriculum reform. In a multimedia special report published by the NSF, *Math: What's the Problem?* key spokespeople delivered commentary on among others, the value of mathematical proficiency and the under-performance of students when making global comparisons. The report emphasised the value of mathematics as a language for understanding the natural world.

Tony Chan, the Assistant Director for Mathematics and Physical Sciences Directorate at NSF:

You look at all the big problem(s) that society faces from energy to climate change how human beings interact all of them have a mathematical component to it. Math, it's just a language of understanding the natural world. So, the more you understand, the more you want to predict, the more you want to understand the structure behind different components of that natural system. It's useful. I think once people appreciate that, then I think they will be more motivated to learn about mathematics. (NSF, 2009)

The utility of mathematics is underscored to an even greater extent by David Bressoud, president-elect of the Mathematical Association of America, at the time, in the extract below:

Mathematics, at its heart, is really looking at the patterns in the world around us, numerical patterns, spatial patterns, especially and understanding those patterns and we're naturally pattern observers. That's part of the human nature. I mean, that's built into our DNA, that we look around the world around us and we try to understand what's likely to happen. (NSF, 2009)

When considering the views of the NSF on the role of mathematics, especially in making sense of the world, it certainly provides a general global trend motivating a curriculum shift where there is emphasis on an empirical encounter with mathematics rather than defining it as a body of knowledge characterised by formal definitions

and logical argumentation. The definition of mathematics provided by the RNCS in South Africa is consistent with views displayed by the NSF in this regard:

Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. (DoE, 2002: 4)

The South African curriculum has also recontextualised the idea of pattern as a useful resource with which to explore and elaborate school mathematics. Both the fields of mathematics and mathematics education strongly endorse the use of pattern as a resource for developing mathematical thinking skills. The RNCS states that:

Investigating patterns and relationships allows the learner to develop an appreciation of the aesthetic and creative qualities of Mathematics. These investigations develop mathematical thinking skills such as generalising, explaining, describing, observing, inferring, specialising, creating, justifying, representing, refuting and predicting. (DoE, 2002: 9)

The curriculum explicitly specifies that the student should demonstrate participation through a more subjective engagement with mathematics which pattern enables because

identifying and analysing regularities and change in patterns and relationships ... enable learners to make predictions and solve problems. (DoE, 2002: 9)

These extracts provided from the RNCS indicate that great value is attached to the utility of pattern as a means for promoting strategies for predicting and generalising to solve problems.

Appendix C

C.1 School improvement studies

I reflect on two methodological resources deployed in school improvement studies, namely the notions of *opportunities-to-learn* (OTL) and *cognitive demand*.

Reeves (2005) and Reeves and Muller (2005: 107) use the notion of opportunity-to-learn (OTL) as a means for arguing that learner achievement is directly related to content and skills that are made available in the classroom, describing it as ‘the degree of overlap between the content of instruction and that tested.’ International research on OTL suggests it to be a more complex construct that includes more than content exposure and emphasis, but also curricular pacing, curriculum sequencing and pacing as well as curricular coherence (McDonnell, 1995; Smith, Smith & Bryk, 1998). In their research in 24 under-resourced schools in the Western Cape, Reeves and Muller (2005) collected data on two dimensions of OTL in the Grade 6 mathematics curriculum, namely, content coverage and emphasis as well as curricular pacing. They noted that while the South African reform curriculum presented ‘potential for improving the quality of learners’ OTL, their potential for reducing inequality in OTL may depend on additional guidance to schools and teachers in ensuring within and across grade content complexity and across grade developmental complexity’ (Reeves & Muller, 2005: 125-126). In essence, their study claims that OTL and teacher feedback on student responses provided showed a significant positive correlation to achievement in mathematics, compared to teaching style – whether teacher-centred or learner-centred – which reflected no significant improvement in learner achievement.

Standardised systemic testing of South African student performance generally point to the issue of low levels of conceptual demand and this concern is identified in a number of studies (Schollar, 1999; Vinjevoold and Roberts, 1999; Hoadley, 2007; Carnoy *et al.*, 2011).

Carnoy *et al.* (2011: 101, italics in original) describe that in addition to the topics and sub-topics engaged with,

[...] lessons should involve the kind and level of thinking required of students on a particular topic or mathematical task, which enriches [...]. We refer to this aspect as the level of *cognitive demand*. Even though the level of cognitive demand relates to the learner, it is the teacher that controls and directs the required level for his or her students’

Carnoy *et al.* (2011) draw on Stein *et al.*’s (2000)³⁶ classification of higher and lower cognitive demand that relate to memorisation, procedures without connections, procedures with connections, and doing mathematics.

³⁶ Stein *et al.*’s (2000) classification of higher and lower cognitive demand include: memorization (recollection of facts, formulae, or definitions), procedures without connections (performing algorithmic type of problems and have no connection to the underlying concept or meaning, procedures with connections (use of procedures with the purpose of developing

In essence then, the levels of cognitive demand resonate with the procedural-conceptual knowledge distinction and this provides a useful way to read what emerges from the context of teaching and learning.

Carnoy *et al.*'s (*ibid.*: 101) research in a sample of sixty Grade 6 mathematics classes in lower-socio-economic areas in the North-West Province claimed that approximately 90% of lessons required students to '[...] recall rules and definitions, or perform algorithms with no understanding of the underlying concepts. Almost no lessons we observed "did mathematics" – that is, explained the underlying mathematics or engaged students in understanding the underlying mathematics of the concepts they were studying.' This analysis also highlighted an emphasis on procedural teaching, where learners had to recall procedures rather than being prompted to make the necessary conceptual links and provide explanations for the underlying concepts governing the procedures.

The reasons suggested for poor mathematics performance in the empirical studies above range from inadequate teacher training, poorly qualified teachers, low levels of teachers' mathematical content knowledge and as a result fewer OTL and low levels of cognitive demand i.e. procedural ways of reasoning that focused on mechanistic and 'meaningless' manipulations. The classroom-based studies, as well as the school improvement studies presented seem to describe the mathematics produced in pedagogic contexts either in terms of the procedural-conceptual distinction or the academic-everyday distinction. Curriculum reform in South Africa, in an attempt to introduce constructivist teaching ideas in reaction to authoritarian, traditional teaching styles synonymous with the previous apartheid curriculum, saw the infusion of more 'everyday knowledge' into the curriculum and textbooks as a means to enhance 'conceptual reasoning'. James (1995) as cited in Long (2005: 59) states that:

[...] reform movements, for example a Western Cape Departmental initiative from the late 1980s and early '90s discouraged teachers from teaching procedures and claimed that with sound conceptual understanding children would develop their own algorithms. [...] Lack of insight into the pedagogical theories underpinning the reform movement caused confusion even among experienced teachers.

Curriculum reform in South Africa, therefore, sought to emphasise more conceptual means of reasoning as a means for arriving at the intended content. Schollar (2004), however, claims that a poor understanding on the part of teachers regarding constructivism has resulted in learners having neither conceptual nor procedural knowledge.

deeper levels of understanding concepts or ideas and doing mathematics (complex and non-algorithmic thinking, students explore and investigate the nature of the concepts and relationships. (in Carnoy *et al.*, 2008: 46)

C.2 Regulation of the computational activity

In a study, conducted by Usiskin (2012: 502), relating to ‘understanding of a concept in mathematics from the standpoint of the student’ he specifies ‘at least five dimensions to this understanding: the skill-algorithm dimension, the property-proof dimension, the use-application (modeling) dimension, the representation-metaphor dimension, and the history-culture dimension.’ Usiskin (*ibid.*: 520) describes that understanding mathematics through the lens of mathematics education varies remarkably for policy makers, mathematicians, teachers and students:

The policy maker needs to understand the importance of that piece (of mathematics) to the student at a given time and place. The mathematician needs to understand the potential for the invention of new concepts, the consideration of new and previously unsolved problems, and the discovery of new results. The teacher needs to have a variety of understandings related to pedagogy, concepts, problems, and connections and generalizations of what is done in the classroom. [...]

It should be noted that Usiskin’s dimensions of understanding are helpful in describing the regulation of mathematical activity even though he does not apply the dimensions of understanding in the manner that I describe the regulation of computational activity. Understanding a mathematical concept, according to Usiskin (*ibid.*: 520), is realised when students are able to master the five above-mentioned dimensions of understanding, emphasising the significance of the first four aspects of understanding for the teaching and evaluation of mathematics learning. I will now provide a brief description of Usiskin’s interpretation of the five dimensions of understanding.

Skills and algorithms associated with the concept not only relate to using a particular algorithm and obtaining the correct solution, but also to having access to a variety of algorithms and using the most efficient ones to effect the correct solution. Usiskin (*ibid.*: 507) argues that the procedural understanding associated with skill-algorithm understanding is underestimated in terms of its value when compared to conceptual understanding. An application of this type of understanding, however, entails a series of decision making moments which certainly require a fair amount of skill.

Properties and mathematical justifications (proofs) involving the concept relates to an understanding that has its origin in mathematical properties underpinning the solution. Usiskin (*ibid.*: 508, 503) proposes that property-proof understanding resonates with conceptual understanding as well as with Skemp’s (1976) relational understanding. Usiskin (*ibid.*: 508) does, however, state that the trajectory from understanding mathematical properties to skill-algorithm understanding is not inevitable.

Uses and applications of the concept entail an understanding of both the algorithms and mathematical properties associated with the concept, as well as an understanding of when to apply the concept. Usiskin (*ibid.*: 508-509)

argues that use-application understanding is quite different to both skill-algorithm and property-proof understanding – often requiring a different kind of thinking. Students, generally, need to be taught *when* to apply concepts i.e. in the appropriate context.

Over and above the three types of understanding of a mathematical concept already discussed, Usiskin (*ibid.*: 510) argues that an understanding of how to represent a mathematical concept, by way of concrete objects, symbolic representations or metaphors, reflects real understanding. He (*ibid.*: 511) coined the term *representation-metaphor understanding* of the concept, stating that

concrete or pictorial (pictorial) representations precede acquisition of the other types of understandings [...] if students are brought carefully to understand (in the representational sense) what they are doing, then they will ultimately be better at skill.

Understanding the *history-culture dimension of the concept* is the fifth element described by Usiskin (*ibid.* : 515) and he admits that although it may be different from the four types of understanding already discussed, it has its value ‘to those who believe in a genetic approach to learning, that is, a progression of learning activities that parallels the historical development of the subject.’

Usiskin (*ibid.*: 516) proposes that an individual’s understanding of a ‘concept’ such as, for example, multiplication of fractions or congruence, can be analysed using his modalities of understanding, stating that: ‘A concept has associated skills, properties, uses, representations and history.’ Usiskin’s (*ibid.*) dimensions of understanding are helpful in understanding how computational activity is regulated in this particular empirical setting.

As a means for further understanding how computational activity is regulated in school mathematics, Dowling’s (2010: 13) four strategies are useful in this regard and they entail the deployment of:

- i. discursive definitions, principles, theorems and so forth;
- ii. visual exemplars, most obviously in the area of geometry;
- iii. formal nomenclatures (the decimal representation of number, for example) and heuristics; and
- iv. instrumentation (calculators, computers, geometric instruments, and so forth).

Dowling (*ibid.*) reconceptualises these strategies in terms of discursiveness and modes of action as depicted in Figure C.1.

Mode of Action	Semiotic Mode	
	Discursive	Non-Discursive
Interpretive	<i>theorem</i>	<i>template</i>
Procedural	<i>procedure</i>	<i>operational matrix</i>

Figure C.1: Mode of action articulated with semiotic mode (Dowling, 2010: 13)

This theoretical resource is relevant for my research study as it also provides a means for describing the regulation of computational activity of students and teachers - ultimately giving insight into how their activity is regulated. If one were to describe Figure C.1 more explicitly, a Mode of Action may be Interpretive or Procedural and the Semiotic Mode may be Discursive³⁷ or Non-discursive. In essence, then, a Strategy entails a Mode of Action and a Semiotic Mode and Strategies may be classified as Theorems, Procedures, Templates or Operational Matrices. The following combinations therefore exist: a Theorem may be described as Interpretive and Discursive, a Template as Interpretive and Non-discursive, a Procedure as Procedural and Discursive and an Operational Matrix is described as Procedural and Non-Discursive. These strategies are helpful for describing different ways that pedagogic activity may be regulated.

I now present a comparison between Davis' (2011b) categories of ground discussed in Chapter Three, Usiskin's (2012) dimensions of understanding and Dowling's (2010) strategies in mathematical practice in Table C.1.

Table C.1: Comparison of a selection of categories of ground (Davis, 2011b), dimensions of understanding (Usiskin, 2012) and strategies in mathematical practice (Dowling, 2010)

Categories of ground (Davis 2011b)	Usiskin's (2012) dimensions of understanding	Dowling's (2010) strategies
Propositional ground	Property-proof	Theorems
Algorithmic	Skill-algorithm	Procedure
Iconic ground	Representation-metaphor	Template / Operational matrix

The comparison presented in this Table C.1 is an attempt to construct an analytical framework to generate data in this particular empirical setting. I have aligned Davis' (*ibid.*) 'propositional ground' with Usiskin's (*ibid.*)

³⁷ Discursive reflects that which is realised in language as opposed to non-discursive which reflects that which is not realised in language.

‘property-proof dimension of understanding’ as well as with Dowling’s (*ibid.*) ‘theorem’ strategy since they refer to mathematical ideas, principles and definitions that underpin a solution procedure. Davis’ (*ibid.*) ‘algorithmic ground’ resonates with both Usiskin’s (*ibid.*) ‘skill-algorithm dimension of understanding’ as well as Dowling’s (*ibid.*) ‘procedure’ strategy in so far as they all reference computational procedures for obtaining correct solutions. Davis’ (*ibid.*) ‘iconic ground’³⁸, together with Usiskin’s (*ibid.*) ‘representation-metaphor dimension of understanding’ and Dowling’s (*ibid.*) ‘template and character distribution matrix’ strategies are similar in that they point to a reliance on what the final solution may look like.

³⁸ Davis’ (2011b) iconic ground describes the use of iconic resources in pedagogy, which involves regulation of the production of knowledge statements by referring to iconic similarity of expressions.

Appendix D: Procedural-conceptual distinction in mathematics education

Appendix A provides a trajectory of curriculum reform, tracing its transformation from a content-focused, authoritarian curriculum during the apartheid era to an interim curriculum that focused on ‘making sense of mathematics’ and ‘constructing meaning’. An attempt to render mathematics in a more ‘meaningful way’ where the student is considered as an active participant in the construction of knowledge, as opposed to a more ‘traditional’ approach that encouraged rote learning of procedural knowledge, has resulted in a reform curriculum that places emphasis on conceptual knowledge.

The procedural-conceptual distinction is, therefore, useful to this study since it provides insight into how mathematics education views the learner in relation to the constitution of school mathematics both prior to curriculum reform as well as during the general milieu of curriculum reform. The difference between ‘meaningless, mechanical, rote learning’ compared to more ‘conceptual exploration’ and ‘meaningful’ forms of mathematics where students have access to the reasons is very prevalent in mathematics education, further entrenching the procedural-conceptual distinction.

A number of other theorists, situated in both the field of psychology of education (Heibert & Lefevre, 1986; Skemp, 1976; Ma, 1999) and the field of sociology of education (Dowling, 1998) have also produced versions of the procedural-conceptual distinction (see Table D.1).

Table D.1: Procedural-conceptual dichotomy in mathematics

Theorist	Type of knowledge/understanding	
	Procedural	Conceptual
McLellan & Dewey (1895)	Skills	Understanding
Skemp (1976)	Instrumental understanding	Relational understanding
Heibert & Lefevre (1986) Steinbring (1989) Rittle-Johnson, Siegler & Alibali (2001)	Procedural knowledge	Conceptual knowledge
Sfard (2008)	Calculational discourse	Conceptual discourse
Dowling (1998)	Proceduralising texts	Principling texts
Ma (1999)	Pseudo-conceptual understanding based on procedural knowledge	Conceptual understanding as the basis of procedural knowledge

Hiebert & Lefevre (1986: 3, 7) define conceptual knowledge as 'knowledge that is rich in relationships' and procedural knowledge as 'rules or procedures for solving mathematical problems' describing the two knowledge forms as being distinct, yet interconnected.

Related to the procedural-conceptual distinction is Skemp's (1976: 22) differentiation between instrumental understanding (knowing how) and relational understanding (knowing how and why). Ma (1999), in her analysis of teachers' mathematical knowledge, also distinguishes procedural and conceptual understanding. Ma (1999: 118, 122) describes American teachers' practice as predominantly being controlled by an algorithm resonating with procedural or pseudo-conceptual understanding whereas Chinese teachers were able to 'justify an explanation with a symbolic derivation, give multiple solutions to a problem, and discuss relationships among the four basic operations of arithmetic.' Ma (*ibid.*: 124) describes Chinese teacher's explanations as displaying a 'profound understanding of fundamental mathematics', and as a well-organised mental package of highly-connected concepts and procedures. When Ma (*ibid.*: 108) discusses the mathematical rationale for an algorithm she states: 'Know how, and also know why.' In essence then, Ma's research claims that Chinese teachers' conceptual understanding was the basis for their procedural understanding when compared to that of American teachers.

Dowling (1998), who is located in the field of sociology of education, distinguishes between proceduralising and principling discourses resonating with the procedural-conceptual distinction employed by Heibert & Lefevre (1986), Skemp (1976), Ma (1999) and Steinbring (1989). Dowling's³⁹ (1998) proceduralising-principling definitions provide a description of the discourse produced by pedagogic agents and his work illuminates the effect of pedagogy on learner acquisition; especially the effect of distributing different forms of knowledge to different groups of learners. For Dowling (1998), procedural forms of knowledge restrict learners' access to mathematics, while principled forms of knowledge has the potential for apprenticing learners into mathematical discourse.

³⁹ It is worthwhile noting that Dowling's (1998) study does not focus on the learner but on the knowledge distributed to the learner.

Appendix E: Analysis for School P1

School P1 Lesson 1

Generating evaluative events

My initial analysis, using the protocol laid out in Chapter Four, produced the segmentation of the lessons displayed in Tables E.1 to E.3.

Table E.1: Evaluative events spanning Lesson 1 at School P1 (S P1 L1 EE1-EE3)

Evaluative Event	Type	Length
1 Prime factorisation of natural numbers [00:00 – 01:26]	Expository	01:26
2.1 Multiples of natural numbers [01:26 – 02:50]	Expository	01:24
2.2 Addition of fractions with same denominators [02:50 – 04:50]	Expository	02:00
2.3 Addition of fractions with different denominators [04:50 – 07:35]	Expository	02:45
3.1 How to calculate lowest common multiples [07:35 – 14:08]	Expository	06:33
3.2 Calculating lowest common multiples [14:08 – 19:46]	Exercise	05:38
		19:46

Next, I consider each evaluative event. For each evaluative event, I describe the computational activity of the teacher and students in order to establish what is constituted as the topic contents in the lessons and to identify what content does the work of the explicit use of apposite topic-related definitions and propositions. A detailed exposé of this step follows for each of the evaluative events:

School P1 Lesson 1 EE1: Prime factorisation

Describing the computational activity

The teacher describes two ‘methods’ for generating prime factorisations of natural numbers, referring to them as the ‘factor tree method’ and the ‘ladder method’, the presentations of which are shown in Figure E.1 and Figure E.2, respectively.

Method 1: The ‘factor tree’ method

The particular presentations of computations involved in the prime factorisation of 36, shown in Figure E.1, demonstrate that the procedure starts with the selection of any pair of cofactors for a given natural number, followed by the generation of further cofactors for each previously selected cofactor, until a pair of cofactors having 1 as a cofactor is the only possibility for a particular cofactor.



Figure E.1: An exemplification of the use of the ‘factor tree’ method

Each pair of cofactors $p \times q$ ($p, q \in \mathbb{N}$), which is not of the form $n \times 1, n \in \mathbb{N}$, is recorded in writing by positioning each cofactor at the terminal point of a line segment which forms a ‘branch’ having a previous composite number at its initial point. The generation of this series of downward branching line segments forms an inverted ‘tree’. The complete collection of natural numbers that remain as terminal points of branches but not the initial points of new branches, is expressed as a product of natural numbers that meets the demand for a prime factorisation of a given natural number.

The procedure requires a student to use their knowledge and experience of multiplication and multiplication tables to select an appropriate pair of cofactors for a given natural number. Where a student is unable to produce an appropriate pair of cofactors, s/he is likely to try to select an appropriate divisor of the given natural number and attempt to produce the necessary cofactors by using division over the natural numbers.

A closer inspection of the computations generated in this evaluative event reveals that the central operation used in the ‘factor tree’ method is multiplication over the natural numbers; that is, $\times(n_i, n_j) \rightarrow n$, where $n, n_k \in \mathbb{N}$. The fact that the teacher has referenced primes, factors and prime factors does not necessarily imply that the ideas indexed by those terms are operative in the students’ solution procedures. This method does not require the student to select prime divisors and the notions of a *factor*, *prime number* or *prime factor* are not necessary regulative resources for realising the required solution. Students merely require a vague knowledge of whole/counting/natural numbers and some familiarity with multiplication and/or multiplication tables in order to generate a solution. The criteria employed in the procedure do not depend on the notions of primes, divisors,

factors and prime factors. It is interesting to note that the specific existential features of primes are not used to guide the activity. Rather, what we arrive at computationally as the final terminal values of the ‘branches’ are referred to as primes.

What the factor tree method does is arrive at the smallest proper divisors of a natural number and its factors. The teacher’s factor tree method does not require students to know that the smallest positive proper divisor of a natural number is necessarily prime nor what a prime number is. The factor tree method generates the prime divisors as the outcome of the procedure rather than as a central regulative resource and constitutes the topic of prime factorisation as multiplication of natural numbers. Secondly, the factor tree method stands in place of the proposition that states that every positive integer (except the number 1) can be uniquely represented as a product of one or more primes.

Method 2: The ‘ladder’ method

The ladder method (Figure E.2) starts explicitly with the definition and knowledge of primes and divisibility. Students make conscious decisions about choosing the smallest prime i.e. two, and they have to consider division as a possible operation to be performed. Students test for potential prime divisors up front and there is a definite idea of what the mathematical objects are when examining the criteria. The central operation in the procedure is division i.e. $\div(n_i, p_i) \rightarrow n_{i+1}$, where the $n_i, p_i \in \mathbb{N}$ and the p_i are the smallest prime divisors of each of the n_i . So $\div: \mathbb{N} \times \mathbb{P} \rightarrow \mathbb{N}$. This procedure works universally and does not depend on the learner.

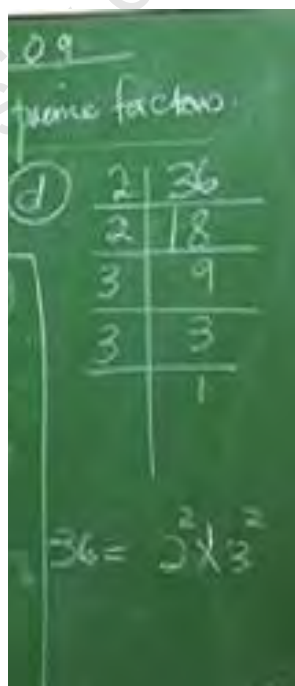


Figure E.2: An exemplification of the use of the ‘ladder’ method

The ladder method entails a recursive process involving division by primes with the quotient of 1 indicating the terminal point of the procedure. In contrast to the factor tree method, the ladder method requires that students know upfront what a prime number is and that the divisors should be prime. It is not clear, though, whether the interrelated notions of divisors, factors and prime factors are made explicit to students. Therefore, it not clear from the method that every divisor of the natural number is a factor of the number. As such, the ‘ladder method’ most likely constitutes the topic of prime factorisation as division of natural numbers.

As a means of summarising the two methods presented by the teacher, I now provide a description of the objects that are arguments (input or domain) for operations and the associated values (output or codomain) for the constitution of the notion of prime factorisation in Table E.2:

Table E.2: The computational activity for S P1L1EE1

S P1 L1 ⁴⁰	EE1 ⁴¹ [00:00-01:26] Prime factorisation [Fragment]	Input (domain)	Output (codomain)	Operation
Method 1: Factor tree method		$n \in \mathbb{N}$	\mathbb{N}	Multiplication over the natural numbers i.e. $\times(n_i, n_j) \rightarrow n$, where $n, n_k \in \mathbb{N}$. (Davis, 2011)
Method 2: Ladder method		A whole number and its minimal prime divisor. That is $n \in \mathbb{N}$, $r_i \in \mathbb{P}$, where $n \mid r_i$ and r_i is the smallest such prime. Implicit domain: $\mathbb{N} \times \mathbb{P}$	Whole number expressed as the product of powers of primes where $n \in \mathbb{N}$ and $r_i \in \mathbb{P}$. Implicit codomain: \mathbb{N}	Division i.e. $\div(n_i, p_i) \rightarrow n_{i+1}$, where the $n_i, p_i \in \mathbb{N}$ and the p_i are the smallest prime divisors of each of the n_i . So, $\div: \mathbb{N} \times \mathbb{P} \rightarrow \mathbb{N}$. (Davis, 2011)

Curriculum and textbooks

When examining the RNCS in this regard, no formal definition of prime factorisation, prime numbers or factors is provided. It only states that multiples and factors need to be recognised, classified and represented:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

⁴⁰ S P1 L1 represents School P1 Lesson 1 and so on.

⁴¹ EE1 represents evaluative event 1 and so on.

Recognises, classifies and represents the following numbers in order to describe and compare them: [...]

- multiples and factors; [...] (DoE, 2002: 69)

As a result, it seems that the teacher's main resource for teaching this topic is derived from the definitions provided by textbook, *Preparing for High School Maths*:

Factors are the whole numbers which divide exactly into the number. (Bull & Hepworth, 2008: 20).

Every composite number can be shown as the product of a string of prime numbers. These are known as prime factors. (Bull & Hepworth, 2008: 24).

In the extract provided in Figure E.3 on the definition and method for computing prime factors of a composite number, the factor tree method is prescribed as a means for arriving at the required solution and two examples are offered.

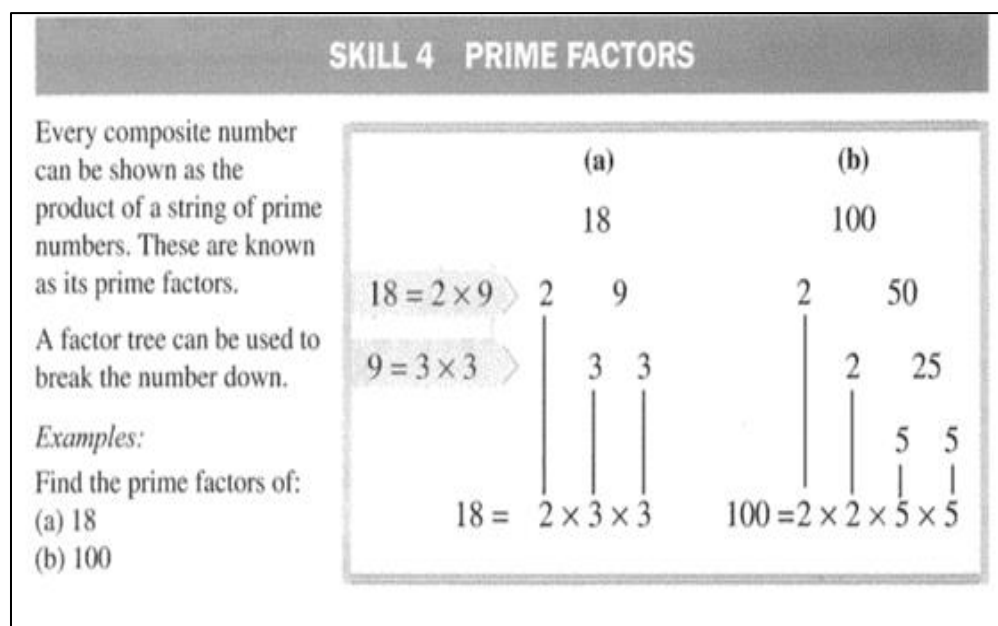


Figure E.3: Prime factors (Bull & Hepworth, 2008: 24)

However, in contrast to the teacher's factor tree method, the textbook starts with the smallest prime factor. Thus the textbook's prime factorisation mirrors the teacher's ladder method.

The mathematics encyclopaedia

When trying to arrive at what is constituted as mathematics and how this is realised, the mathematics encyclopaedia provides valuable insight. The proposition that that follows is central to the notion of prime factorisation – yet it not prescribed in both the curriculum or in textbooks as being a necessary regulative resource:

Suppose that the smallest positive proper divisor, p , of $n \in \mathbb{N}$ is not prime, then there must exist a natural number, $s \in \mathbb{N}$, such that $1 < s < p$, where $s|p$, since p is composite. So, $s|p \Rightarrow s|n$ because $p|n$, contradicting the definition of p as the smallest positive proper divisor of n . Therefore, p must be prime.

The smallest divisor of a natural number is necessarily prime – this is not taught. This proposition is implicit and is rather rendered as a *computational resource* in the form of the ‘factor tree method’ and the ‘ladder method’. The formal definitions of a factor and prime factor found in the mathematics encyclopaedia are non-existent in both the curriculum as well as the textbook being used:

Factor:

We say that $k \in \mathbb{N}$ is a *factor* or *divisor* of $m \in \mathbb{N}$ if there exists $s \in \mathbb{N}$ such that $m = ks$. We write $k \mid m$. Trivially 1 and m are factors of m ; any other factor is called a *proper factor*. (Stewart & Tall, 1977: 165, italics in original.)

Prime factor:

Suppose that n is a natural number greater than 2 and that p_1, p_2, \dots, p_k are the distinct prime factors of n written in increasing order; that is $p_1 < p_2 < \dots < p_k$. For each prime factor p_i of n , let m_i be the number of times that p_i occurs as a factor of n in its prime factorization. Then a prime factorization of n can be expressed in the following form:

$$n = p_1^{m_1} \cdot p_2^{m_2} \cdot \dots \cdot p_k^{m_k}$$

This is called the standard or canonical prime factorization of n . (Usiskin *et al.*, 2003: 220)

The topic, prime factorisation, is constituted in a manner that pays no attention to the notion of primes, factors or divisors. The two *methods* for computing prime factors are central to the topic and the textbook states that ‘(a) factor tree can be used to break a number down’ (Bull & Hepworth, 2008: 24). It is interesting that the textbook describes prime factorisation in this informal manner (see Figure E.3) rather than describing the utility of a factor tree in expressing a composite number as the product of its prime factors.

Regulation of the computational activity

In essence, the regulation of this evaluative event relates to both the ‘factor tree method’ and the ‘ladder method’ being *computational resources* for producing the required solution.

For the ‘factor tree’ method, knowledge of the primes is not a pre-requisite for successful completion of the procedure. When factorising a composite natural number, factors are either composite or prime. One continues applying the same rule to composite numbers, resulting in primes i.e. a recursive application of this proposition. The central operation employed is multiplication over the natural numbers.

For the ‘ladder method’ the central operation employed is division and knowledge of the list of primes (not necessarily the notion of a prime) is necessary for prime factorisation. I now provide a summary of the regulation of the topic, prime factorisation, by presenting a summary of procedures used, the content realised through the procedures, means of regulation as well as the content substituted by the procedures for this topic in Table E.3:

Table E.3: The constitution of prime factorisation

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Prime factorisation	Factor tree method	Computational resource	Multiplication of whole numbers	Definition of prime numbers Smallest proper divisor is prime
Prime factorisation	Ladder method	Computational resource Reliance on what solution looks like	Division of whole numbers	Relations between divisors and factors

School P1 Lesson 1 EE2.1: Multiples

Describing the computational activity

I now describe the computational activity that unfolds in this evaluative event in more precise terms by focusing on the procedures used by the teacher for computing multiples. The teacher suggests two methods for calculating multiples in the transcript extract that follows:

- Teacher: Okay, then in the lab now, we started with multiples and we said that the multiples are the answers that you get ... When you do what?
- Learner: Times.
- Teacher: When you?
- Learner: Times.
- Teacher: Times what?
- Learners: Times ... By the um .. By the um ...[inaudible].
- Learner: By the prime number.
- Teacher: By the prime number? [Speaks with rising intonation and frowns, indicating her disagreement.]
- Learner: [Many learners call out at once.] No. No.
- Teacher: No. Okay. Let's say I ask you for multiples of five. [Teacher cleans a section of chalkboard as she speaks.] Remember we did .. factors. Hey? So now we're doing .. multiples.
- Learner: Natural numbers. [It appears that the learner is reading from the textbook. See Figure 3.]
- Teacher: So I'm asking you now for the multiples of five.
- Learner: [Many learners call out at once.] Five. Five. Five, ten. Five. [Teacher gestures with her hands, indicating that she wants learners to count off multiples of five. – See Figure 4.1.]
- Teacher: It's five, ten ..
- Learner and Teacher: Ten, fifteen, twenty.
- Teacher: [Interjecting.] So we said .. How do we get to those answers? We multiply it by?
- Learner: Multiply. Multiply by five.

Teacher: We multiply it by?

Learner: Five.

Learner: By one.

Teacher: By one.

Learners: By two. By three.

Teacher: By two. By three. .. The answers are the multiples. So the multiples of five will be? ... Five. [Teacher writes /5/, followed by /5, 10, 15, 20, 25/ as learners call out the multiples of five.] (S P1 L1 EE2.1 lines 12-34.)

The first method relies on the use of the multiplication times table and the second method primarily involves the operation of multiplication. The procedure for the two methods can be restated in the following manner:

Method 1: Read off or recite multiplication times table for the natural number a .

Method 2: Given a number, a (implicitly a natural number), multiply that number by 1, then 2, then 3, then 4, and so forth, until you obtain the number of multiples that are required. The central operation of the procedure is multiplication over the natural numbers, resulting in the production of a finite sequence of n numbers, like: 5, 10, 15, 20, ..., $5n$ for some $n \in \mathbb{N}$

In other words, what we have is a function, f , from the natural numbers (domain) to the natural numbers (codomain). That is, $f(n) \rightarrow an$, where $a, n \in \mathbb{N}$. The particular rule (simple multiplication), which effects the mapping from \mathbb{N} to \mathbb{N} and so generates the required sequence of multiples, has a particular number, a , as one argument, and $n \in \mathbb{N}$ as the other. That is, $x(a, n) \rightarrow an$, where $a, n \in \mathbb{N}$

Consider Figure E.4 where the teacher gestures with her hands, signalling that she wants learners to count off multiples of five. This, perhaps, unintentional approach also gives insight into how the notion of a multiple is constituted.



Figure E.4: How to calculate multiples of five: 'five', 'ten' and 'fifteen'

As a means of summarising the two methods just presented, I now present a description of the objects involved in the computational activity in terms of the domain and codomain for the two methods for computing multiples in Table E.4:

Table E.4 : The computational activity for S P1L1EE2.1

Lesson 1	EE2.1[01:26-02:50]	Input (domain)	Output (codomain)	Operation
Multiples				
Method 1		$a \in \mathbb{N}$	The timestable for a .	Multiplication
Method 2		$a \in \mathbb{N}$ and \mathbb{N}	Sequence: $(an) \mathbb{N}$	Multiplication

Curriculum and textbooks

I present extracts from both the RNCS and the textbook used in this particular pedagogic encounter with multiples.

Multiples and factors are included in the curriculum but only in terms of being able to recognise, classify and represent them with no reference being made to defining explicitly what they are:

OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them:

- integers;
- decimals, fractions and percentages; [...]
- multiples and factors; [...] (DoE, 2002: 69)

The textbook used by the teacher, *Preparing for High School Maths*, presents the definition of a multiple either as ‘a number (that) can be found in the ‘multiplication table’ of that number’ or ‘the multiples of a number are obtained by multiplying it by the natural numbers’ as presented in Figure E.5. and Figure E.6. (Bull & Hepworth, 2008: 20). Both definitions for multiple provided by the textbook are defined in a computational manner and both are supplemented with an example.

2 NUMBER THEORY

DEFINITIONS :

A **prime number** has only two factors – itself and one.

A **composite number** has more than two factors.

Factors are the whole numbers which divide exactly into the number.

Common factors are those factors which are shared between two or more numbers.

Multiples of a number can be found in the ‘multiplication table’ of that number.

Example: The first 6 multiples of 10 are 10, 20, 30, 40, 50, 60.

Figure E.5 Definitions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

SKILL 5 MULTIPLES

The multiples of a number are obtained by multiplying it by the natural numbers.

Example:

Find the first five multiples of the number 8.

Solution:

The first five multiples of 8 are:
8, 16, 24, 32, 40

1st multiple is $1 \times 8 = 8$

2nd multiple is $2 \times 8 = 16$

3rd multiple is $3 \times 8 = 24$

4th multiple is $4 \times 8 = 32$

5th multiple is $5 \times 8 = 40$

Figure E.6: Multiples in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

When relating the two methods for finding a multiple prescribed by the teacher with the ‘definitions’ provided by the textbook in Figure E.5 and Figure E.6, it appears that the textbook is her primary resource since they are so similar. The curriculum provides no clarity regarding the definition of a multiple.

The mathematics encyclopaedia

I describe how the teacher's constitution of the notion of multiple relates to the notion of multiple in the encyclopaedia.

Typically, a mathematical definition of multiple is encountered in a description of a cluster of inter-related ideas: factors, co-factors, divisors, divisibility, products and, usually, integers. Outside of dictionaries, appropriate definitions are found in texts on number theory, an example of which follows.

In the case in which the quotient resulting from the division of a by b is an integer, denoting it by q , we have $a = bq$, i.e. a is equal to the product of b by an integer. We will then say that a is *divisible by b* or that b *divides a* . Here a is said to be a *multiple* of b and b is said to be a *divisor* of the number a . The fact that b divides a is written as: $\frac{a}{b}$. (Vinogradov, 1954: 1-2; italics in original.)

The definition maps out the set of inter-related existential features of a number when it can be described as a multiple. In terms of the definition, the recognition of one number as a multiple of another entails an activation of a web of ideas, as indicated earlier, that are much richer than the idea of multiples as the answers we get when we multiply by one, by two, by three, and so forth.

In this instance, the notion of a multiple is constituted in a manner very different to what is found in the mathematics encyclopaedia. No explicit formal definition is provided by the RNCS, textbook or teacher and students are encouraged to define a multiple either by using the multiplication times table as a resource or by using multiplication as the central operation.

I now shift to the next level of analysis where I discuss the regulation of this evaluative event. This will provide even more clarity when attempting to provide a description and analyses of what emerges in pedagogic contexts where the curriculum, texts and pedagogy have downplayed explicit attention to the formal description associated with names.

Regulation of the computational activity

Given that the teacher indicated counting numbers through gesture (Figure E.4), it would not be unreasonable for learners to think of a sequence of counting numbers, each of which is multiplied by five as the concept of a multiple. Multiplication tables serve as central regulative resources rather than the definition of a multiple and the relations between multiple, factor or divisor. The multiplication tables would therefore be considered to be a *computational resource* when using my analytic framework to read the data. Table E.5 summarises the procedure used, the content realised through the procedure, content substituted by the procedure as well as the form of regulation for the constitution of a multiple.

Table E.5: The constitution of a multiple

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Multiples	Multiplication tables, Multiplication	Computational resource	Multiplication tables	Definition of a multiple and relations between multiple, factor, divisor

School P1 Lesson 1 EE2.2: Addition of fractions having the same denominators

Describing the computational activity

It should be noted that EE2.2 and EE2.3 i.e. addition of fractions with the same and different denominators is an attempt to display the utility of the lowest common multiple (EE3.1 and EE3.2) and serves as an introduction to this topic. Consider an extract from this evaluative event which describes the utility of multiples for calculating the lowest common multiple:

Teacher: [...] Now why do we need to know multiples? Because we might eventually be asked (writes on board), what is a quarter plus a quarter?

Learner: A half.

Learner: Two quarters.

Teacher: A half?

Learner: Yes

Learner: No.

Teacher: Is it?

Learner: No.

Learner: No, miss

Teacher: Okay, somebody's saying it's a half.

Learner: A whole, ma'am.

Teacher: Somebody's saying two quarters.

Learner: Shh.

Learner: A whole.

Teacher: Shh, right let's see.

Learner: A half.

Learner: Three quarter.

Learner: The denominator stays the same. (S P1L1EE2.2 lines 38-55)

After a few learners have provided a variety of answers for $\frac{1}{4} + \frac{1}{4}$, ranging from $\frac{1}{2}$, $\frac{2}{4}$, a whole, $\frac{3}{4}$ and 'the denominator stays the same', the teacher provides the following procedure for adding two fractions with the same denominator:

Teacher: The denominator stays the same. Darren, Darren what's a denominator?

Learner: Miss?

Teacher: Yes?

Learner: The number at the bottom.

Teacher: The number at the bottom of the fraction. He's saying it's two quarters because the number at the bottom stays the same and then we add the top numbers (...) (S P1 L1 EE2.2 lines 56-60)

The transcript extract suggests that when the denominators are the same, one simply adds the numerators of the two fractions. Fractions and its constituent parts, numerators and denominators, are defined by the teacher and students in a manner that does not correspond with what is found in the mathematics encyclopaedia i.e. the

number at the bottom of a fraction constitutes the denominator (line 59). The answer that results from $\frac{1}{4} + \frac{1}{4}$, transforms addition over the rational numbers to addition over the natural numbers when the numerators are added independently and the problem then reverts back to addition over the rational numbers when the final answer is given as a fraction. Table E.6 that follows presents the computational activity of what I have just described:

Table E.6 : The computational activity for S P1L1EE2.2

Lesson 1	EE2.2 [02:50 – 04:50]	Input (domain)	Output (codomain)	Operation
Addition of fractions having the same denominators				
e.g. $\frac{1}{4} + \frac{1}{4}$		\mathbb{Q}	\mathbb{N} \mathbb{Q}	Addition

Curriculum and textbooks

The RNCS prescribes the use of appropriate algorithms for working with fractions and no precise definitions of numerators, denominators or equivalent fractions are provided:

- Recognises, describes and uses: [...]
 • algorithms for finding equivalent fractions; [...] (DoE, 2002: 69)

As mentioned earlier, this evaluative event was an introduction to the the topic, lowest common multiple, so the textbook does not provide any reference or detailed definitions of fractions and the associated operations.

The mathematics encyclopaedia

The definition of a denominator found in the mathematics encyclopaedia is very different to what is often presented in pedagogy⁴², including this evaluative event:

Denominator (*Mathworld* definition):
 The number q in a fraction p/q .

Fractions are defined as:

[...] a number consisting of one or more equal parts of a unit. It is denoted by the symbol $\frac{a}{b}$, where a and $b \neq 0$ are integers. The numerator a of $\frac{a}{b}$ denotes the number of parts taken of the unit; this is divided by the number of parts equal to the number appearing as the denominator b . A fraction may also be considered as the ratio produced by dividing a by b . (Encyclopaedia of Mathematics⁴³)

Both the textbook as well as the RNCS do not provide this amount of clarity regarding fractions, its components and associated operations.

⁴² 'The number at the bottom of the fraction.' (line 59) S P1 L1 EE2.2

⁴³ URL: <http://www.encyclopediaofmath.org/index.php?title=Fraction&oldid=24256>

Regulation of the computational activity

The form of regulation governing this evaluative event is the *computational resource* for adding fractions with the same denominator. This computational resource, which entails adding the numerators when the denominators are equal, overshadows the notion of mathematical object been dealt with i.e. fractions. The computational activity, already described, demonstrates this quite clearly as the teacher shifts back and forth between rational number addition and natural number addition. Since the addition of fractions with the same denominator is a means for introducing the lowest common multiple, I will present a description of the constitution of the lowest common multiple when it is dealt later in this lesson in EE 3.1.

School P1 Lesson 1 EE2.3: Addition of fractions having different denominators

Describing the computational activity

After providing a procedure for addition of fractions having the same denominator, the teacher describes the relevance of multiples for adding fractions with different denominators in the transcript extract that follows and uses an example in Figure E.7 to further elaborate this topic:

- Teacher: Right, now why do I need multiples? Because when I add a quarter to a quarter, when these are the same [pointing to the denominators], then it's easy to add, hey? But what if I ask you what is that plus that? [Pointing to $\frac{1}{2} + \frac{1}{3}$] See Figure E.4
- Learner: Inaudible.
- Teacher: Then, is it, can I give an answer immediately?
- Learner: No, miss.
- Teacher: Is it? Why not?
- Learner: They haven't the same denominator, miss.
- Learners: [All talk at once – inaudible.]
- Teacher: By finding the lowest.
- Learners and teacher: Common multiple. (S P1L1EE2.3 lines 82-91)



Figure E.7: A method for finding the lowest common multiple

Once it has been established that the lowest common multiple is necessary for successfully completing problems of this nature, the following procedure is provided by the teacher:

Procedure for addition of two fractions with the different denominators:

1. Generate multiples of both denominators.
2. Select the lowest, common multiple of both denominators.
3. Transform $\frac{1}{2}$ to $\frac{3}{6}$ by multiplying both the numerator and denominator in the fraction, $\frac{1}{2}$, by 3
4. Transform $\frac{1}{3}$ to $\frac{2}{6}$ by multiplying both the numerator and denominator in the fraction, $\frac{1}{3}$, by 2.
5. Once the denominators are the same i.e. $\frac{3}{6} + \frac{2}{6}$, the numerators can be added and the denominator remains the same.

A description of the computational activity for $\frac{1}{2} + \frac{1}{3}$, could be described as initially being located in the realm of the rational numbers. However, after a series of calculations (see steps 3 and 4 above), which treat the numerator and denominator as separate entities, i.e. as natural numbers and no longer as rational numbers, the problem is completed and the final answer is rational. Once the lowest common multiple of both denominators has been identified the numerator and denominator of each fraction is multiplied separately by it so that the denominators are identical. Once this requirement has been fulfilled, the numerators can be added independently. What we have here is addition of two rational numbers, $\frac{3}{6} + \frac{2}{6}$, where the numerators are calculated with independently as natural numbers in order to arrive at the final answer $\frac{5}{6}$ which is rational. I have provided a summary of the computational activity in the Table E.7:

Table E.7: The computational activity for S P1L1EE2.3

Lesson 1	EE2.3 [04:50-07:35]	Input (domain)	Output (codomain)	Operation
Addition of fractions having different denominators	e.g. $\frac{1}{2} + \frac{1}{3}$	\mathbb{Q}	\mathbb{Q}	<ol style="list-style-type: none"> 1. Generate multiples of both denominators. 2. Select the lowest, common multiple of both denominators. 3. Transform $\frac{1}{2}$ to $\frac{3}{6}$ by multiplying both the numerator and denominator in the fraction, $\frac{1}{2}$, by 3 4. Transform $\frac{1}{3}$ to $\frac{2}{6}$ by multiplying both the numerator and denominator in the fraction, $\frac{1}{3}$, by 2. 5. Once the denominators are the same i.e. $\frac{3}{6} + \frac{2}{6}$, the numerators can be added and the denominator remains the same. $(\mathbb{Q}, \times) \rightarrow (\mathbb{N}, \times)$ – Step 3 & 4 $(\mathbb{Q}, +) \rightarrow (\mathbb{N}, +) \rightarrow (\mathbb{Q}, +)$ – Step 5

In this evaluative event, the following definition of ‘common’ unfolds as the teacher introduces the lowest common multiple:

Teacher: [...] Now how do I find a lowest common multiple? What does common mean?
Learners: [All talk at once – inaudible].
Teacher: Used the most.
Teacher: Used the most frequently.
Learner: Appears in both.
Teacher: Appears in both. Right. [...] (S P1L1EE2.3 lines 92-97)

It is interesting how this definition refers to a very general description of the word ‘common’ rather than both the teacher and the student describing the lowest common multiple of two integers as the smallest positive integer that is divisible by both of these integers

Curriculum and textbooks

The curriculum also proposes the use of an algorithm for finding equivalent fractions – but does not make details of this algorithm explicit. No mention is also made of the topic, lowest common multiple:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, describes and uses: [...]

- algorithms for finding equivalent fractions; [...] (DoE, 2002: 69)

The textbook provides the following definition of the lowest common multiple:

Lowest Common Multiple: The lowest common multiple of a group of numbers is the smallest multiple common to all the numbers. (Bull & Hepworth, 2008: 25)

The board snapshots in Figure E.7 presented for this evaluative event are similar to the method for computing the lowest common multiple prescribed by the textbook (follows later) in Figure E.10. So the textbook is relied on to a great extent by the teacher as a resource.

The mathematics encyclopaedia

The following definitions of lowest common multiple, fractions, denominator and multiple, found in the mathematics encyclopaedia are, once again, different from what is found in pedagogic contexts in the transcripts presented:

Lowest Common Multiple also known as Least Common Multiple: Given two integers a and b , the least common multiple of a and b , written $\text{lcm}(a,b)$, is the smallest natural number m such that both a and b are factors of m . (Usiskin *et al.*, 2003: 211)

Denominator – see L1 EE2.2

Multiples – see L1 EE2.1

Fraction – see L1 EE2.2

Regulation of the computational activity

When adding fractions with different denominators, the computational activity shows that the choice of the appropriate algorithm serves as a *computational resource* for producing the required solution. This evaluative event is meant to illustrate the utility of the lowest common multiple when adding fractions with different denominators. The series of calculations in the solution procedure does not consider the fraction as a single entity with specific properties. Rather, the numerator and denominator are operated on independently and the fraction is re-assigned its rational number status at the conclusion of the procedure. Multiplication times tables for generating multiples for calculating the lowest common multiple is another means of regulation employed in this evaluative event and could also be identified as a *computational resource*. As already mentioned, addition fractions with the different denominators is also means for introducing the lowest common multiple and I will present a description of the constitution of the lowest common multiple the next evaluative event i.e. EE3.1.

School P1 Lesson 1 EE3.1: How to calculate the lowest common multiple

Describing the computational activity

Prime factorisation (S P1 L1 EE1) and multiples (S P1 L1 EE2.1) were dealt with to serve as a basis for teaching the highest common factor and the lowest common multiple of two numbers, respectively. A rationale for finding the lowest common multiple is first provided in S P1 L1 EE2.2 – EE2.3 when adding fractions with the same denominators and adding fractions with different denominators before the actual presentation of the topic in S P1 L1 EE3.1 – EE3.2. An introduction to the topic proceeds as follows:

Teacher: Okay, so that is the reason why it's important for us to know our multiples because if we don't know multiples then we can't find lowest common multiples and then we find it difficult to add fractions where denominators are different. [...] (S P1 L1 EE3.1 line 125)

The teacher is explicit that students use a particular method and layout for their solution procedure in Figure E.8 and Figure E.9 and she points to it, emphasising that they list the multiples, circle the lowest common one and summarise their result :

Teacher: [...] Okay, this is how you're going to do all the sums. Yes? (S P1 L1 EE3.1 line 141)

Teacher: No, I don't want answers, use that method. Write down the multiples of both numbers then you circle the common one. (S P1 L1 EE3.1 line 204)

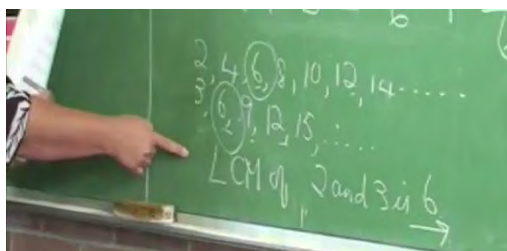


Figure E.8: The spatial distribution of symbols in finding the lowest common multiple of 2 and 3

The same layout for the solution procedure in Figure E.9 is used in another worked example when calculating the lowest common multiple of 12 and 8:

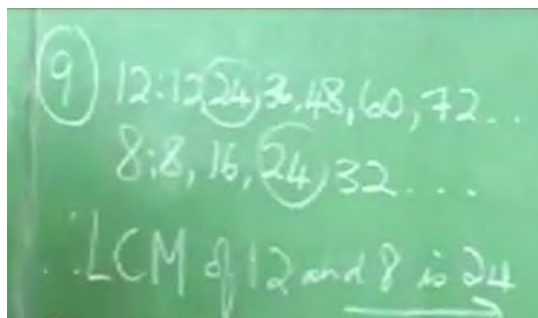


Figure E.9: Finding the lowest common multiple of 12 and 8

The solution procedure prescribed by the teacher is similar to what is found in the textbook (Figure E.10.) and it appears that the spatial distribution of symbols, i.e. the explicit instructions for the layout of this solution procedure, regulate finding the lowest common multiple. The procedure provided in Figure E.8 and Figure E.9 for adding fractions with different denominators can be structured as follows:

1. Generating multiples of a : $\times (a, b_n)$ where $a \in \mathbb{N}$ and $b \in \mathbb{N} \equiv a \times b_1; a \times b_2; a \times b_3; a \times b_4; a \times b_n$
2. Generating multiples of c : $\times (c, b_n)$ where $c \in \mathbb{N}$ and $b \in \mathbb{N} \equiv c \times b_1; c \times b_2; c \times b_3; c \times b_4; c \times b_n$
3. Find the common multiple
4. Find the lowest common multiple

The computational activity for the above procedure may be restated in the following manner in Table E.8:

Table E.8: The computational activity for S P1L1EE3.1

Lesson 1	EE3.1 [07:35-14:08]		Input (domain)	Output (codomain)	Operation
How to calculate lowest common multiples			$a \in \mathbb{N}$ and $b \in \mathbb{N}$ $c \in \mathbb{N}$ and $b \in \mathbb{N}$	\mathbb{N}	$\times (a, b)$ Multiplication $\times (c, b)$ Multiplication

Curriculum and textbooks

When examining the textbook reference (Figure E.10) relating to this topic, it presents a definition and procedure similar to the one the teacher uses. Consider the textbook definition and procedure that follows in Figure E.10:

Lowest common multiple (LCM)

The lowest common multiple of a group of numbers is the smallest multiple common to all the numbers.

Example:
Find the LCM of 3, 6 and 8

Step 1
List the multiples of the numbers.

Step 2
Select the lowest multiple which is on all the lists.

Multiples of 3	{ 3, 6, 9, 12, 15, 18, 21, 27, 30, 33... }
Multiples of 6	{ 6, 12, 18, 30, 36, 42... }
Multiples of 8	{ 8, 16, 32, 40, 48... }

Lowest common multiple is 24

Figure E.10: Lowest common multiple in *Preparing for High School Maths* (Bull & Hepworth, 2008: 25)

The curriculum does reference this topic and, hence, the textbook and the prescribed solution procedure that it presents in Figure E.10 seems to be the key resource used by the teacher for calculating the lowest common multiple.

Mathematics encyclopaedia

The definitions provided in the mathematics encyclopaedia for Multiples – L1 EE2.1 and Lowest Common Multiple – L1 EE3.1 are not stated in the curriculum, the textbook or in any of the transcript records.

Regulation of the computational activity

The teacher's procedure outlined in the computational activity and illustrated in Figure E.8 and E.9 substitutes for the definition of lowest common multiple found in the encyclopaedia. The teacher's method echoes the method prescribed by the textbook in Figure E.10. It is evident that the procedure for calculating the lowest common multiple serves as a *computational resource* and hence regulates this evaluative event. A *reliance on what the solution should look like* also regulates this event and great emphasis is placed on this when the teacher says: 'Okay, this is how you're going to do all the sums. Yes?' (S P1 L1 EE3.1 line 141). I have provided an account of the constitution of the lowest common multiple in Table E.9:

Table E.9: The constitution of the lowest common multiple

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Lowest common multiple (LCM)	Multiplication tables, counting	Computational resource Reliance on what solution looks like	Multiplication tables	Definition of LCM Relations between factor, multiple, divisor

School P1 Lesson 1 EE3.2: Calculating the lowest common multiple

Describing the computational activity

See L1 EE3.1

Curriculum and textbooks

See L1 EE3.1

The mathematics encyclopaedia

See L1 EE3.1

Regulation of the computational activity

See L1 EE3.1

School P1 Lesson 2

Generating the evaluative events

Table E.10: Evaluative events spanning Lesson 2 at School P1 (S P1 L2 EE1-EE6)

Evaluative Event	Type	Length
1.1 Filling in missing multiples and number patterns [00:00 – 02:52]	Exercise	02:52
1.2 How to check if a number is a multiple of another [02:52 – 04:43]	Exercise	01:51
2 Lowest common multiple [04:43 – 08:44]	Expository	04:01
3 Highest common factor of two numbers [08:44 – 20:29]	Exercise	11:45
4 Preferencing ladder method for finding highest common factor between two numbers [20:29 – 42:41]	Expository	23:12
		42:41

School P1 Lesson 2 EE1.1: Filling in missing multiples and Number Patterns

Describing the computational activity

This lesson commences with a homework exercise being checked on the board⁴⁴. Students were required to fill in missing multiples of a natural number after having been given two or three multiples of that number as shown in Figure E.11:

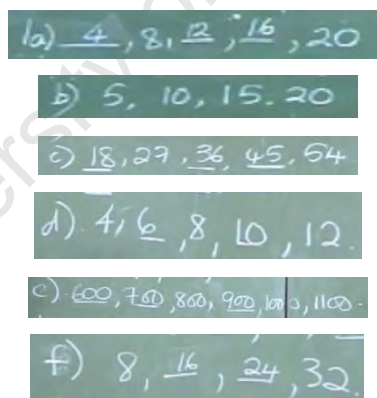


Figure E.11: Filling in missing multiples

The lesson commences with the teacher checking Number 1a -f in Figure E.11 and she suggests two methods for filling in the missing multiples. Consider the following transcript extracts as I identify these two methods:

Number 1a:

Teacher: [...] One a. The number pattern, it's ... [writes on board] and remember ... you must fill in missing multiples so what, ... which multiples do we have?
 Learner: It's err...

⁴⁴ The snapshots provided in Figure E.11 are the final answers after the missing multiples have been filled in.

Teacher: Multiples of four. [Writes answers on board.] (lines 7-9)

Number 1c:

Teacher: So which ... which multiplication table was this? [Referring to 1c.]

Learner: Nine.

Teacher: These are multiples of?

Learners and Teacher: Nine. (S P1 L2 EE 1.1 lines 27 – 30)

The two methods presented in this transcript extract can be summarised as follows:

Method 1:

1. Fill in missing multiples based on two given multiples in order to generate a number sequence.
2. Stop when sequence ends and use the multiplication times table as a resource. OR

Method 2:

Reciting times tables.

Table E.11 describes the computational activity involved in filling in missing multiples:

Table E.11: The computational activity for S P1L2EE1.1

Lesson 2	EE1.1[00:00-02:52]	Input (domain)	Output (codomain)	Operation
Filling in missing multiples and number patterns				
Method 1: Using a number sequence or pattern		\mathbb{N}	\mathbb{N}	Multiplication
Method 2: Using multiplication times table		\mathbb{N}	\mathbb{N}	Multiplication

The curriculum and textbooks

See L1 EE2.1

The mathematics encyclopaedia

See L1 EE2.1

As in Lesson 1, when multiples are referenced in this lesson, no formal definition of this entity emerges in transcript records, the textbook or the curriculum.

Regulation of the computational activity

The number pattern is suggested as a means for filling in the missing multiples. The other alternative means for arriving at the answer is to use the multiplication times tables. Both of these methods would be considered to be regulating this evaluative event and could be classified as *computational resources* which stand in place of the formal definition of a multiple and the relations between multiple, factor and divisor. I have already presented an account of the constitution of a multiple in L1 EE2.1 in Table E.5.

School P1 Lesson 2 EE1.2: How to check if a number is a multiple of another

Describing the computational activity

Consider the following transcript extract where the teacher asks students how to check whether a particular number is a multiple of another:

Teacher: And is nine thousand one hundred and eighty a multiple of nine?
Learner: Yes, Miss.
Learner: Yes, Ma'am.
Teacher: How do we know that ... that this so?
Learners: [All call out various answers at the same time – inaudible.]
Teacher: How do we calculate it?
Learners: Divide by. [Learners give different answers – inaudible.]
Learner: Times.[Other learners talking ... inaudible.]
Teacher: Shh.
Learner: Divide by nine.
Teacher: And what answer did you get?
Learner: One thousand and twenty.
Teacher: One thousand and ...
Learner: Twenty.
Teacher: One – o – two – zero, okay. So it's yes, ... [writes on board]. So nine thousand one hundred and eighty is a multiple of nine ... and he said to get to that answer, divide thousand one hundred and eighty by nine ... and we get an answer of one thousand and twenty. So yes, it is a multiple of nine. [...] (S P1 L2 EE 1.2 lines 81-95)

The emphasis appears to be on how one calculates whether one number is a multiple of another and the method for doing this takes precedence over what the definitions of a multiple might be. A procedure or calculation for doing this could be stated in the following manner:

1. Given two numbers (implicitly natural numbers), check whether first number is a multiple of the second by dividing the second number by the first, where first number is smaller than second number,
2. If there is no remainder, it is a multiple (Implicit).

I present the computational activity for EE2.1. in Table E.12:

Table E.12: The computational activity for S P1L2EE1.2

Lesson 2	EE1.2[02:52-02:43]	Input (domain)	Output (codomain)	Operation
How to check if a number is a multiple of another		\mathbb{N}	\mathbb{N}	Division

Curriculum and textbooks

See L1 EE2.1

The mathematics encyclopaedia

See L1 EE2.1

Regulation of the computational activity

See L1 EE2.1 and L2 EE1.1

School P1 Lesson 2 EE2: Lowest common multiple

Describing the computational activity

In Lesson 1 EE3.1 the teacher suggested a method for finding the lowest common multiple which closely resembled what the textbook prescribed. In this particular evaluative event she references this method once again and does an example to demonstrate it. (See Figure E.12)

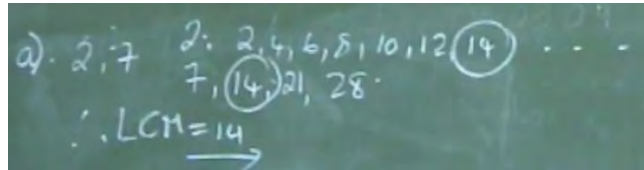


Figure E.12: Method 1: Finding the lowest common multiple of 12 and 8

After presenting this example, she discounts this method as being ‘long and tedious’ and offers another method that entails simply writing down answers for the lowest common multiple (S P1 L2 EE2 line 119). Consider the following transcript extract:

- Teacher: Right. So we did, we used this method hey, we circled it [circles on board] and we said therefore the LCM is?
- Learner: Fourteen.
- Teacher: Right. Now this is a long and tedious method but you did it yesterday. Today we’re gonna use a different method. ... So we’re just gonna write down answers. Only if you don’t have the answer, am I gonna show you the multiples, circle the common one and show you how we got the answer. [...] (S P1 L2 EE2 lines 117-119)

It seems as though the second method, in Figure E.13, suggested by the teacher is actually just students reading answers (which had been done for homework and most probably using Method 1) from their notebooks and the teacher seeing no reason to repeat all the steps for computing the lowest common multiple. In essence, then, Method 2 is not an alternative procedure but simply a more concise presentation of the answer.

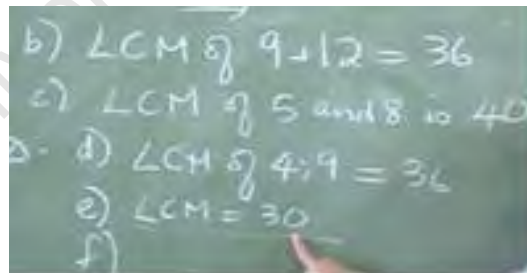


Figure E.13: Method 2: Finding the lowest common multiple

The two methods suggested by the teacher can be restated as follows and I also present a summary of the computational activity for this evaluative event in Table E.13:

Method 1: Long and Tedious (Refer to L1 EE3.1)

Finding the lowest common multiple of sets of numbers:

1. Given a set of numbers (implicitly a natural numbers), list multiples of those numbers (implicitly natural numbers).
2. Circle the lowest multiple which appears in both lists.

Method 2:

1. Present final answers from homework already completed.

Table E.13: The computational activity for S P1L2EE2

Lesson 2	EE2[04:43 – 08:44]	Input (domain)	Output (codomain)	Operation
Lowest common multiple		$a \in \mathbb{N}$ and $b \in \mathbb{N}$ $c \in \mathbb{N}$ and $b \in \mathbb{N}$	\mathbb{N}	$\times (a,b)$ Multiplication $\times (c,b)$ Multiplication

Describing the computational activity

See L1EE3.1

Curriculum and textbooks

See L1 EE3.1

The mathematics encyclopaedia

See L1EE3.1

Regulation of the computational activity

See L1EE3.1

School P1 Lesson 2 EE3: Highest common factor of two numbers

Describing the computational activity

This evaluative event commences with the teacher checking a homework exercise on finding the highest common factor of six and forty-four. It appears that this topic must have been covered already in a prior lesson/s seeing as the series of three lessons are not consecutive. She provides two methods for calculating the highest common factor (S P1 L2 EE3-EE4), one method for smaller numbers and another for bigger numbers:

Method 1: Finding the highest common factor of smaller numbers

1. Given two numbers (implicitly natural numbers), list all the factors of both numbers (implicitly natural numbers) from smallest to biggest.
2. Choose the highest factor which is common.

She used Method 1 to calculate the highest common factor of six and forty-four as can be seen in Figure E.14:

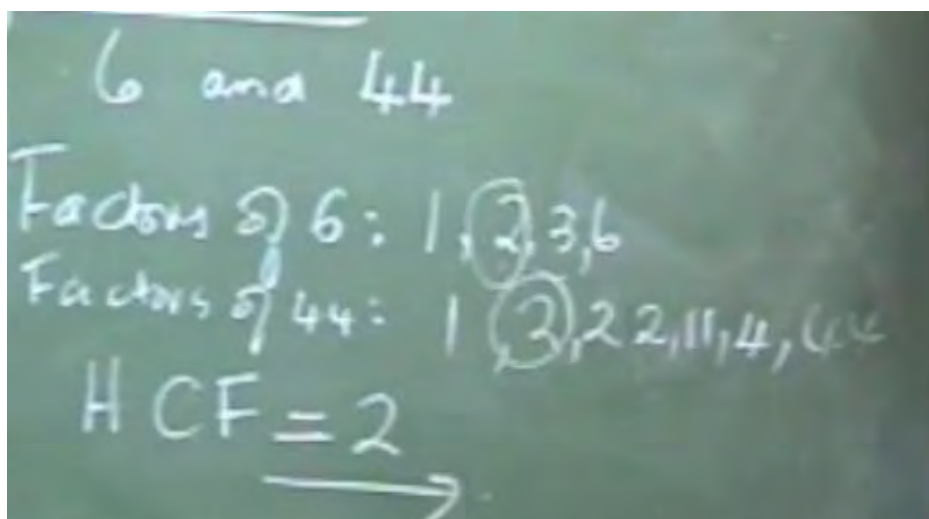


Figure E.14: Finding the highest common factor of six and forty-four

During this explanation, the teacher also provided her definitions for a 'factor' and term 'common':

Teacher: Right, firstly, do you agree that the factors of six are one, two, three and six?
 Learner: Yes
 Teacher: Right, because all of them will divide into six without giving a remainder. [...] (SP1L2EE3 lines 173-175)

OR

Teacher: Right, who still doesn't understand how to do the factors of a number? ... Your hand up?
 Learner: No, Miss.
 Teacher: Okay, we all know how to get the factors. We ask ourselves which numbers multiplied with each other give us that number or we say which numbers can divide into that number and not give a remainder. So what are the factors of forty-two? (S P1 L2 EE3 lines 215-217)

Teacher: Okay, remember people, firstly it must be common. Common means it must appear in both sets so ... (S P1 L2 EE3 line 200)

Consider the following transcript extract for why Method 2 is suggested as a means for dealing with bigger numbers:

Teacher: Now you see that the bigger the number becomes, the more factors you have ... so ... surely there must be a method where ... you're gonna take less time ... and it's actually going to be easier. Because some of you are going to skip numbers [...]. So, there must, the other method is the method that we use where we factorise a number ... into the product of its prime factors. [...] (S P1 L2 EE3 line 295)

For Method 2, only the prime factors are required, as opposed to all the factors in Method 1. The fact that the ladder method is used, implies that a knowledge of the prime numbers is required and Figure E.15 depicts this quite clearly. The other, longer Method 1 entailed listing all the factors in ascending order. It is implicit that students are using the rules of divisibility in this method. Method 1 is not encouraged by the teacher, since factors may be omitted. I now present the steps for Method 2:

Method 2: Bigger numbers (takes less time)

1. Given two numbers (implicitly natural numbers) and express as the product of its prime factors using the ladder method⁴⁵ rather than the factor tree method.
2. Express numbers as the product of prime factors by reading from the 'ladder'.
3. Identify the common prime factors in both products in order to obtain highest common factor.
4. Calculate the highest common factor from the list of common prime factors.

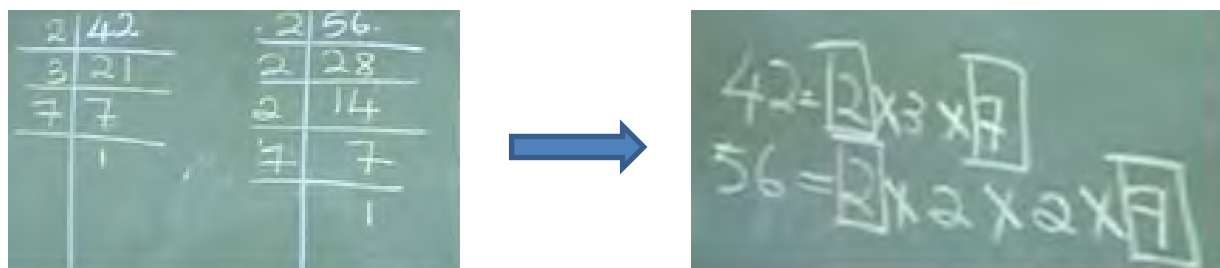


Figure E.15: Method 2 for finding the highest common factor of 42 and 56 using the ladder method

So the highest common factor of 42 and 56 is $2 \times 7 = 14$.

Consider the computational activity for Methods 1 and 2 in the Table E.14:

Table E.14: The computational activity for S P1L2EE3

Lesson 2	EE3[08:44 – 20:29]	Input (domain)	Output (codomain)	Operation
Highest common factor of two numbers				
Method 1: Smaller numbers (using the factor tree method)		N	N	Multiplication
Method 2: Bigger numbers (using the ladder method)		N	N	Division

The curriculum and textbooks

The curriculum provides no definition or method for calculating the highest common factor. Only factors, which are an essential component of the highest common factor, are referenced in the curriculum extract that follows:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them:

- [...] multiples and factors; [...] (DoE, 2002: 69)

⁴⁵ Both the ladder method and the factor tree method for obtaining prime factors of a natural number were discussed at length in EE1 of Lesson 1.

Factors are only referenced in terms of one being able to ‘recognise, classify and identify’ them rather than define them (DoE, 2002: 69). The textbook’s reference to the highest common factor is a lot more comprehensive than what the curriculum provides. It provides a ‘definition’ as well as an example with a step by step guide for how to calculate the highest common factor in Figure E.16. It is interesting that the textbook prescribes a method that does not correspond with the teacher’s. The teacher prefers calculating the highest common factor using the ‘ladder method’, i.e. Method 2, which only requires prime factors. The textbook prescribes Method 1, i.e. ‘factor tree method’ (Figure E.16), as a means for calculating the highest common factor where all the factors are listed.

Highest common factor (HCF)

The highest common factor is the largest of the common factors.

Example

Find the highest common factor of 8 and 12.

Step 1

List the factors of both numbers in numerical order.

Step 2

Locate the common factors.

Step 3

Find the highest common factor.

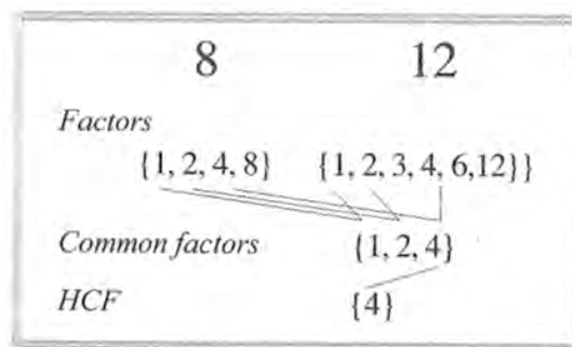


Figure E.16: Highest common factor in *Preparing for High School Maths* (Bull & Hepworth, 2008: 23)

On closer inspection of the teacher’s Method 1, which she identifies as the ‘factor tree method’ in lines 299 & 301, it is apparent that it is not really the ‘factor tree method’. The factor tree method starts with any two factors and then finds factors of factors. Method 1 in Figure E.16 starts with the first pair of factors, i.e. 1×12 , 2×6 , 3×4 , which entails order compared to the ‘factor tree method’ which does not entail any order.

The mathematics encyclopaedia

The mathematics encyclopaedia’s description of the highest common factor is similar to what is found in the textbook and what the teacher prescribes. Consider the following formal definition of highest common factor:

Highest common factor: [...] if a and b are integers, not both zero, then the greatest common factor of a and b , $\text{gcf}(a,b)$ is the unique natural number such that

1. $\text{gcf}(a,b)$ is a factor of both a and b ;
2. If d is any integer that is factor of both a and b , then d is a factor of $\text{gcf}(a,b)$.

For example, $\text{gcf}(-24;30) = 6$ and $\text{gcf}(15;-8) = 1$ (Usiskin *et al.*, 2003: 208-209)

It seems as though the *method* for computing the highest common factor, either Method 1 (factor tree method) or Method 2 (ladder method), which the teacher prescribes, is emphasised in the textbook over and above any formal definition that may be offered by the mathematics encyclopaedia.

Regulation of the computational activity

Both the teacher and the textbook prescribe ‘methods’ for calculating the highest common factor. The ‘factor tree method’ - which lists all the factors of a particular number, as well as the ‘ladder method’ - which lists only the prime factors of a number, are *computational resources* that stand in place of the formal definition of the highest common factor. The ‘ladder method’, which is the teacher’s preferred computational resource, has a structuring effect on *what the solution will look like* because of the very nature of the ladder template in Figure E.15. Consider the constitution of the highest common factor in Table E.15:

Table E.15: The constitution of the highest common factor

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Highest common factor (HCF)	Factor tree method Ladder method	Computational resource Reliance on what solution looks like	Multiplication of whole numbers	Definition of HCF Relations between factor, multiple, divisor

School P1 Lesson 2 EE4: Finding the Highest Common Factor of two numbers using the ‘ladder method’

Describing the computational activity

Finding the highest common factor of, for example, 96 and 108 (bigger numbers) is displayed in two ways – Method 1 (Figure E.17) involves listing all the factors of both numbers in ascending order while Method 2 (Figure E.18 and Figure E.19) entails expressing both numbers as the product of their prime factors.

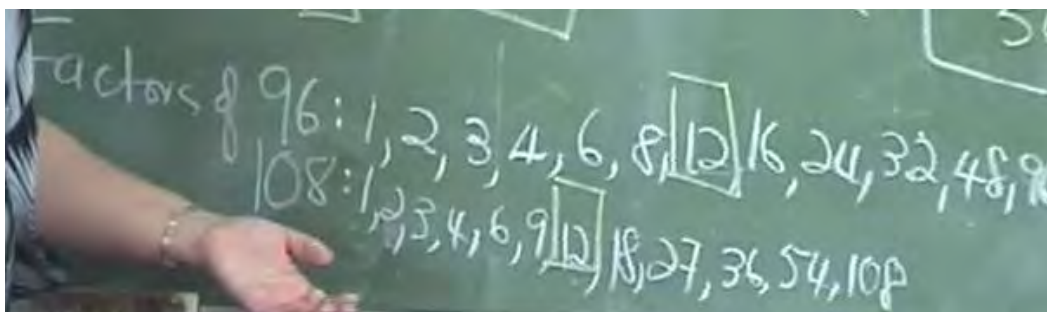


Figure E.17: Method 1: Listing all the factors of 96 and 108

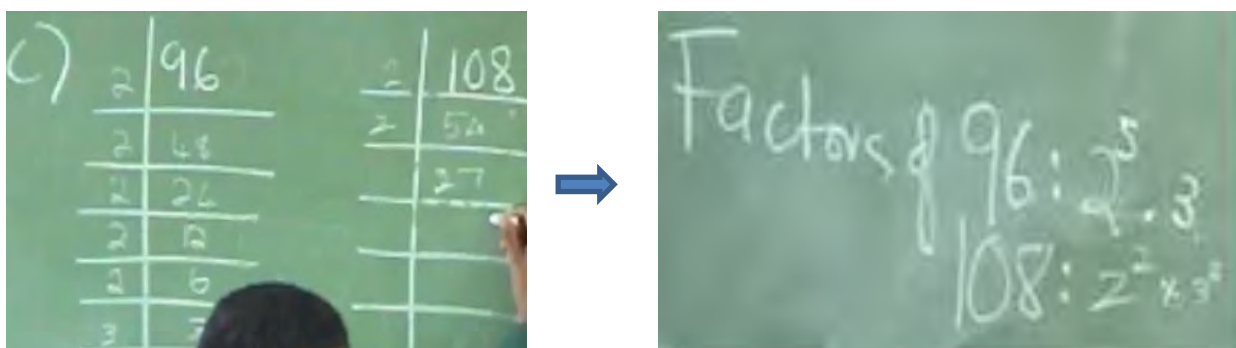


Figure E.18: Method 2: Using the ladder method to list the prime factors of 96 and 108

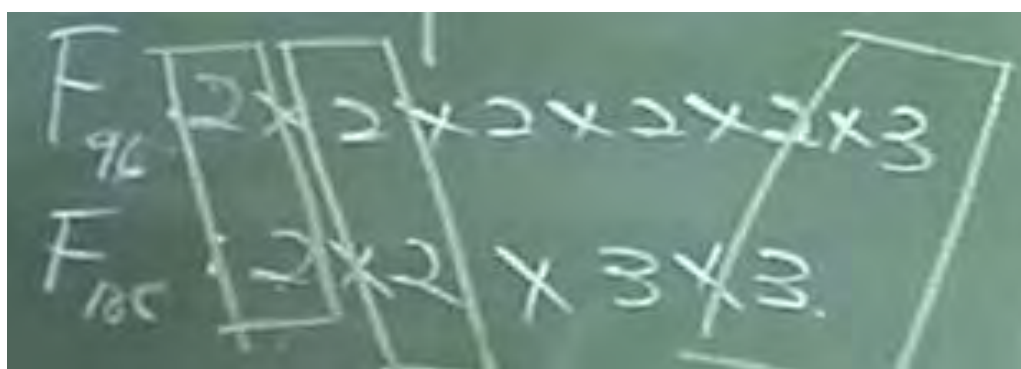


Figure E.19: Method 2: Finding the highest common factor of 96 and 108

In Method 2 the teacher emphasises the use of the ladder method of prime factorisation, as opposed to Method 1 where all the factors are listed. Method 2 is the preferred procedure that the teacher uses since it avoids the problem of learners leaving out factors which she indicates might happen if one lists the factors. Consider the factors of 108 which were listed by the teacher in Figure E.19 and that *she* has in fact left out one of the prime factors of 108. It should have read $2 \times 2 \times 3 \times 3 \times 3$ instead of $2 \times 2 \times 3 \times 3$. This is possibly just a careless error on the part of the teacher, but she is not corrected by any of the students. She still manages to compute the HCF correctly.

She draws the ladder template on the board (Figure E.18 and E.20), saying:

Teacher: Who did HCF of ninety-six and one-o-eight? Who did the highest common factor of ninety-six and one-o-eight? ... Right, let's do it. Let's use this method. Right, take out your pens ... and your page and let's do it together. And I'm coming to check to see how you do it, hey? We're going to do factors of ninety six using this method. The ladder one. [...] (S P1 L2 EE4 line 365)

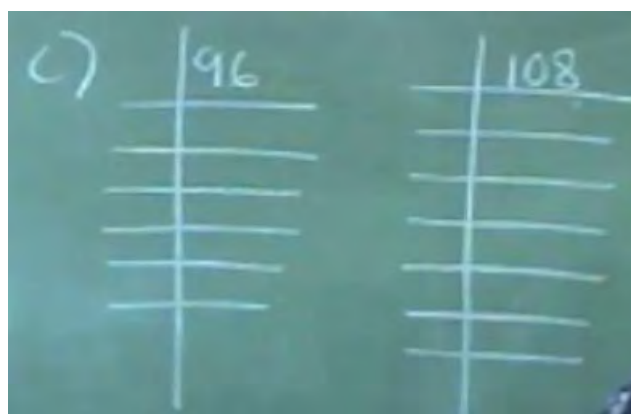


Figure E.20: Ladder templates

Consider a summary of the prime factorisations used in Method 2:

$96 = 2^5 \times 3$, $108 = 2^2 \times 3^3$, from which it is immediately apparent that $2^2 \cdot 3 = 12$ is the highest common factor,

$$\left. \begin{array}{l} 96 = 2^5 \times 3 \\ \text{i.e. } 108 = 2^2 \times 3^3 \end{array} \right\} 2^2 \cdot 3 = 12$$

The procedure described in this evaluative event can be described in more formal terms:

$$\text{Max} [F(96) \cap F(108)] = \text{HCF} (96, 108)$$

$$F(n) = \{m \in \mathbb{N} \text{ s.t. } m|n \text{ where } n \in \mathbb{N}\}$$

$$\text{So, } F(96) = \{m \in \mathbb{N} \text{ s.t. } m|96 \text{ where } n \in \mathbb{N}\}$$

$$F(96) = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96\}$$

$$F(108) = \{1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108\}$$

$$F(96) \cap F(108) = \{1, 2, 3, 4, 6, 12\}$$

$$\text{Max} [F(96) \cap F(108)] = \text{Max} [1, 2, 4, 6, 12] = 12$$

$$\therefore \text{HCF} = 12$$

The ladder method and the arrangement of symbols in the ladder template in Figure E.20 appear to be an important regulative resource for proceeding and generating prime factors of both 96 and 108. Prime factors are then listed in Figure E.18 and highest common factor is identified in Figure E.19. Method 2 is described as: ‘[...] Right, we’re going to use this method, it’s much easier. ... Easier, simpler, quicker. [...]’ (S P1 L2 EE4 line 502) compared to Method 1 where the teacher says: ‘Do you see how long this method is? And you could easily skip one or two numbers [...]’ (S P1 L2 EE4 line 497).

In elaborating the notion of the highest common factor (S P1 L2 EE4), the content that comes to be is the outcome of some computational procedure, i.e. Method 1 or Method 2, rather than the topic regulating the process upfront. The ‘factor tree method’ and the ‘ladder method’ for calculating the highest common factor stand in place of any formal definition of the object, highest common factor. Various attempts at constructing a description of what the highest common factor might be for the teacher relies on an ‘everyday language description’ when she references ‘common songs’ in the extract that follows:

- Teacher: So what have you got? Right, now ... write down the prime factors here and here and show me what’s common. Common means they appear in both. Circle. Every time a number appears in both, circle that number and see what ... (S P1 L2 EE4 line 414)
- Teacher: So if we um ... if we’re talking about um ... something that we’ve got in common. Songs that we like in common, then it’s songs that appears the most, that we both like. But in this case if we say, something is in common, something is common, then it means it appears in both. [...](S P1 L2 EE4 line 474)

The question then arises: What do these kinds of attempts at providing a definition achieve in the context of pedagogy? It seems that rather than the nature of the object regulating the computation, the computation regulates the notion of the object, highest common factor, and there is no attempt to define the object in any precise way.

I now present a summary of the procedure used in the two methods presented by the teacher as well as an overview of the computational activity in the table that follows in Table E.16:

Method 1: ‘Long’ ‘And you could easily skip one or two numbers ...’ – line 497

1. List all the factors of the given numbers from smallest to biggest.
2. Find the highest, common factor.

Method 2: ‘Easier, simpler, quicker’ – line 502

1. Draw the ladder template for both numbers.
2. Start dividing each number by the smallest prime factor (two, three, five and so on) and stop when the final answer is one.
3. Express both numbers as the product of their prime factors.
4. Circle the factor that appears in both products.
5. Identify the biggest factor from the factors common to both numbers.

Table E.16: The computational activity for S P1L2EE4

Lesson 2	EE4 [20:29-42:41]		Input (domain)	Output (codomain)	Operation
Highest common factor of two numbers using the ladder method					
Method 1: Listing all the factors to find HCF			N	N	Multiplication
Method 2: Listing prime factors to find HCF			N	N	Division

The curriculum and textbooks

See L2 EE3

The mathematics encyclopaedia

Factors – L1 EE1

Prime factors – L1 EE1

Prime numbers – L1 EE1

Highest common factor – L2 EE3

Regulation of the computational activity

L2 EE3

School P1 Lesson 3

Generating the evaluative events

Table E.17: Evaluative events spanning Lesson 3 at School P1 (S P1 L3 EE1-EE2)

Evaluative Event	Type	Length
1.1 Practical problem solving: Making a bicycle tassle [00:00 – 23:50]	Exercise	23:50
1.2 Using ‘mathematical knowledge’ – HCF to solve problem [23:50 – 35:20]	Exercise	11:30
2 Practical problem solving: Train arrival times [35:20 – 52:30]	Exercise	17:10
		52:30

School P1 Lesson 3 EE1.1: Practical problem solving: Making a bicycle tassle

Describing the computational activity

The series of lessons culminates in two practical activities set up by the teacher in Lesson 3 which focuses on an application of the highest common factor and lowest common multiple already dealt with in Lessons 1 and 2. The first ‘problem solving activity’ for making a leather bicycle tassle entails students been given three strips of paper of equal length. They were required to produce a bicycle tassle using strips of paper⁴⁶, by firstly measuring lengths of 9cm, 12cm and 18cm and then cutting the strips into shorter equal lengths so that there was no remainder. Consider the instructions given for solving the problem in the transcript extract that follows:

Teacher: [...]Number two. David has three strips of colourful leather and they measure nine centimetres, [writes on board] twelve centimetres and eighteen centimetres. Right, so he’s got three strips of colourful leather. The strips measure, one of them measure nine centimetres, another measure

⁴⁶ The strips of paper had been pre-cut into equal lengths by the teacher.

- twelve and another measure eighteen. Now, he wants to cut them into shorter, equal lengths to make a tassel to hang on his bicycle handle bars. (S P1 L3 EE1 line 36)
- Teacher: [...] What is the longest length that he can cut the strips into so that no pieces are left over? (S P1 L3 EE1 line 42)
- Teacher: [...] Now, you're going to ... you got the problem in front of you. You're going to get the strips of paper. You're going to work out, whether it is by calculation or by folding, you're going to figure out into what the longest length can be for these strips to make the tassel but there must be no pieces left over. (S P1 L3 EE1 line 44)
- Learner: [Raises his hand.] (S P1 L3 EE1 line 45)
- Teacher: And remember, all of them must be the same length. Now if you've got an answer, don't shout it out. You must convince your group that the answer that you got is the right one. Explain it to them. Show it to them, right? And then we will compare the results of each group. So every group will get these and together we must sit and figure it out. And people you must try to be involved in this problem, right? Contribute. See, and I know it's hot [opens door], ... but you must try to figure out something, okay? So somebody must take out a page maybe a calculator. You definitely need a ruler, a pair of scissors. If you don't have a scissors, fold, tear. Right, so let's see. ... Um, I'll give each group six and you can see if you can figure it out. [Hands out strips of paper.] ... Remember the question is in front of you. If you're struggling, read it again and again. [...] (S P1 L3 EE1 line 46)

So the instructions given by the teacher involve the following steps:

1. Given three strips of leather⁴⁷ with lengths of 9cm, 12 cm and 18cm.
2. Cut all the strips into shorter, equal lengths so that there is no remainder.
3. Fold and tear the strips of paper or use calculation as a resource for finding the longest length that each of these strips should measure.

Table E.18 describes the computational activity for this evaluative event:

Table E.18: The computational activity for S P1L3EE1.1:

Lesson 3	EE1.1[02:56-23:50]	Input (domain)	Output (codomain)	Operation
Practical problem solving: Making a bicycle tassel		N	N	Cut / fold / tear strips into shorter equal lengths

Curriculum and textbooks

The problem solving activity presented in this evaluative event is not found in the textbook, *Bull and Hepworth*. It is an extra worksheet, comprising of three questions, used as a resource by the teacher. The first question, not dealt with on the worksheet, requires students to calculate the lowest common multiple and highest common factor of two numbers and the next two questions are the problem solving activities discussed in this evaluative event as well as in the next evaluative event. Also see L2 EE3.

⁴⁷ The equal strips of paper represent strips of leather.

Mathematics encyclopaedia

The following entities, not found in the RNCS, are referenced in the transcript and it is generally assumed that students understand their meanings:

Remainder (*Mathworld* definition): In general, a remainder is a quantity "left over" after performing a particular algorithm. The term is most commonly used to refer to the number left over when two integers are divided by each other in integer division.

Equal lengths (*Mathworld* definition): Two quantities are said to be equal if they are, in some well-defined sense, equivalent. Equality of quantities a and b is written $a = b$.

Both the highest common factor (L2EE3) and the lowest common multiple (L1EE3.1) have been discussed in previous evaluative events.

Regulation of the computational activity

The topic to be acquired in this evaluative event is the highest common factor. A physical model is set up and students engage in folding and tearing strips of pre-cut paper to meet the specifications of the problem. Students do resort to using calculation procedures to find the longest length that each of the strips should measure so that there is no remainder. In this evaluative event students don't have the slightest inkling that finding the highest common factor of the three values might assist them in finding the required solution.

School P1 Lesson 3 EE1.2: Using 'mathematical knowledge' – HCF to solve problem

Describing the computational activity

The teacher encouraged students to use folding and tearing the strips of paper or calculations as a resource for finding the longest length that each of these strips should measure. After approximately thirty minutes of students engaging in setting up a physical model, and suggesting various methods for finding the solution the following interaction ensues:

Learners: Miss, you add it all up.
Teacher: You add it all up.
Learner: Add it all up and divide by thirteen.
Teacher: By thirteen?
Learner: Huh?
Learner: By three.
Learner: Divide by thirteen, you get three, Miss.
Teacher: Okay, let's see. That's what? Twenty-one and thirty-nine divide by ...
Learner: Three
Teacher: Thirteen. Why did we divide by thirteen?
Learner: Then you get ... then you get. [Inaudible]
Teacher: But why did you choose thirteen to divide it by?
Learner: Because all together you get thirteen pieces. Miss.
Teacher: But how, but how did you know you must get thirteen pieces?
Learner: Miss, because we ... [Inaudible]
Learners: Miss, miss, miss.
Teacher: Because you divide it by three. That's why you got to thirteen. Okay, so you divide.
What can we use? What have we been doing lately people? We've been?
Learners: The LCM, Miss.

Teacher: Finding the LCM and the HCF.
 Learner: Highest common factor.
 Teacher: Highest common.
 Learners and Teacher: Factor (S P1 L3 EE1.2 lines 196-217)

Because this is a practical activity, any accomplishment with a reasonable solution is acceptable. In this instance, the idea that the piece of leather needs to be cut into equal, shorter lengths results in a spontaneous response on the part of the students to divide. The content which the teacher is after i.e. highest common factor and the criteria associated with the realisation of this mathematical content is not dealt with explicitly, rather it is meant to emerge as an outcome of the activity. It is interesting to note that four different methods are presented for achieving the solution:

Method 1: (Learner) – line 178

Use division

Method 2: (Learner) – line 196

Add up all the lengths and divide by three.

Method 3: (Teacher) – line 214 & 215

Find the highest common factor.

Method 4: (Teacher) – line 370 (see the transcript extract that follows)

1. Cut strips into 3cm each.
2. Present answer using 'English' rather than 'mathematics' by providing a detailed explanation

Teacher: So where's your method? How'd you get to this? ... Right people, shh, if you did not use, Darren, are you listening? If you did not use this method but in words you said, cut the strip into pieces to measure three centimetres each etcetera, then you write that as your answer, right? If you didn't do it mathematically like this but you solved it using English ... then that's fine but then I want to see that answer. Don't just tell me three and twenty-four. I want to see how you arrived at your answer. (S P1 L3 EE1.2 line 372)

Table E.19 is an overview of the computational activity of the problem solving activity that was meant to be an application of the topic, highest common factor:

Table E.19: The computational activity for S P1L3EE1.2:

Lesson 3	EE1.2[23:50-35:20]	Using 'mathematical knowledge' – HCF to solve problem	Input (domain)	Output (codomain)	Operation
HCF-bicycle tassle problem	Method 1		N	N	Division
	Method 2		N	N	Addition and division
	Method 3		N	N	Division
	Method 4		N	N	'using English'

Curriculum and textbooks

See L2 EE3

See L3 EE1

Mathematics encyclopaedia

Highest common factor – L2 EE3

Prime factors – L1 EE1

Factors – L1 EE1

Regulation of the computational activity

This evaluative event, where the topic to be acquired is the highest common factor, is regulated by both physical means, where students devise a model to arrive at the answer, as well as calculation i.e. addition and division. When the teacher reminds students of topics they have just covered, they realise that the highest common factor could possibly assist them. These calculations could be classified as *computational resources*. However, there is also an appeal by the teacher to solve the problem using ‘English’ rather than ‘mathematically’ (S P1 L3 EE1.2 line 370) and this evaluative event could also be categorised as being regulated by *general descriptions of mathematical concepts* rather than precise, formal definition. The constitution of the highest common factor has already been presented in L2 EE3 in Table E.14.

School P1 Lesson 3 EE2: Practical problem solving: Train arrival times

After solving the bicycle tassle problem in EE1.2, and the teacher alluding to the link between this particular problem solving activity with the one they had just covered in questions based on the lowest common multiple and highest common factor (lines 210-212), she reads another question from the same worksheet. The first question (not covered in this lesson) on the worksheet required students to calculate the lowest common multiple and highest common factor of numbers and the second problem was the bicycle tassle problem solving activity discussed in EE1.2. The third problem solving activity involved different train arrival times at Park Station. A train from Soweto arrived at Park Station every 6 minutes while a train from Springs arrived every 8 minutes. Students were faced with the problem of calculating how long it would take for the two trains to arrive at Park Station simultaneously if this had just happened. Immediately after the teacher had presented the question, a student shouted out an answer. The transcript extract that follows shows that he correctly recognised that the lowest common multiple was the means for solving the problem:

Teacher:	Number three. I want you all to look at your page ... or listen. A train from Soweto arrives at Park Station every six minutes. ... The train from Soweto arrives every six minutes. ... The train from Springs arrived every eight minutes. Right, so there's two trains arriving at Park Station; one from Soweto, one from Springs. The one from Soweto comes every six minutes, the one from Springs every eight minutes. Now the trains from Soweto and Springs have just arrived at
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the same time, right now, they've both come to the same station. How long will it be before this is going to happen again when they both come?

Learner: Twenty-four minutes, Miss.

Teacher: Twenty-four minutes. How'd you get your answer?

Learner: Yes, Miss.

Another Learner: Right.

Teacher: You say after twenty-four minutes and they'll be there again. Why?

Learner: It's the lowest common multiple.

Teacher: Right, because now we have found the lowest common multiple. Okay. So in your maths notebooks, let's write, let's write down these sums. [Cleans board.] ... Your heading, highest common factor, lowest common multiple. (S P1 L3 EE1 lines 310 – 317)

Table E.20 presents an overview of the computational activity of the train arrival problem solving activity that was meant to be an application of the topic, lowest common factor:

Table E.20: The computational activity for S P1L3EE2:

Lesson 3	EE2[35:20-52:30]	Input (domain)	Output (codomain)	Operation
Practical problem solving: Train arrival times		$a \in \mathbb{N}$ and $b \in \mathbb{N}$	\mathbb{N}	$\times (a,b)$ Multiplication
		$c \in \mathbb{N}$ and $b \in \mathbb{N}$		$\times (c,b)$ Multiplication

Curriculum and textbooks

See L2 EE3

See L3 EE1

Mathematics encyclopaedia

Lowest common multiple – L1 EE4

Prime factors – L1 EE1

Factors – L1 EE1

Regulation of the computational activity

A student gave the correct answer immediately after the teacher presented this problem solving activity and stated that the lowest common multiple was the means of regulation for arriving at the answer. The constitution of the lowest common multiple has already been presented in L1 EE3.1 Table E.8.

I have presented an overview of the constitution of mathematics for Lessons 1, 2 and 3 for School P1 by presenting the computational activity for each evaluative event as well as a synopsis of how this relates to what is found in the RNCS, the textbook used and the encyclopaedia of mathematics. I have also discussed the final stage of my analytic framework which describes the regulation of the computational activity using the schema of analysis developed in Chapter Four.

Appendix F: Analysis for School P2

School P2 Lesson 1

Generating evaluative events

Table F.1: Evaluative events spanning Lesson 1 at School P2 (S P2 L1 EE1-EE2)

Evaluative Event	Type	Length
01 Plotting co-ordinates [00:00 – 18:26]	Expository	18:26
02 Plotting co-ordinates (Homework) [18:28 – 21:52]	Exercise	03:24 21:50

School P2 Lesson 1 EE1: Plotting co-ordinates

Describing the computational activity

Lesson 1 commenced with the teacher confirming that students were already competent in identifying and reading off points in the Cartesian plane, stating that they were about to learn how to plot points:

- Teacher: [...] Okay. We already did the co-ordinate, nhe? {right?} You know how to read ordered pairs in the Cartesian plane, nhe? {right?} (S P2 L1 EE1 line 10)
- Teacher: Because we already know how to read, nhe? {right?} in the Cartesian plane. Now I want you to plot in the Cartesian plane [...] (S P2 L1 EE1 line 16)

Thereafter, a procedure for plotting points was presented quite explicitly by the teacher in the following transcript extract and Figure F.1:

- Teacher: [...] you always start by x because it is an order pair, nhe? {right?}, so you must start by x always and follow, y nhe? {right?} (S P2 L1 EE1 line 20)
- Teacher: Here is the point, nhe? {right?}, point A has two and one, that means the values are two and one, and that two is for what? Which value? x and y , nhe? {right?} So this is x and y . You must plot there, the values of x , the value of x is two and one, and then one, where is one? Where they meet in the point. Where? (S P2 L1 EE1 line 21)



Figure F.1: A set of co-ordinates (S P2 L1 EE1)

This procedure presented by the teacher entailed a selection of the first number to represent x and the second number to represent y . She says: '[...]Where they meet in the point. [...] (S P2 L1 EE1 line 21)' to signal the point of intersection of the line $x = 2$ and $y = 1$. She does not indicate that $x = 2$ and $y = 1$ represent linear functions and that the point of intersection of these two straight lines is the point $(2; 1)$. She uses the grid drawn on the chalkboard, indicates where $x = 2$ and $y = 1$ is and traces the intersection of the two 'lines' on the board.

So the gesture ‘tracing’ and ‘counting’ is used to substitute for the above. Plotting points, it seems, is constituted as counting with the co-ordinate pair specifying horizontal displacement and vertical displacement from the rigid.

The procedure for reading and plotting points in the transcript extracts presented can be re-stated as follows:

Procedure for reading points:

When reading a co-ordinate pair, the first co-ordinate represents x and the second y .

Procedure for plotting points:

1. Read the co-ordinate pair, the first co-ordinate representing x and the second y .
2. The position ‘where they meet in’ (S P2 L1 EE1 line 21) the Cartesian plane is the point, actually signaling the point of intersection of the straight lines $x = 2$ and $y = 1$.

Procedure for joining points:

Join the points plotted in alphabetical order to form a shape.

Even though the procedure for plotting points was spelt out quite clearly, confusion still arose later when plotting points:

- Teacher: ... we see that we’ve done a mistake, nhe? {right?}, of mixing x and y , don’t mix x and y so you must start by x always and follow y , nhe? {right?}’ (S P2 L1 EE1 line 33-35)
- Teacher: So the first number is x which is two and the second number is y which is one, and what else? Most of you, you mix x and y . Any question so far?’ (S P2 L1 EE1 line 44)

Table F.2 presents an account of the computational activity for the procedure for plotting points by describing the objects that are arguments (input or domain) for operations and the associated values (output or codomain):

Table F.2: The computational activity for S P2L1EE1

Lesson 1	EE1[00:00 – 18:26]	Input (domain)	Output (codomain)	Operation
Plotting co-ordinates				
E.g. Plotting (2; 1)		$x \in \mathbb{Z}$, $y \in \mathbb{Z}$	$(x; y)$ where $x, y \in \mathbb{Z}$	<u>Operation/operation-like manipulation:</u> <ol style="list-style-type: none"> 1. Selection of first number – x 2. Selection of second number – y 3. Count 2 from the origin as indicated by the value of x of the co-ordinate pair. 4. Count 1 from the origin as indicated by the y-value of the co-ordinate pair. 5. Trace the lines from $x = 2$ and $y = 1$. 6. The point (2; 1) is where the lines meet.

Curriculum and textbooks

After reflecting on the classroom interaction on how the notion of a point in a Cartesian plane is presented, it is interesting to reflect on the RNCS' account of how this topic should be constituted:

Learning Outcome 3 SPACE AND SHAPE (GEOMETRY)

The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions.

We know this when the learner:

Uses transformations (rotations, reflections and translations) and symmetry to investigate (alone and/or as a member of a group or team) properties of geometric figures.

Locates positions on co-ordinate systems (ordered grids), Cartesian plane (first quadrant) and maps, and describes how to move between positions using:

- horizontal and vertical change;
- ordered pairs;
- compass directions. (DoE, 2002: 83)

The curriculum does provide a certain amount of clarity in terms of what it requires that learners should know on this topic i.e. 'Locates positions on co-ordinate systems (ordered grids), Cartesian plane (first quadrant) [...]' by making use of 'ordered pairs' (DoE, 2002: 83). The textbook also provides a description of an ordered pair in Figure F.2, i.e., 'x is always followed by y' but great emphasis is placed on the procedure for plotting the ordered pair in Figure F.3, rather than providing clarity on its formal definition:

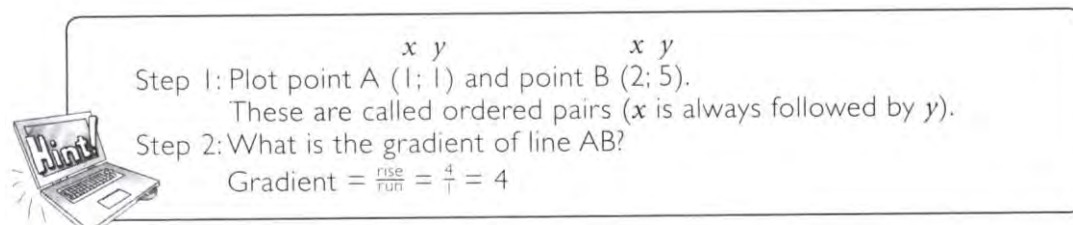


Figure F.2: The definition of an ordered pair in *Oxford Successful Mathematics* (du Preez *et al.*, 2006: 202)

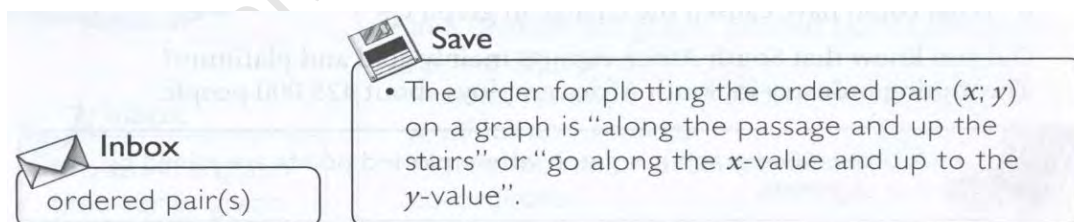


Figure F.3: Plotting an ordered pair in *Oxford Successful Mathematics* (du Preez *et al.*, 2006: 203)

The 'order for plotting' in Figure F.3 really refers to which number represents x and which one represents y (du Preez *et al.*, 2006: 203). Figure F.3 also describes plotting an ordered pair $(x; y)$ as 'along the passage and up the stairs' which may serve as substitutes for lines $x = 2$ and $y = 1$.

Now consider the following exercise extract from the textbook, *Oxford Successful Mathematics*, which students worked on in class:

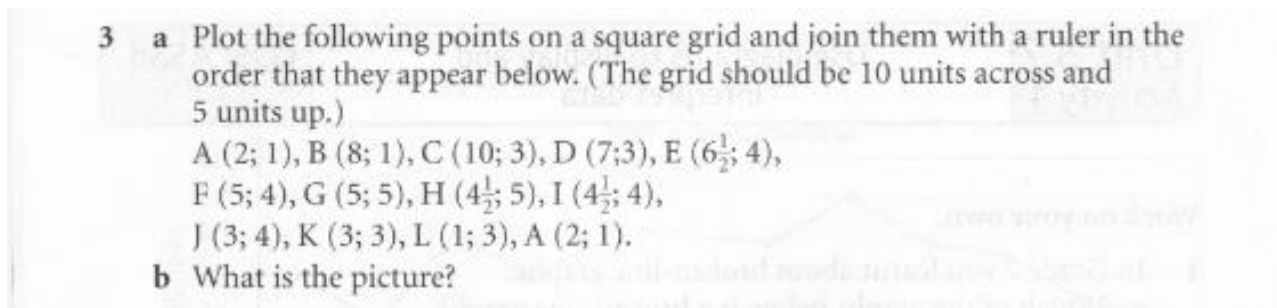


Figure F.4: An exercise on plotting points in *Oxford Successful Mathematics* (du Preez *et al.*, 2006: 203)

These textbook extracts are helpful in illuminating how it is that the central object of concern, a point in a plane, emerges. The definitions provided in Figure F.2 and Figure F.3 emphasise procedures for realising the topic to be acquired. Once points have been plotted in the exercise in Figure F.4, the next question in this exercise asks ‘What is the picture?’ There seems to be an implicit idea that the drawing of a boat in Figure F.5 is the desired outcome rather than the mathematical objects announced.



Figure F.5: The final picture (S P2 L1 EE1)

If the picture was a marker for both the student and the teacher that the criteria for producing the required solution had been achieved, rather than the notion of a Cartesian plane being saturated with an infinite number of points, it may not be uncommon for students to unintentionally swap x and y co-ordinates in Figure F.6 and still produce a valid shape i.e. a reflection in the line $y = x$ of the intended shape.

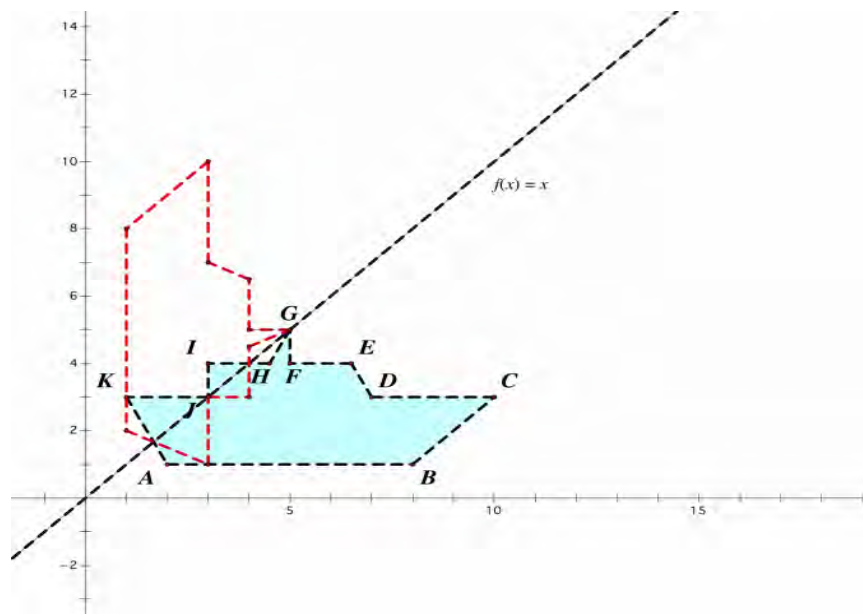


Figure F.6: An unintended swapping of x and y co-ordinates

The existential nature of the object, a point in the Cartesian plane, is not defined in any precise way when examining the textbook extracts (Figures F2 and F.3) as well as in the teacher's attempt at providing a definition of a point in a Cartesian plane (S P2 L1 EE1 line 20 -21). Rather, the object emerges as the outcome of a procedure, i.e. where x and y meet. Even though the procedure was re-iterated by the teacher it still presented problems later on when plotting points.

Mathematics encyclopaedia

Neither the textbook, nor the curriculum provided a formal definition of a set of points / co-ordinates in the Cartesian plane as can be found in the mathematical encyclopaedia:

Cartesian plane/Cartesian point/Ordered pairs :

[...] the Cartesian plane is the set of \mathbb{R}^2 of all ordered pairs of real numbers. Accordingly, a Cartesian point is an ordered pair of real numbers. A Cartesian line is the set of points $(x;y)$ that satisfy an equation equivalent to a linear equation in the standard form $Ax + By + C = 0$, where A and B are constant real numbers that are both not zero. (Usiskin *et al.*, 2003: 571 – 572)

Co-ordinate:

[...] the most common coordinate system is the rectangular (Cartesian) coordinate system, [...] by using x and y as variables for the rectangular coordinates [...] (Usiskin *et al.*, 2003: 88)

The Cartesian plane represents a mapping from \mathbb{R} to \mathbb{R} and it is interesting that definitions of ordered pairs or co-ordinates found in the encyclopaedia make no reference to methods of computing as is prevalent in the textbook 'definitions'.

Regulation of the computational activity

It appears that the symbolic representation of the points in the Cartesian plane, i.e. the shape of the boat, seem to regulate the mathematics more than formal definitions of co-ordinates or ordered pairs found in the mathematics encyclopaedia. This would imply that there is a *reliance on what the solution may look like*. *Computational*

resources, i.e. the procedure for plotting points, for realising the required content are equally important in this evaluative event and, this, to a large extent regulates the way both the textbook and the teacher present the topic. Table F.3 provides an account of how the concept of a co-ordinate is constituted by providing a summary of procedure used, the content realised through the procedure, the means of regulation as well as the content substituted by the procedure for this topic:

Table F.3: The constitution of a co-ordinate

Announced topic	Procedures used	Means of regulation	Realised content	Substitutes for
Plotting co-ordinates	<ol style="list-style-type: none"> 1. Selection of first number – x 2. Selection of second number – y 3. Count x units from the origin as indicated by the value of x in the co-ordinate pair. 4. Count y units from the origin as indicated by the y-value of the co-ordinate pair. 5. Trace the lines from $x = a$ and $y = b$. 6. The point $(a; b)$ is where the lines meet. 	<p>Computational resource</p> <p>A reliance on what solution may look like</p>	<p>Order of plotting points</p> <p>Pictorial representation</p>	Definition of a co-ordinate

School P2 Lesson 1 EE2: Plotting co-ordinates (Homework)

Describing the computational activity

The board extract in Figure F.7 depicts the homework exercise given to students to copy in their notebooks and complete using the procedure for plotting points, discussed in EE1:

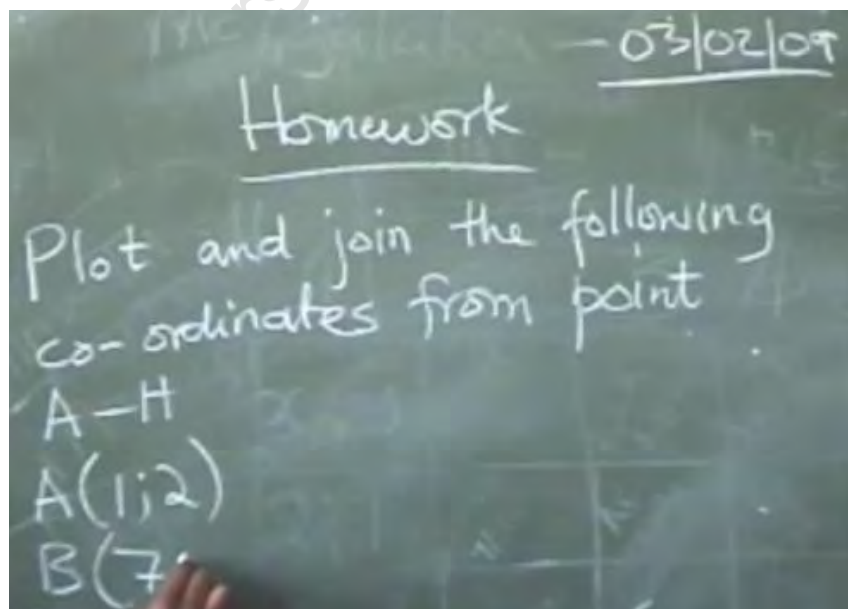


Figure F.7: Homework: Plotting points and joining them

I now present an account of the computational activity in Table F.4:

Table F.4: The computational activity for S P2L1EE2

Lesson 1	EE2[18:28 – 21:52]	Input (domain)	Output (codomain)	Operation
Plotting co-ordinates (Homework)		$x \in \mathbb{Z}, y \in \mathbb{Z}$	(x, y) where $x, y \in \mathbb{Z}$	<u>Operation/operation-like manipulation:</u> <ol style="list-style-type: none"> 1. Selection of first number – x 2. Selection of second number – y 3. Count x units from the origin as indicated by the value of x in the co-ordinate pair $(a; b)$. 4. Count y units from the origin as indicated by the y-value of the co-ordinate pair $(a; b)$. 5. Trace the lines from $x = a$ and $y = b$. 6. The point $(a; b)$ is where the lines meet.

Curriculum and textbooks

See L1 EE1

Mathematics encyclopaedia

Co-ordinates: See L1 EE1

Point: See L1 EE1

Regulation of the computational activity

See L1 EE1

The homework exercise given to students in this evaluative event is regulated by the *computational procedure* for plotting points discussed in EE1 as well as the instruction to join points A to H in alphabetical order. Lesson 2 discusses the regulation of this homework exercise in more detail.

School P2 Lesson 2

Generating evaluative events

Table F.5: Evaluative events spanning Lesson 2 at School P2 (S P2 L2 EE1)

Evaluative Event	Type	Length
01 Plotting and joining co-ordinates [00:00 – 21:21]	Expository	21:21
The entire lesson is spent on plotting points and drawing picture.		21:21

School P2 Lesson 2 EE1: Plotting and joining co-ordinates

Describing the computational activity

This evaluative event focuses on checking the previous day's homework (S P2 L1 EE2) which covered plotting and joining eight co-ordinate points. Eight students came up to the board, each plotting a point in Figure F.8:

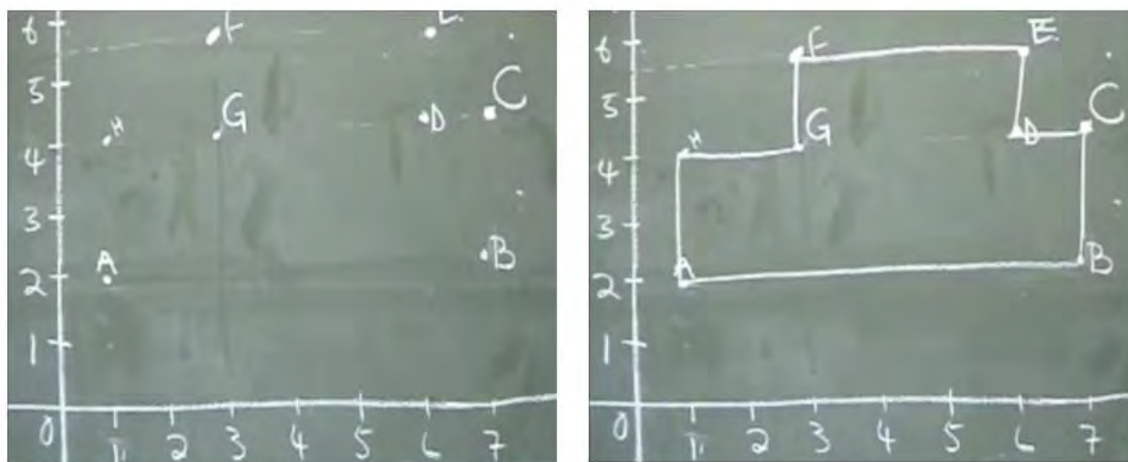


Figure F.8: Joining the dots

The procedure provided by the teacher could be stated as follows:

1. Plot co-ordinate pairs, the first co-ordinate represents x and the second y .
2. Join the co-ordinate pairs in alphabetical order to form a shape, noting that the grid will ensure accuracy (line 44).

Now consider the computational activity of this evaluative event in Table F.6:

Table F.6: The computational activity for S P2L2EE1

Lesson 2	EE1[00:00 – 21:21]	Input (domain)	Output (codomain)	Operation
Plotting and joining co-ordinates		$x \in \mathbb{Z}, y \in \mathbb{Z}$	$(x; y)$ where $x, y \in \mathbb{Z}$	<u>Operation/operation-like manipulation:</u> <ol style="list-style-type: none"> 1. Selection of first number – x 2. Selection of second number – y 3. Count x units from the origin as indicated by the value of x of the co-ordinate pair $(a;b)$. 4. Count y units from the origin as indicated by the y-value of the co-ordinate pair $(a;b)$. 5. Trace the lines from $x = a$ and $y = b$. 6. The point $(a; b)$ is where the lines meet.

Towards the end of this evaluative event at 20:17 the following interaction regarding plotting co-ordinates ensued:

Teacher: [...] Ok class! We finish doing co-ordinates, nhe? {right?} For tomorrow, for the lesson tomorrow will do transformation, nhe? {right?} (S P2 L2 EE1 line 112)

Teacher: Tomorrow we will do transformation. We finish with co-ordinates. That is all. (S P2 L2 EE1 line 113)

It is interesting that the teacher expresses that transformations are unrelated to the concept of a co-ordinate, stating that they are separate entities. Perhaps this is why students often need to be re-taught the same topics repeatedly and are unable to understand the interrelatedness of the concepts in mathematics.

Curriculum and textbooks

See L1 EE1

Mathematics encyclopaedia

Co-ordinate: See L1 EE1

Point: See L1 EE1

Regulation of the computational activity

The means of regulation in this evaluative event relies on choosing the appropriate procedure for plotting points. So the *computational resource* for plotting points provided by both the teacher and the textbook, *Oxford Successful Mathematics* (du Preez *et al.*, 2006: 202-203), is key in regulating how this content is realised for

students. In both Lesson 1 and Lesson 2, the procedure for plotting points replaces the formal definition of a co-ordinate pair in a Cartesian plane.

School P2 Lesson 3

Generating evaluative events

Table F.7: Evaluative events spanning Lesson 3 at School P2 (S P2 L3 EE1-EE2)

Evaluative Event	Type	Length
01 Transformation Geometry - translations [00:00 – 08:49]	Expository	08:49
02 Translating an object three units right [08:49 – 19:12]	Expository	10:23
		19:12

School P2 Lesson 3 EE1: Transformation geometry – translations

Describing the computational activity

Consider the following extract from Lesson 3 on Transformation Geometry, specifically translations, where a pedagogic space is set up by the teacher for students to encounter a mathematical concept by being presented with a very general description of the topic, ‘translations’. It differs from the lesson on multiples in School P1 (Appendix E Evaluative Event 2.1) where no description was provided initially. Rather, the notion of a ‘multiple’ was meant to emerge as a result of calculation. The evaluative event that follows provides a different approach for how teachers reference mathematical objects and processes in the absence of its formal definition:

- Teacher: What do you know about transformation? What is the meaning of transformation? Can you tell me? When we are talking about transformation, think about that word. What is the meaning of that? Hmm? Transformation, hmm? Anyone? Transformation? Do you want to try? Anyone? Do you want to try? [Learners do not respond to any of the teacher’s questions.] No one? Ok! Transform! When we are talking about transform what is the meaning of that class? In English, transform? Hmm? Okay. Okay. Transformation is a change, nhe? {right?} It is a change in position. Transformation is a change in position. If I am moving here and I move to another position, nhe? {right?}
- Learners: Yes, madam. [Class choruses.]
- Teacher: So it is transformation, nhe? {right?} Okay! It is a change, you can write down the meaning of transformation. [Teacher writes the ‘definition’ on the board, while reading from her notes, and instructs students to write it down.] It is a change in position, shape and or size nhe? {right?} So the object changes in position, size and shape. So there are three types of transformation, therefore three types, nhe? {right?} there is one, nhe? {right?} reflection, you know one of the types of transformation? Okay! What is it?
- Learner: Rotation.
- Teacher: Ok, rotation. Good. And what else?
- Learners: Translation.
- Teacher: Hmm? (S P2 L3 EE1 lines 1-13)

The teacher provides a definition of a transformation in Figure F.9 as a ‘change in position’ (and so excludes null transformations) and uses herself as an illustration of an object that undergoes a change in position when she

says: ‘If I am moving here and I move to another position, nhe? {right?}’ (S P2 L3 EE1 line 1). She makes no reference to what will be transformed when she provides a ‘definition’ of transformation on the board in Figure F.9.

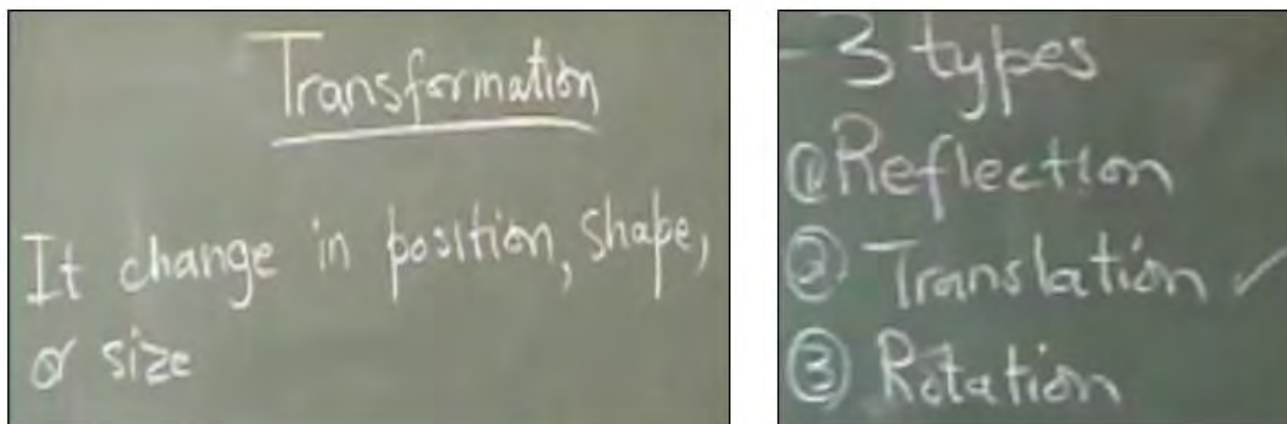


Figure F.9: A ‘definition’ of transformation (S P2 L3 EE1)

The short extract from this evaluative event (S P2 L3 EE1 lines1-8) just presented provides an interesting insight into how, in this instance, a teacher deals with the mathematical concept of transformation which she describes generally in terms of its properties only. This description could refer to a transformation of anything. Her description of translation resonates with metaphorical moving and the Cartesian plane is used to discuss the movement of the object. A pedagogic context is thus set up for learners to encounter general descriptions of mathematical objects or processes with little, if any reference to its formal mathematical properties. The teacher references mathematical objects in the same way that she would reference everyday objects and the outcome or desired solution is dependent on the activity of the learner, rather than stating the existential features of mathematical objects and processes upfront.

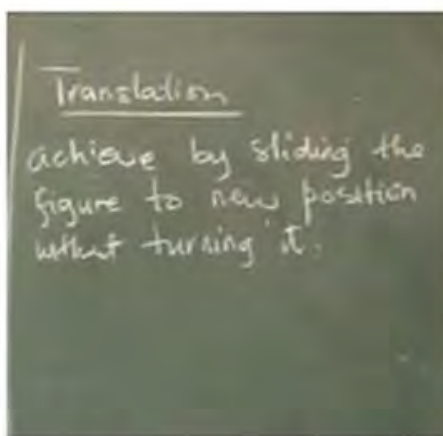
In this evaluative event there definitely is an attempt to provide a ‘definition’, but it remains vague, even though its intended purpose, possibly, was to remove any ambiguity. In order to further illustrate the concept of the first type of transformation, i.e. translation, the teacher provides a general description of translation (S P2 L3 EE1 lines 13-14) and uses of a board duster to trace the movement of an object (Figure F.10):

Teacher: When we are talking about translation. Put down the pen, nhe? {right?}, and listen. Translation. Put down the pen and try to listen, nhe? {right?}. Translation is when you achieve, ah, is when you slide an object, in fact, when you slide an object from one position to another position, without changing, ah. Without changing any shape of the object, nhe? {right?} Like if I got this duster here and then I want to move this duster to another position. Maybe I want to move it down nhe? {right?} I’m take it, straight down, nhe? {right?} I want change the position of duster, so it’s therefore translation. Don’t change anything in there. Translation, so you achieve by sliding the figure to a new position, nhe? {right?} without turning it. Are we together?

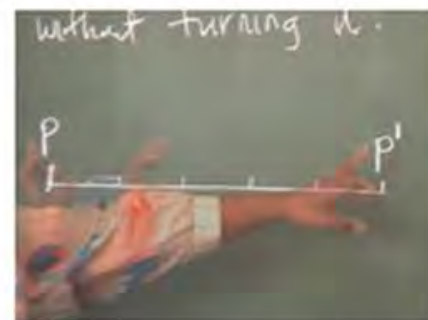
Learners: Yes [Learners’ chorus.] (S P2 L3 EE1 lines 20-23)



Physical action



Linguistic description of physical action



Attempted symbolic description accompanied by physical action

Figure F.10: An attempted description of the act of translating an object (S P2 L3 EE1)

The various attempts at providing a definition in Figure F.10 are generally procedures for achieving a translation rather than a formal definition. It is interesting that the teacher perceives translation as only a vertical or a horizontal movement: 'Translation, so you achieve by sliding the figure to a new position, the? {right?} without turning it. Are we together?' (S P2 L3 EE2 lines 13) In essence, then, when translating an object, the object slides up or down or left or right by a certain number of units without changing the orientation of the object. This could also have been explained by the teacher as each ordered co-ordinate pair being transformed by certain number of units but this is not how she expresses herself.

The teacher provides a procedure for achieving translation rather than providing a definition of this concept and the attempted description in Figure F.10 resounds with Tall and Vinner's (1981) description of a *concept image*. The concept image consists of all the cognitive structures associated with the concept and may include mental images and associated properties which do not necessarily correspond to the *concept definition*. Tall and Vinner (1981: 153) distinguish between a *personal* concept definition and a *formal* concept definition, 'the latter being a concept definition which is accepted by the mathematical community at large.' A combination of the teacher's physical action, the linguistic description and an attempted symbolic action of translation in Figure F.10 resounds with Tall and Vinner's *personal concept definition* rather than their formal concept definition. Tall (2009: 7) provides a *formal concept definition* of translation: 'a translation of an object on a plane is an action in which each point is moved in the same direction by the same magnitude' which is at odds with the definition provided by the teacher in her linguistic description in Figure F.10. Tall's definition also differs from that of the mathematics encyclopaedia (see later discussion). He refers to 'action' rather than function. So if $A(x_1; y_1)$ is

translated by five units to the right and two units down, then the co-ordinates of the image point A' ($x_1 + 5$; $y_1 - 2$). The point A does not physically move.

I now present an account of the computational activity of the evaluative event just discussed in Table F.8:

Table F.8 The computational activity for S P2L3EE1

Lesson 3	EE1[00:00 – 08:49]	Input (domain)	Output (codomain)	Operation
Transformation geometry - translations		$(x;y)$ where $x, y \in \mathbb{Z}$	$(x \pm m; y \pm n)$ where $x, y, m, n \in \mathbb{Z}$	<u>Operation/operation-like manipulation:</u> <ol style="list-style-type: none"> 1. Identify where to start. 2. Count the number of spaces that the object has to ‘move.’ 3. Move the object so that the space between the displaced object and original object equals the translated space. <p>Teacher: The question will comes and says that translate that P by 5 units to the right. So you are here, this is your starting point and count 5 units to the right <u>nhe?</u> {right?} Count.</p> <p>Learners & Teacher: 1 2 3 4 5</p> <p>(S P2 L3 EE1 lines 15-16)</p>

Curriculum and textbooks

See L1EE1 for an account of what the RNCS prescribes with regard to translations.

The textbook, *Preparing for High School Maths*, was used in addition to *Oxford Successful Mathematics*. A definition of translation was provided and, like the definition provided by the teacher in Figure F.9, the idea of what a translation might be was described in very general terms – unlike what was found in the mathematical encyclopaedia. This definition in Figure F.11 was immediately followed by an example of how to apply the concept.

Translation (straight-line movement)

The shape is shifted in a direction with all corners moving the same distance. The shape is unchanged.

Example:

Translate the shape shown 4 units horizontally to the right.

The original shape (object) has been shifted 4 units to the right without changing shape.

The best way to translate a shape is to move the bottom corner left 4 units and then draw in the rest of the shape.

The original shape, also known as the object, has been shifted 4 cm to the right without changing shape.

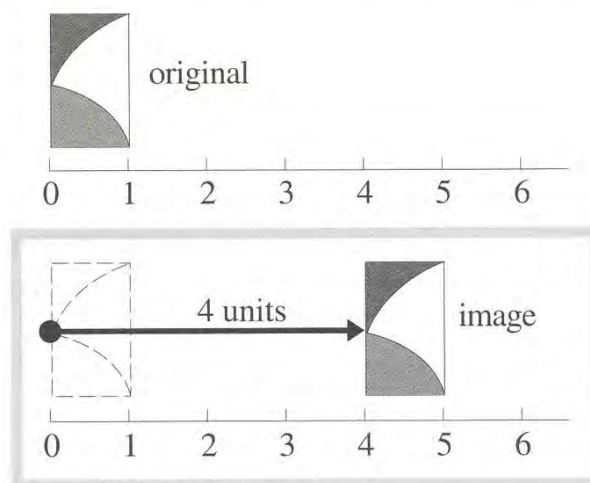


Figure F.11: The definition of translation in *Preparing High School Maths* (Bull & Hepworth, 2008: 120)

Consider Figure F.11 which describes a method for translating the shape four units to the right. It suggests that ‘(t)he best way to translate a shape is to move the bottom corner left 4 units and then draw in the rest of the shape’ (Bull & Hepworth, 2008: 120). Perhaps this is the source of the teacher’s confusion since the textbook does not stipulate which bottom corner is being referred to.

Mathematics encyclopaedia

The mathematics encyclopaedia provides definitions of the concepts in Transformation Geometry in a manner very different to what is presented in textbooks, pedagogy and the curriculum. Consider the following formal definitions found in the mathematics encyclopaedia:

Transformation:

A transformation of the plane or plane transformation is a 1-1 function with the plane as its domain and codomain. (Usiskin *et al.*, 2003: 298)

When T is a transformation and $T(\alpha) = \beta$ we say the transformation maps α to β . We call α the preimage and β the image. [...] In the Cartesian plane, a transformation of the plane is a 1-1 mapping from \mathbb{R}^2 onto \mathbb{R}^2 . (Usiskin *et al.*, 2003: 298)

Reflection:

Let m be a line in a plane. For each point P in that plane the reflection image of P over line m , denoted by $r_m(P)$, is the point Q such that (i) $Q = P$ if P is on m , and (ii) m is the perpendicular bisector of \overline{PQ} if P is not on m . (Usiskin *et al.*, 2003: 313)

Translation:

Let $T_{\overrightarrow{AB}}$ be the translation specified by the directed line segment \overrightarrow{AB} , where $A = (p, q)$ and $B = (r, s)$.

Then $T_{\overrightarrow{AB}}(x, y) = (x', y') = (x + h, y + k)$, where $h = r - p$ and $k = s - q$. (Usiskin *et al.*, 2003: 304)

That is, when $b = \overrightarrow{AB}$, the translation T_b is the vector form of $T_{\overrightarrow{AB}}$ with $b = (h, k)$. We sometimes call this translation $T_{h,k}$; it has the effect of sliding each point of the plane h units to the right and k units up. Thus $T_{h,k}(x, y) = (x + h, y + k)$. We call h and k the horizontal component and vertical component of the translation by the vector b . If h is negative, then the slide is to the left. If k is negative, the slide is down. If $h = 0$, the T is a vertical translation. If $k = 0$, then T is a horizontal translation. If both $h = 0$ and $k = 0$, then T maps each point onto itself. It is the identity transformation or zero translation. (Usiskin *et al.*, 2003: 304)

Rotation:

Let C be a point and φ be a real number such that $[-180^\circ < \varphi \leq 180^\circ]$. Then the φ -rotation about C , denoted $R_{C,\varphi}$, is the plane transformation that maps C onto itself, and every other point P of the plane onto the point Q such that (i) $PC=QC$ and (ii) $m\angle PCQ = |\varphi|$ where $\varphi > 0, \Delta PCQ$ is counterclockwise oriented and if $\varphi < 0, \Delta PCQ$ is clockwise oriented. (Usiskin *et al.*, 2003: 307)

Image:

When T is a transformation and $T(\alpha) = \beta$ we say the transformation maps α to β . We call α the preimage and β the image. (Usiskin *et al.*, 2003: 298)

Regulation of the computational activity

The means of regulation governing this evaluative event relies on a very *general description of the mathematical concept* of transformation, rather than its formal mathematical definition. Reconsider the following transcript extract where the teacher requests the ‘English’ ‘meaning’ of this topic:

Teacher: What do you know about transformation? What is the meaning of transformation? Can you tell me? When we are talking about transformation, think about that word. What is the meaning of that? Hmm? Transformation, hmm? Anyone? Transformation? Do you want to try? Anyone? Do you want to try? [Learners do not respond to any of the teacher’s questions.] No one? Ok! Transform! When we are talking about transform what is the meaning of that class? In English, transform? Hmm? Okay. Okay. Transformation is a change, nhe? {right?} It is a change in position. Transformation is a change in position. If I am moving here and I move to another position, nhe? {right?} (S P2 L3 EE1 lines 1-4)

Table F.9 provides an account for how the notion of translation is constituted in this evaluative event.

Table F.9 The constitution of the topic: ‘translation’

Announced topic	Procedures used	Means of regulation	Realised content	Substitutes for
Translations	Object slides up or down or left or right by a certain number of units without changing the orientation of the object	General description of mathematical concepts i.e. linguistic description Computational procedure for translating objects	Sliding a figure to a new position without turning it Counting	Definition of a translation

School P2 Lesson 3 EE2: Translating an object three units right

Describing the computational activity

Now consider the following board images (Figure F.12) and transcript extracts of attempts at translating an object three units to the right in a Cartesian plane (S P2 L3 EE2):

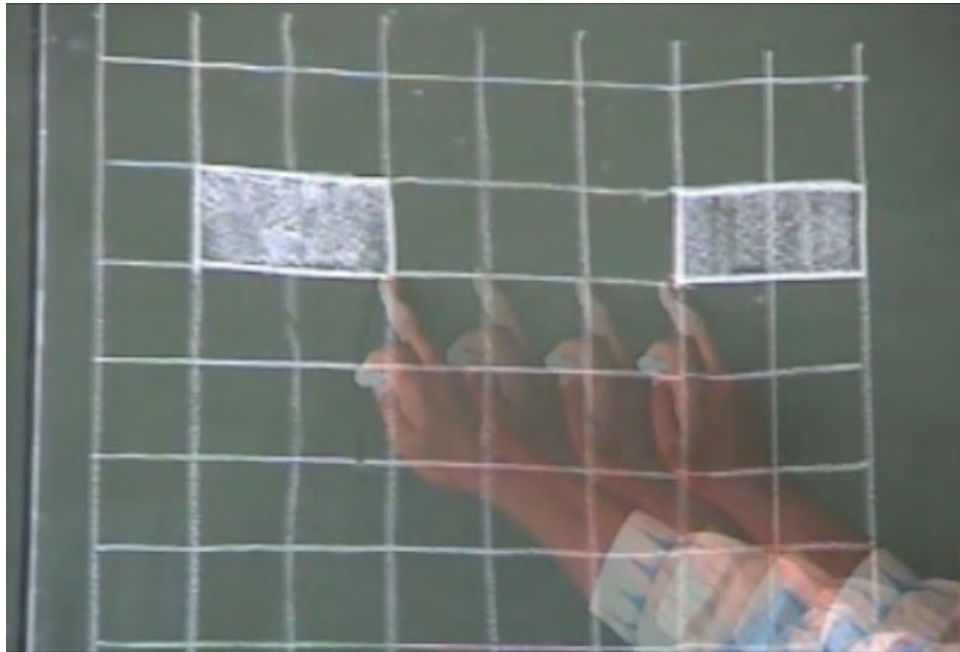


Figure F.12 The teacher indicating that she measured out a displacement of three units even though the object is displaced by five units (S P2 L3 EE2)

- Learners and Teacher: Translate these shapes three units right
Teacher: Ok then, we do that shade in here. Is that clear to everyone?
Learners: Yes.
Teacher: Translate this shape three units down, nhe? {right?} to the right, nhe? {right?} Ok you are here, nhe? {right?}
Learners: Yes
Teacher: So you will count how many units?
Learners and Teacher: Three units
Teacher: Three units to the right, nhe? {right?}
Learner: Yes
Teacher: So where are you going to start? Here, nhe? {right?} and count how many?
Learner and Teacher: One, two, three, nhe? {right?}
Teacher: So it means that object will be here, nhe? {right?} From here you count one, two, three. Are we together, bethunana {people}?
Learners: Yes, Miss (S P2 L3 EE2 lines 46-57)

In order to describe the teacher's procedure clearly, I have provided clarity with regard to the terminology that I will be using in Figure F.13 to describe her procedure as she translates an object three units to the right:

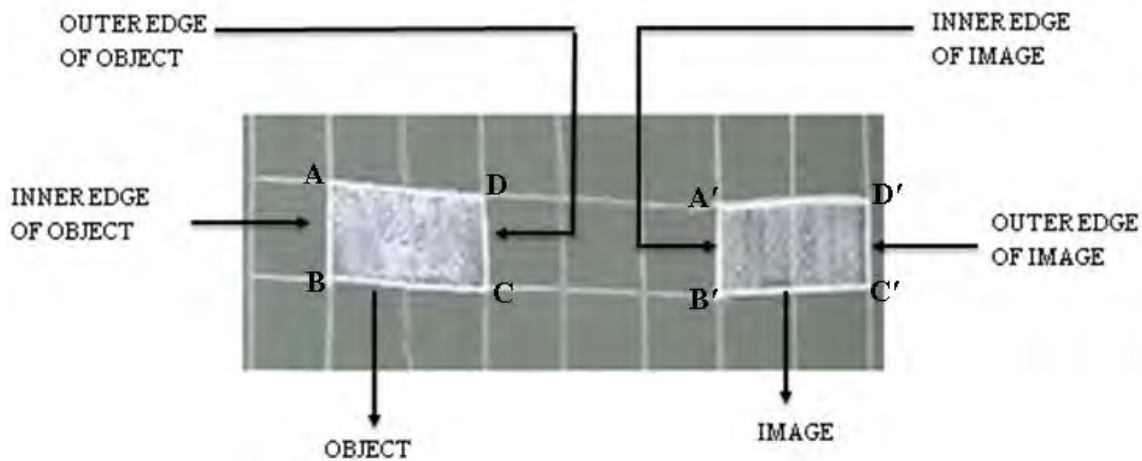


Figure F.13: An incorrect translation of three units to the right (S P2 L3 EE2)

The procedure for translating the object is described by the teacher as follows: Start counting from the outer edge of the object and count three units right i.e. the vertices of the outer edge of an object are translated three units right and this represents the vertices of the inner edge of the image. The procedure can be re-stated as follows:

1. Identify the starting point from which to count. (Teacher points to the bottom right vertex, i.e. C, of object in Figure F.13.)
2. From C start counting the number of units specified. (Note the point B'.)
3. 'Move the object' so that the translated object is positioned as 'starting' from B' i.e. the bottom left vertex is now positioned at B'.

The same procedure is referenced in the example that follows in Figure F.14 where a learner indicates the point from which he starts measuring out a displacement of three units. In Figure F.15 the teacher corrects him and shows him where to start measuring from:

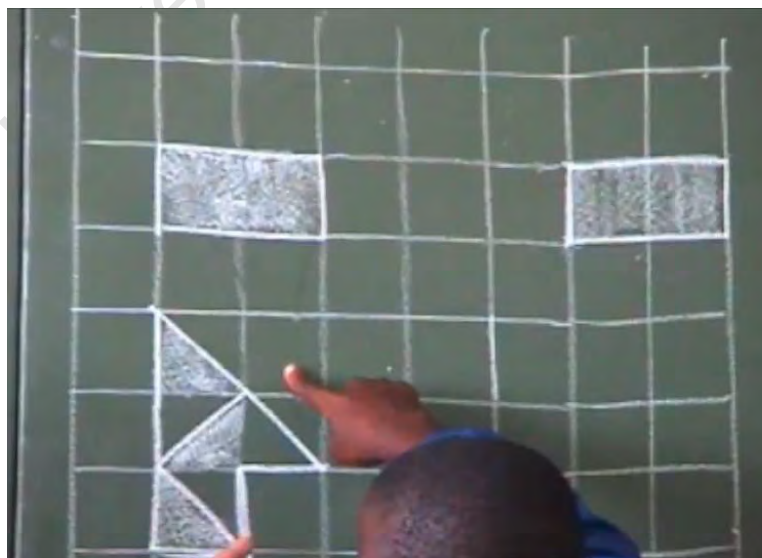


Figure F.14: The starting point for measuring out a displacement of three units for a Learner 1

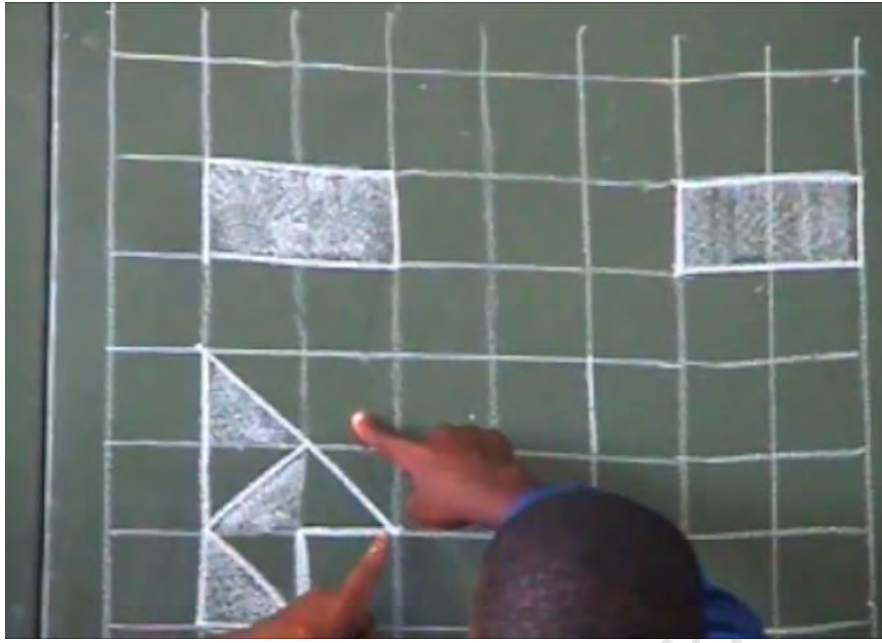


Figure F.15: The starting point for measuring out a displacement of three units indicated by the teacher

Learner 1 now attempts to use the teacher's criteria in Figure F.15 for translating the object three units to the right but the teacher dismisses his attempt as being incorrect in Figure F.16.

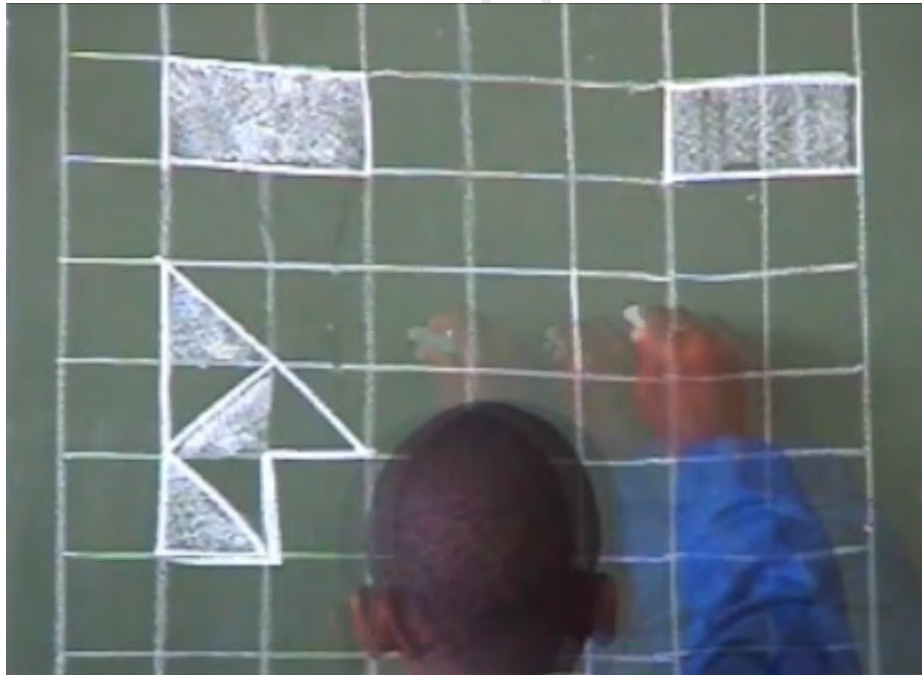


Figure F.16: Learner 1's second attempt is dismissed as being incorrect

A second learner now attempts counting out a displacement of three units in Figure F.17, following the teacher's explanation, and her attempt is accepted as being correct. After Learner 2 has described her suggested solution

the teacher, once again, indicates the position from which to start drawing the result of the required translation in Figure F.18 as well as how to measure out a displacement of three units in Figure F.19.

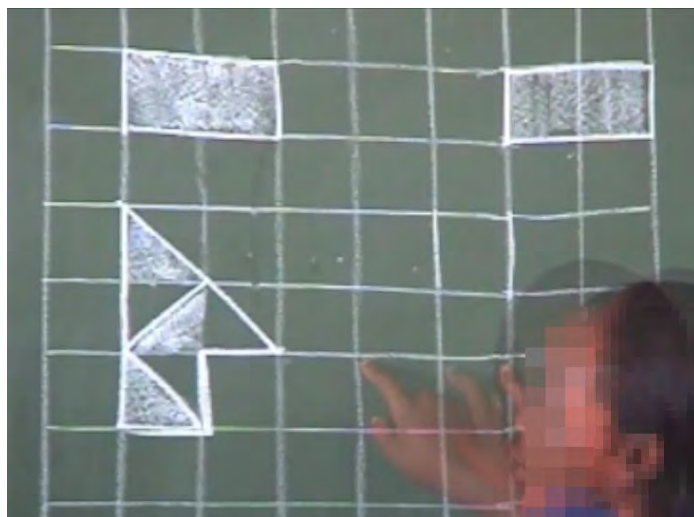


Figure F.17: Learner 2's attempt at translating an object three units to the right

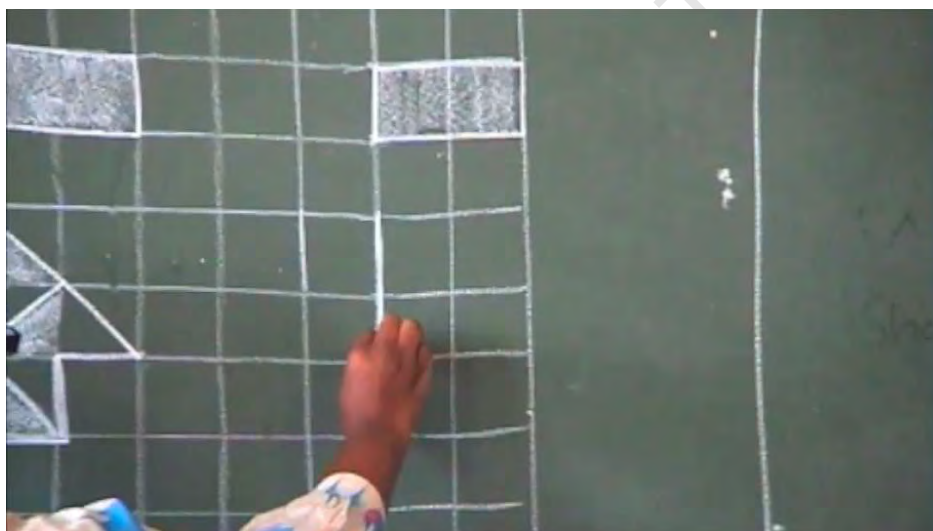


Figure F.18: Teacher indicates the position from which to start drawing the result of the translation

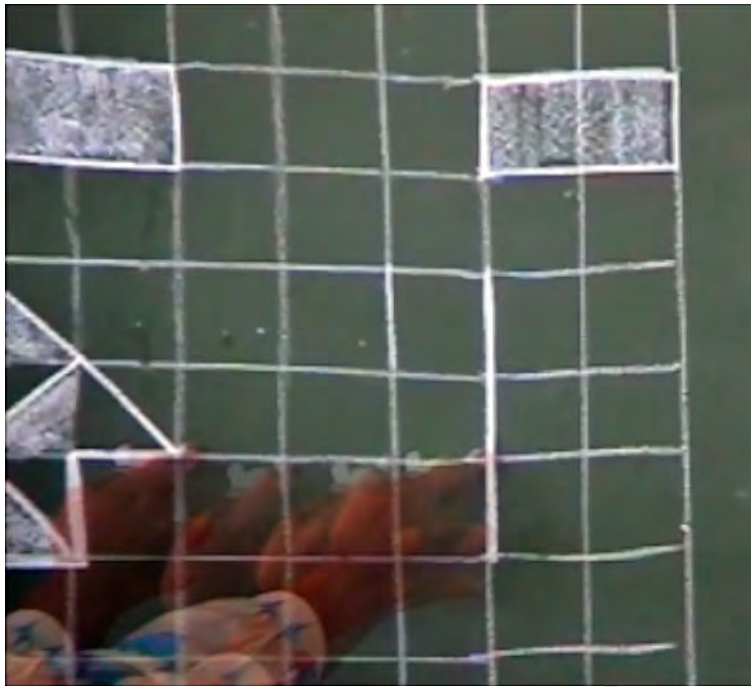


Figure F.19: Teacher re-iterating how to count out three units

The teacher checks that the object has been translated three units by counting the number of blocks between the object and the image in Figure F.19. This count is three units which proves to her that the correct translation has been effected.

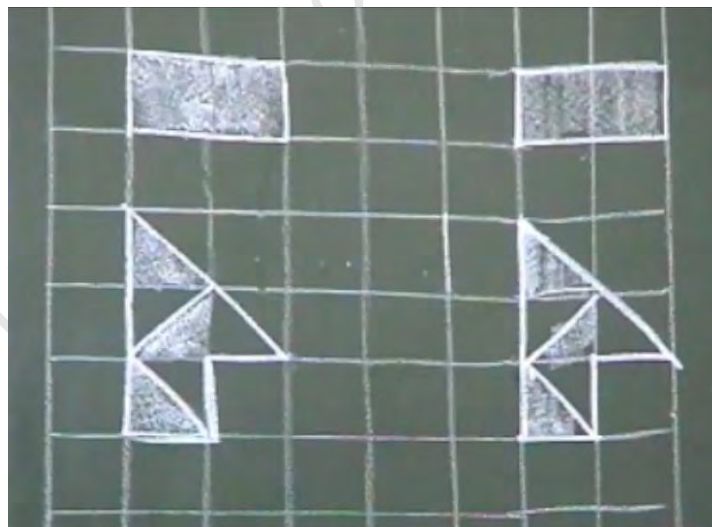


Figure F.20: The teacher's final solution

Both representations of translating an object three units to the right in Figure F.20 are incorrect – they have been translated by five units to the right. More simplistically described, it appears that the teacher basically counts three units and then moves the shape to the new position which is in fact a translation of five units right. The teacher does not correct herself in this lesson, neither is this inaccuracy picked up by any of the students.

Students continue doing a homework exercise on translations from the textbook incorrectly because of the procedure described by the teacher.

Consider the computational activity of this evaluative event in Table F.10:

Table F.10: The computational activity for S P2L3EE2

Lesson 3	EE2[08:49 – 19:12]	Input (domain)	Output (codomain)	Operation
Translating an object three units right		$(x;y)$ where $x, y \in \mathbb{Z}$	$(x \pm m; y \pm n)$ where $x, y, m, n \in \mathbb{Z}$	<u>Operation/operation-like manipulation:</u> <ol style="list-style-type: none"> 1. Identify where to start 2. Count out the three spaces that the object has to ‘move.’ 3. Move the object so that the space between the displaced object and original object equals the 3 spaces.

Curriculum and textbooks

See L3 EE1

The GET phase of the RNCS has, to a large extent backgrounded the use of formal definitions and propositions of mathematical objects and processes from the curriculum. As discussed in Lesson 1 and 2, the RNCS in its Assessment Standards for LO 3 (Space and Shape), requires the following from a learner to signal competency in this particular area:

The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions.

We know this when the learner:

Uses transformations (rotations, reflections and translations) and symmetry to investigate (alone and/or as a member of a group or team) properties of geometric figures.

Locates positions on co-ordinate systems (ordered grids), Cartesian plane (first quadrant) and maps, and describes how to move between positions using:

- horizontal and vertical change; [...] (DoE, 2002: 83)

The chapter covering Geometry in the textbook, *Preparing for High School Maths*, commences with a series of diagnostic tests on basic geometry i.e. sizes of angles, classification of triangles, quadrilaterals and solids, moving shapes and symmetry. The diagnostic test taken from the textbook, meant to cover Transformation geometry, is titled ‘Moving shapes’ and covers questions on rotations, reflections and translations of shapes.

Figure F.21 is the question relating to translations and these two shapes were the ones which were translated incorrectly by the teacher as displayed in Figure F.20.

2. Translate these shapes
3 units right.

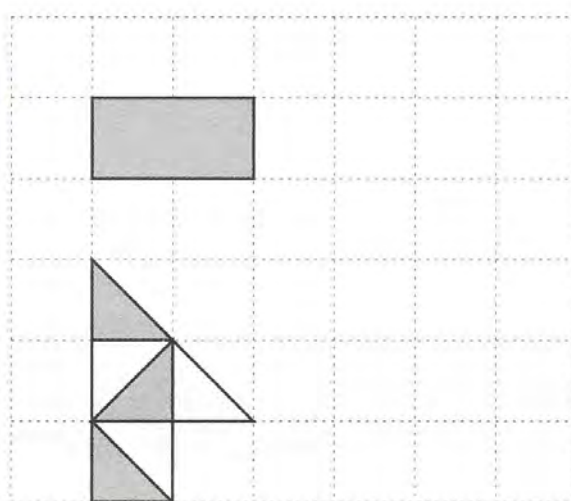


Figure F.21: A diagnostic test on translation in *Preparing High School Maths* (Bull & Hepworth, 2008: 109)

The textbook provides solutions (see Figure F.22) to all of the problems and if the teacher had checked, she would have picked up her inaccuracy.

2.

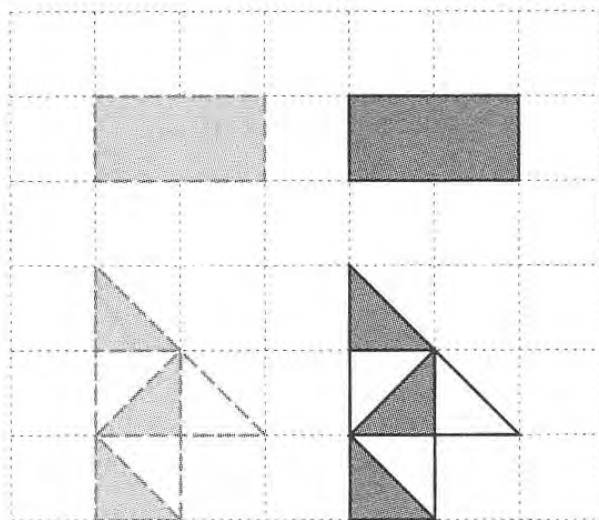


Figure F.22: The solution to No. 2 in *Preparing High School Maths* (Bull & Hepworth, 2008: 231)

Mathematics encyclopaedia

Translate: see L3 EE1

Units: a single magnitude or number considered as the base of all numbers

Regulation of the computational activity

The means of regulation provided by the teacher in arriving at these incorrect solutions include counting from the outer vertex of the object and she always asks the question: ‘Where do you start?’

- Teacher: And also that one you need to do. I mean to move, translate by 3 units to the right. Who can try and tell me where it can stand? The object. Count on the board and show me where. Where do you start? But the object ends here. There is supposed to be 2 blocks for this. Yes. But the object all the object ends here so you start where?
- Learner: Here.
- Teacher: Hmmm? So count again.
- Learner: One. Two. Three. (S P2 L3 EE2 lines 44-47)

Her *computational procedure*, which really entails counting three units and then moving the object to the new position, regulates this evaluative event. Students continue with an exercise on translations from the textbook and all their translations are incorrect as a result of the two examples done by the teacher.

My data production and analyses for School P2 show that very little attention is paid to what the mathematics encyclopaedia has to offer in terms of the set of interconnected propositional statements of mathematical relations between mathematical objects. Rather, the means of regulation in this particular empirical setting relies on computational resources i.e. choosing the appropriate procedure for plotting points, a reliance on what the solution may look like i.e. the symbolic representation of the boat and general descriptions of mathematical concepts i.e. the ‘definition’ of translation:

- Teacher: [...] Translation is when you achieve, ah, is when you slide an object, in fact, when you slide an object from one position to another position, without changing, ah. Without changing any shape of the object, nhe? {right?} Like if I got this duster here and then I want to move this duster to another position. Maybe I want to move it down nhe? {right?} I’m take it, straight down, nhe? {right?} I want change the position of duster, so it’s therefore translation. Don’t change anything in there. Translation, so you achieve by sliding the figure to a new position, nhe? {right?} without turning it. Are we together? (S P2 L3 EE1 lines 61-66)

I have presented an overview of the constitution of mathematics for Lessons 1, 2 and 3 for School P2 by presenting the computational activity for each evaluative event along with selections related to the announced topic from curriculum documents and the textbook as well as an account of the means of regulation in each evaluative event. In Chapter Six, I provide a detailed discussion what is constituted as mathematics for this particular teacher at School P2.

Appendix G: Analysis for School P3

Generating evaluative events

Table G.1: Evaluative events spanning Lesson 1 at School P3 (S P3 L1 EE1-EE3)

Evaluative Event	Type	Length
01 Definition of a prime number [00:00 – 03:46]	Expository	03:46
02 Composite numbers, factors and prime numbers [03:46 – 08:02]	Expository	04:16
03 Highest common factor and lowest common factor [08:03 – 20:20]	Expository	12:16
		20:18

School P3 Lesson 1 EE1: Definition of a prime number

Describing the computational activity

Lesson 1 (S P3 L1 EE1) commenced with the teacher announcing the topic, prime numbers, quite clearly:

Teacher: What is a prime number? Explain what is a prime number? We are busy with prime number, composite number, low common factor and highest common factor. What is a prime number? (S P3 L1 EE1 lines 1-4)

In this lesson there is an attempt on the part of the teacher to provide ‘definitions’ of the topics announced and she writes these definitions on the chalkboard (see Figure G.1). At the commencement of the lesson the teacher asks for the definition of a prime number and then proceeds to give the answer as well. She repeats the statement: ‘prime numbers have only two factors’ six times in a space of three minutes and the definition that she provides of a prime number is immediately followed by an example:

Teacher: [...] What is a prime number? [...] (S P3 L1 EE1 line 2)
 Teacher: Prime numbers are the numbers that have only two factors, andithi? {is it not?}
 Learners: Yes.
 Teacher: That have only two factors, examples zethu? {our examples?} What are the examples of prime numbers? Are the numbers that have only two factors. What are the examples of prime numbers?
 Teacher: Uthi ngu-nine {He/she says it is nine}. Is nine a prime number? (S P3 L1 EE1 lines 6-11)
 Learners: No.

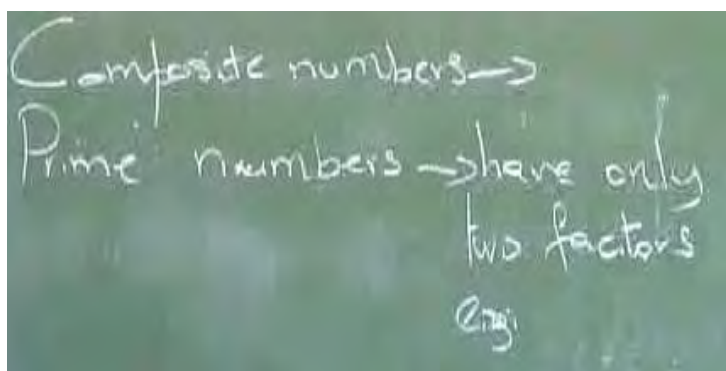


Figure G.1: Teacher’s ‘definition’ of a prime number

After providing the class with this definition in Figure G.1, she re-states the question for what a prime number is and the following interaction ensues:

- Teacher: [...] What is a prime number? [...] (S P3 L1 EE1 line 2)
 Learner: Ndiyiibele, Miss. {I forgot teacher}
 Teacher: Uyilibe! Ubuyazile kuqala? {You forgot!? Did you know it in the first place?}
 Learner: Yes Miss
 Teacher: Prime numbers are the numbers that has only two factors, andithi? {is it not?}
 Learner: Yes
 Teacher: That have only two factors, examples zethu? {our examples} What are the examples of prime numbers? Are numbers that has only two factors. What are the examples of prime numbers? (S P3 L1 EE1 lines 3 - 9)
 Teacher: [...] What is a prime number? A prime number is a number that has only two factors [...] (S P3 L1 EE1 line 14)

It is implicit that a prime number is a number that is divisible by only one and itself. As already stated, the teacher does attempt to provide ‘definitions’ (prime number, factor, composite numbers) and always states the topic upfront: ‘We are busy with prime number, composite number, low(est) common factor and highest common factor.’ (line 2-3) However, there always seems to be a reliance on the use of an example immediately after a definition is supplied - almost as if the example validates the description provided. This phenomenon was noted for the definitions of prime number, factor and composite number. The teacher provides a definition of prime numbers which can be categorised as a general description. So the domain, codomain and operation are not applicable in this instance. (See Table G.2)

Table G.2: Computational activity for S P3L1EE2

Lesson 1	EE1 [00:00 – 03:46]	Input (domain)	Output (codomain)	Operation
Definition of a prime number		Not applicable	Not applicable	Not applicable

Curriculum and textbooks

The RNCS makes no reference to prime numbers whatsoever in Learning Outcome 1 which deals with Numbers, Operations and Relationships.

The textbook in use at this school, but not referenced during this particular lesson, is *Preparing for High School Maths*. It is worthwhile reflecting on the definition of a prime number in this textbook in Figure G.2 and comparing it to what the teacher presents:

SKILL 2 PRIME NUMBERS

Prime numbers have only two factors, themselves and 1.

The set of prime numbers are {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, ..., etc.}

No.	Products	Factors	
2	2×1	[1, 2]	Prime
3	3×1	[1, 3]	Prime
4	$4 \times 1, 2 \times 2$	[1, 2, 4]	Not prime
5	5×1	[1, 5]	Prime
6	$6 \times 1, 2 \times 3$	[1, 2, 3, 6]	Not prime
7	7×1	[1, 7]	Prime

Figure G.2: Prime numbers in *Preparing for High School Maths* (Bull & Hepworth, 2008: 23)

The definition provided by the teacher is similar to that in the textbook so it may well have been used as a resource in her preparation for this lesson. Although she does state that prime numbers have two factors, she does not elaborate what these two factors are and rather uses an example to qualify her definition.

The mathematics encyclopaedia

Consider the formal definition of a prime number found in the mathematics encyclopaedia:

Prime Number:

An integer > 1 is prime if and only if its only integer divisors are itself and 1; otherwise it is composite. (Usiskin *et al.*, 2003: 209)

It is worth noting that the proposition that states that the smallest divisor of a natural number is necessarily prime is not taught or available in textbooks or the curriculum. This proposition is implicit and is rather rendered as a *general description of a mathematical concept* and validated with worked examples.

Regulation of the computational activity

This evaluative event is regulated by the *general description of a mathematical concept* i.e. 'Prime numbers are the numbers that has only two factors [...]' (line 9) rather than by its formal definition found in the mathematics encyclopaedia. Consider the constitution of a prime number in Table G.3:

Table G.3: The constitution of a prime number

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Prime numbers	Textbook definition and example ⁴⁸ i.e. 'Prime numbers have only two factors, themselves and 1.' ⁴⁹	General description of a mathematical concept	Prime numbers	Formal definition of a prime number Smallest proper divisor is prime

School P3 Lesson 1 EE2: Composite numbers, factors and prime numbers

Describing the computational activity

- Composite numbers

Composite numbers are described as follows during the lesson:

- Teacher: [...] What about the composite number? What are the composite numbers? At the back?
- Learner: Are the numbers that have two factors.
- Teacher: Are the numbers that have more than two numbers. Example of the number that have more than two factors? Give me one.
- Teacher: Yes, you. Uthi-four. {He/she says four.} Uthi {He/she says} four is one of the composite numbers, what are the factors of four? So that we can see that four has more than two factors, what are the factors of four? (S P3 L1 EE2 lines 28 - 32)

The teacher then provides a definition to differentiate between prime numbers and composite numbers, explaining that prime numbers only have two factors whereas composite numbers have more than two factors in Figure G.3 and in the transcript extract that follows:

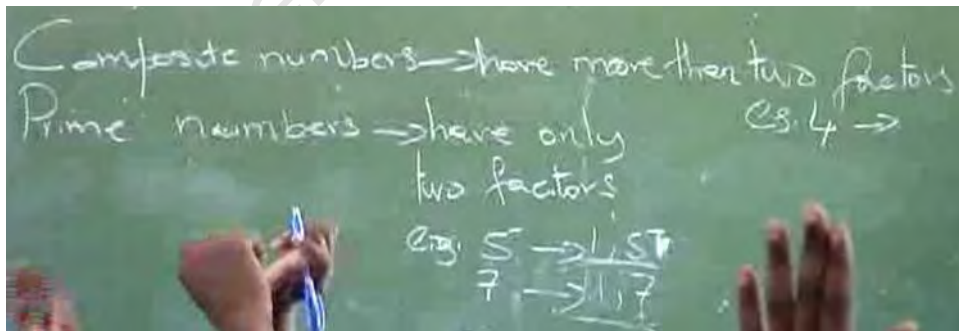


Figure G.3: 'Definition' of a prime number and a composite number (S P3 L1 EE1)

⁴⁸ The textbook definition and example is not a procedure as specified by the column title. As discussed earlier in this chapter, the teacher is providing a 'definition'. These definitions however are recontextualised versions of the encyclopaedic definitions.

⁴⁹ (Bull & Hepworth, 2008: 23)

Teacher: Yintoni umehluko {What is the difference} between those two? A Prime number and a composite? What is a major difference between those two?
 Learner: A prime number have two factors, a composite has more than two factors.
 Teacher: That's the difference between those two, a prime number got only two factors and a composite number gets more than two factors. (S P3 L1EE2 lines 62 – 65)

- **Factors and prime numbers**

As a means of further clarifying that prime numbers have only two factors, a definition of factor is confirmed by both a learner and the teacher in the following transcript extract:

Teacher: [...] What is a prime number? A prime number is a number that has only two factors. What is a factor first? Yintoni i-factor? {What is a factor?} So that we can be able to identify.
 Learner: A factor is a number that gets into another number and left without a remainder.
 Teacher: Is a number that you divide by a number that have no remainder. What are the examples? Five, five is one of the prime number. What are the factors of five? What are the factors of five? Pha ngasemva? {at the back?} (S P3 L1 EE1 lines 18 - 23)

An example follows immediately after the definition is supplied, and it seems as though this example is meant to confirm the definition of a prime number and a factor in some way. Factors of a number are generated randomly by students. As the factors of four (Figure G.4) are listed on the board by the teacher, she writes down one as a factor of four followed by a comma, then two followed by a comma and finally four with no comma. The comma suggests that more factors have yet to be listed.



Figure G.4: Listing factors of 4 (S P3 L1 EE2)

Consider the following transcript extract for how the teacher and learners generate factors of four:

Teacher: [...] four is one of the composite numbers. What are the factors of four? So that we can see that four has more than two factors. What are the factors of four? One?
 Learner: No, two.
 Teacher: It's one and two
 Learner: Four
 Teacher: No, four), Ikhona enye? {Is there another one?}
 Learner: No Miss.
 Teacher: Huh? {what?}
 Learner: No Miss.
 Teacher: Zi-more than two {they are more than two?}
 Learner: Yes (S P3 L1EE2 lines 31 – 40)

Besides factors of four being listed in a way that implicitly suggests when to stop generating factors, factors of twelve are also generated in this manner in Figure G.5. Consider how factors of twelve are generated:

Teacher: Zukisani {calling the learner by name} khanisinike i-factor ka-twelve. {Give us a factor of twelve.}
 Learner: Six
 Teacher: Six, two. Heh? {Huh}
 Learner: Four
 Teacher: Bani?! {What?} Oh! Four

Learner: Three
Teacher: Three?
Learner: One
Teacher: One, pha ekoneni? {At the corner?}
Learner: Twelve
Teacher: Twelve (repeating the answer)
Learner: Eight
Teacher: Is eight a factor of twelve?
Learner: No
Teacher: When we divide twelve by eight, there is a remainder?
Learner: Yes
Teacher: There is a remainder, so we are looking for a number that we can divide and be left with no remainder andithi? {Isn't it?} (S P3 L1EE2 lines 46 – 62)

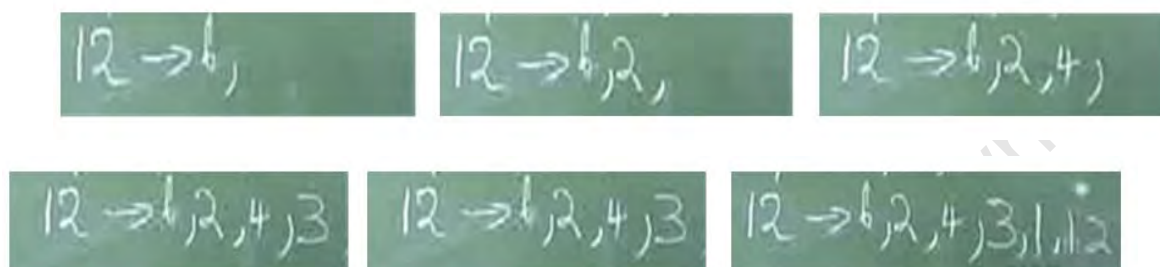


Figure G.5: Listing factors of 12 (S P3 L1 EE2)

The transcript extract presented as well as the board snapshots in Figure G.5 suggest that factor pairs are generated i.e. 6 and 2 followed by 4 and 3 followed by 1 and 12.

The teacher provides a definition of composite numbers, factors and prime numbers which can be categorised as general descriptions. So the domain, codomain and operation are not applicable in this instance. (See Table G.4)

Table G.4: Computational activity for S P3L1EE2

Lesson 1	EE2 [03:46 – 08:02]	Input (domain)	Output (codomain)	Operation
Composite numbers, factors and prime numbers		Not applicable	Not applicable	Not applicable

Curriculum and textbooks

The RNCS is rather vague regarding what composite numbers, factors and prime numbers might be. It only requires that students recognise, classify and represent multiples and factors rather than providing any formal definition:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them: [...]

- multiples and factors; [...] (DoE, 2002: 69)

As already discussed in EE1, the textbook is not referred to in this lesson but may have been used as a resource in lesson preparation so I have provided extracts for its definitions of composite numbers and factors in Figure G.6 and Figure G.7.

SKILL 3 COMPOSITE NUMBERS

Composite numbers are the whole numbers which have more than two factors.

The set of composite numbers is:
 $\{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, \dots \text{etc.}\}$

Except for the number 1 (which has only one factor), all the whole numbers are either prime or composite.

No.	Products	Factors	
2	2×1	[1, 2]	Prime
3	3×1	[1, 3]	Prime
4	$4 \times 1, 2 \times 2$	[1, 2, 4]	Composite
9	$9 \times 1, 3 \times 3$	[1, 3, 9]	Composite
12	$12 \times 1, 2 \times 3$ 3×4	[1, 2, 3, 4]	Composite

Figure G.6: Composite numbers in *Preparing for High School Maths* (Bull & Hepworth, 2008: 24)

The definition for composite numbers in Figure G.6 is accompanied by a list of the set of composite numbers as well as ‘method’ for how one distinguishes between a prime and a composite number. In Figure G.7 the definition of a factor is provided as well as the procedure for finding factors of 8, 9 and 12.

SKILL 1 FACTORS

Factors

A factor is a number which divides into another number exactly with no remainder.

Example:
 Find the factors of 8, 9 and 12.

Step 1
 Write the number pairs which multiply together to give the number. These are the factors.

Step 2
 List these factors in numerical order.

8	9	12
1×8	1×9	1×12
2×4	3×3	2×6
		3×4
{1, 2, 4, 8}	{1, 3, 9}	{1, 2, 3, 4, 6, 12}

Figure G.7: Factors in *Preparing for High School Maths* (Bull & Hepworth, 2008: 22)

Both the definitions provided by the teacher in the transcript extracts for composite numbers and factors are very similar to what is found in the textbook.

The mathematics encyclopaedia

Now consider the formal definitions for these topics found in the mathematical encyclopaedia:

Factor:

We say that $k \in N$ is a *factor* or *divisor* of $m \in N$ if there exists $s \in N$ such that $m = ks$. We write $k \mid m$. Trivially 1 and m are factors of m ; any other factor is called a *proper factor*. (Stewart & Tall, 1977: 165, italics in original.)

Composite Number:

An integer > 1 is prime if and only if its only integer divisors are itself and 1; otherwise it is composite. (Usiskin *et al.*, 2003: 209)

These formal definitions are not commonly found in the RNCS or in high school textbooks.

Regulation of the computational activity

The *general descriptions* of a factor and a composite number provided by the teacher as well as the definitions prescribed by the textbook are quite different to what is offered in the mathematics encyclopaedia. These general descriptions are all that students have access to as a means of regulating any exercise they have to do. The way that the teacher positions the commas when she generates factors also regulates students in terms of whether or not more factors have yet to be listed or whether all have been listed. These means of regulation are not adequate for ensuring that the content meant to be acquired is in fact realised by the student in all instances. Table G.5 represents the constitution of composite numbers and factors:

Table G.5: The constitution of composite numbers and factors

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Composite numbers	Textbook definition and example ⁵⁰ 'Composite numbers are the whole numbers which have more than two factors.' ⁵¹	General description of a mathematical concept	Composite numbers	Formal definition of a composite number

⁵⁰ The textbook definition and example is not a procedure as specified by the column title. As discussed earlier in this chapter, the teacher is providing a 'definition'. These definitions however are recontextualised versions of the encyclopaedic definitions.

⁵¹ (Bull & Hepworth, 2008: 24)

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Factors	Textbook definition and example ⁵² i.e. ‘A factor is a number which divides into another number exactly with no remainder.’ ⁵³	General description of a mathematical concept	Factors	Formal definition of a factor

School P3 Lesson 1 EE3: Highest common factor and lowest common factor

Describing the computational activity

In this evaluative event, the highest common factor (HCF) is dealt with as well as the lowest common factor (LCF). The lowest common factor is not a commonly used term found in texts for teaching and may be the teacher’s own terminology. Factors of twenty are generated and the HCF and LCF of 18 and 24 are calculated. Consider the following transcript extract for an account of how the teacher differentiates between the two concepts, HCF and LCF, without providing a precise formal definition:

- Teacher: Lowest common factor also the highest common factor. Ngubani ozakusixelela ukuba yintoni i-lowestcommon factor? {Who is going to explain to us, what is a lowest common factor?}
- Differentiate between LCF and HCF? Ndinike umehluko {give me the difference} between those two LCF and HCF. Is it the first time you hear those words?
- Learner: No
- Teacher: Ok, come on guys, let’s do it. The difference between those two. What is the lowest common factor? Hayi musani ukuba Zolani. Anifuni ukuthetha? {No, don’t be quiet. Don’t you want to say something?}
- Learner: I think i-number {the number} of the LCF is the smallest number from the factors of that number.
- Teacher: What about the highest? Highest is the opposite of lowest. Let’s look at this, twenty, what are the factors of twenty? (S P3 L1EE2 lines 66 – 74)

An example was used once again to confirm the definition of factors. It is interesting that initially the teacher referred to the ‘highest factor’ of twenty, but later referred to the ‘highest common factor’ of twenty even though it was only one number:

- Teacher: Twenty. Twenty is the highest common factor of twenty. (S P3 L1 EE3 line 106)

⁵² The textbook definition and example is not a procedure as specified by the column title. As discussed earlier in this chapter, the teacher is providing a ‘definition’. These definitions however are recontextualised versions of the encyclopaedic definitions.

⁵³ (Bull & Hepworth, 2008: 22)

A precise formal definition for lowest common factor and highest common factor is not provided by the teacher, rather: ‘Highest is the opposite of lowest. Let’s look at this, twenty, what are the factors of twenty? ...’ (S P3 L1 EE3 line 90 - 91). Generating the factors of twenty is the example meant to validate what was just explained.

The teacher then proceeded to find the highest common factor of two numbers, namely 18 and 24, (see Figure G.8⁵⁴) and provided an algorithm for this procedure where students were required to list factors of both numbers, identify the common factor and then classify factors as being the highest or the lowest. The procedure employed may be stated as follows:

1. List factors of a number
2. Identify the common factor
3. Choose HCF/LCF?

This method used by the teacher is similar to what the textbook prescribes later in Figure G.9, the only difference being that the textbook makes no mention of the lowest common factor.

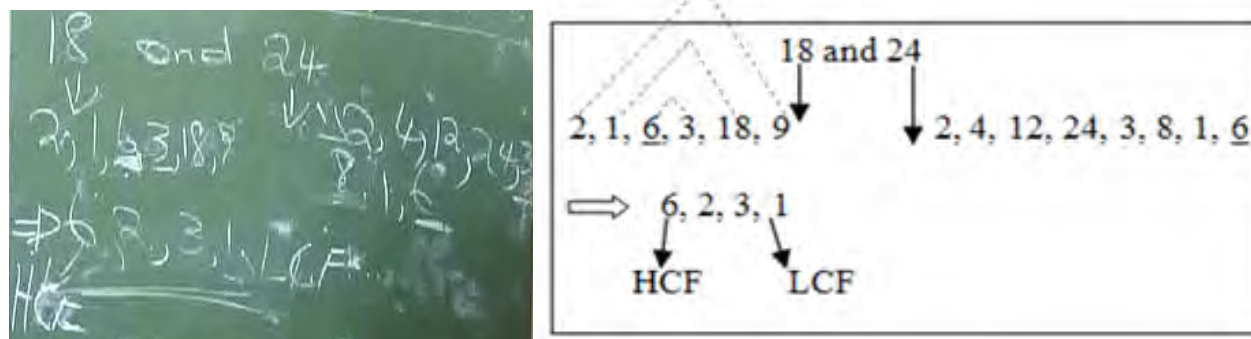


Figure G.8: The highest and lowest common factor of 18 and 24 (S P3 L1 EE3)

It seems as though the teacher is producing factor pairs for 18 in Figure G.8 and a random list of factors for 24.

The teacher provides a definition of highest common factor and lowest common factor which can be categorised as a general description. So the domain, codomain and operation are not applicable in this instance. (See Table G.6)

I now present an account of the computational activity for this evaluative event in Table G.6:

Table G.6: Computational activity for S P3L1EE3

Lesson 1	EE3 [08:03 – 20:20]	Input (domain)	Output (codomain)	Operation
Highest common factor and lowest common factor		Not applicable	Not applicable	Not applicable

⁵⁴ I have re-typed what was written on the board in Figure G.8 and presented it alongside to provide clarity.

Curriculum and textbooks

The curriculum provides no explanation or technique for calculating the highest common factor. Only factors, which are a crucial component of the highest common factor, are referenced in the curriculum excerpt that follows:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them:

- [...] multiples and factors; [...] (DoE, 2002: 69)

Once again, factors are not defined but only referenced in terms of one being able to ‘recognise, classify and identify’ them (DoE, 2002: 69). While the textbook (in Figure G.9) lists factors of a given value in numerical order, the teacher does not (in Figure G.8). The textbook method involves producing factor pairs, then listing the factors. Other than that, the very explicit procedure accompanied by an example provided in the textbook in Figure G.9 as well as the method used by the teacher are important criteria used by the students for proceeding to find the HCF. Consider the textbook definition and procedure for computing the HCF in Figure G.9:

Highest common factor (HCF)

The highest common factor is the largest of the common factors.

Example

Find the highest common factor of 8 and 12.

Step 1

List the factors of both numbers in numerical order.

Step 2

Locate the common factors.

Step 3

Find the highest common factor.

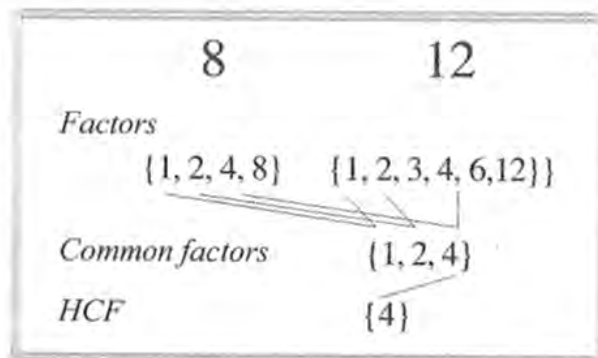


Figure G.9: Highest common factor in *Preparing for High School Maths* (Bull & Hepworth, 2008: 23)

The mathematics encyclopaedia

The mathematics encyclopaedia's description of the highest common factor is similar to what is found in the textbook and to what the teacher prescribes. Consider the following formal definition of highest common factor:

Highest Common factor:

Highest common factor: [...] if a and b are integers, not both zero, then the greatest common factor of a and b , $\text{gcf}(a,b)$ is the unique natural number such that

1. $\text{gcf}(a,b)$ is a factor of both a and b ;
2. If d is any integer that is factor of both a and b , then d is a factor of $\text{gcf}(a,b)$.

For example, $\text{gcf}(-24;30) = 6$ and $\text{gcf}(15;-8) = 1$ (Usiskin *et al.*, 2003: 208-209)

Regulation of the computational activity

The 'method' for calculating the highest common factor prescribed by both the teacher and the textbook serves as a *computational resource* that stands in place of the formal definition of the highest common factor. Consider the constitution of the highest common factor in Table G.7:

Table G.7: The constitution of the highest common factor

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Highest common factor	List factors Identify common factor Select the highest common factor	Computational resource General description of HCF	Multiplication of whole numbers	Definition of HCF Relations between factor, multiple, divisor

School P3 Lesson 2 EE1: Types of fractions – equivalent and proper fractions

Table G.8: Evaluative events spanning Lesson 2 at School P3 (S P3 L2 EE1-EE3)

Evaluative Event	Type	Length
01 Types of fractions – equivalent, proper fractions and improper fractions [00:00 – 04:04]	Expository	04:04
02 Addition of fractions [04:04 – 24:19]	Expository	20:25
03 Algorithm for addition of fractions [25:58 – 30:03]	Expository	05:05
		29:34

Describing the computational activity

This lesson commences with a review of the types of fractions dealt with up until that point. These included equivalent fractions, proper fractions, converting improper fractions to mixed numbers as well as converting mixed numbers to improper fractions. Students had just completed doing an exercise presented in Figure G.10 from the textbook, *Preparing for High School Maths*, on equivalent fractions and cancelling. This is a short evaluative event that merely recaps these topics mentioned, but I will provide some background from the textbook which may provide some insight into how this topic may have been presented.

SKILL 1 Equivalent fractions and cancelling

Find the missing numbers to make these equivalent fractions:

$$1. \frac{7}{8} = \frac{?}{16} \quad 2. \frac{11}{12} = \frac{?}{36} \quad 3. \frac{13}{15} = \frac{39}{?} \quad 4. \frac{4}{9} = \frac{36}{?} \quad 5. \frac{5}{11} = \frac{?}{99}$$

Cancel these fractions to their lowest value:

$$6. \frac{12}{18} \quad 7. \frac{18}{20} \quad 8. \frac{6}{21} \quad 9. \frac{15}{45} \quad 10. \frac{20}{26}$$

Figure G.10 – The exercise on equivalent fractions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 36)

In this exercise presented in Figure G.10, students were meant to find the missing numbers in order to make equivalent fractions. The textbook provides a method for doing this:

To make an equivalent fraction, multiply the top and bottom (numerator and denominator) by the same number (Bull & Hepworth, 2008: 36)

Besides presenting a method, which is described in two steps in Figure G.11, the textbook also uses two examples to clarify how to calculate equivalent fractions:

Examples:

(a) $\frac{4}{5} = \frac{?}{15}$ (b) $\frac{7}{8} = \frac{84}{?}$

Fill in the missing numbers to make equivalent fractions.

Step 1
Find the number which has been used to multiply the first part of the fraction.

Step 2
Use it to complete the second multiplication.

	Step 1	Step 2
(a)	$\frac{4}{5} = \frac{?}{15}$ $\times 3$	$\frac{4}{5} = \frac{12}{15}$ $\times 3$
(b)	$\frac{7}{8} = \frac{84}{?}$ $\times 12$	$\frac{7}{8} = \frac{84}{96}$ $\times 12$

Figure G.11 – Examples on equivalent fractions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 36)

The method for calculating equivalent fractions in Figure G.11 instructs the student to operate on the numerator and denominator independently, treating them as whole numbers rather than calculating on the fraction in its entirety. Step 1 of Figure G.11 states: ‘Find the number which has been used to multiply the first part of the fraction’ – making no reference to the constituent parts of the fraction i.e. the numerator and denominator.

Consider Table G.9 which provides an account of the computational activity for this evaluative event which displays that \mathbb{Q} are the initial input and output objects, but the intermediary objects are whole numbers:

Table G.9: Computational activity for S P3L2EE1

Lesson 2	EE1 [08:03 – 04:04]	Input (domain)	Output (codomain)	Operation
Types of fractions		\mathbb{Q}	\mathbb{Q}	<p><u>Equivalent fractions:</u></p> <p>(\mathbb{N}, \times) - i.e. both the numerator and the denominator are operated on separately as \mathbb{N} and are multiplied by the same value to calculate equivalent fractions</p> <p><u>Converting mixed numbers to improper fractions:</u></p> <p>$(\mathbb{N}, \times) \& (\mathbb{N}, +)$ -</p> <p>The integer component of the mixed number is multiplied by denominator of fraction component and this product is added to the numerator of the fraction. This numerator is expressed over the denominator, finally returning to the realm of rational numbers.</p>

Curriculum and textbooks

The RNCS emphasises the algorithm for finding equivalent fractions rather than any mention being made of a formal definition of equivalent, proper or improper fractions:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, describes and uses:

- algorithms for finding equivalent fractions; [...] (DoE, 2002: 69)

The textbook which may have been used prior to this lesson, seeing as it was used a resource to do an exercise, provides procedures for dealing with each of these types of fractions in Figure G.12, G.13 and G.14. Equivalent

fractions in Figure G.12 are referenced quite frequently by students when they add fractions with different denominators later in EE2.

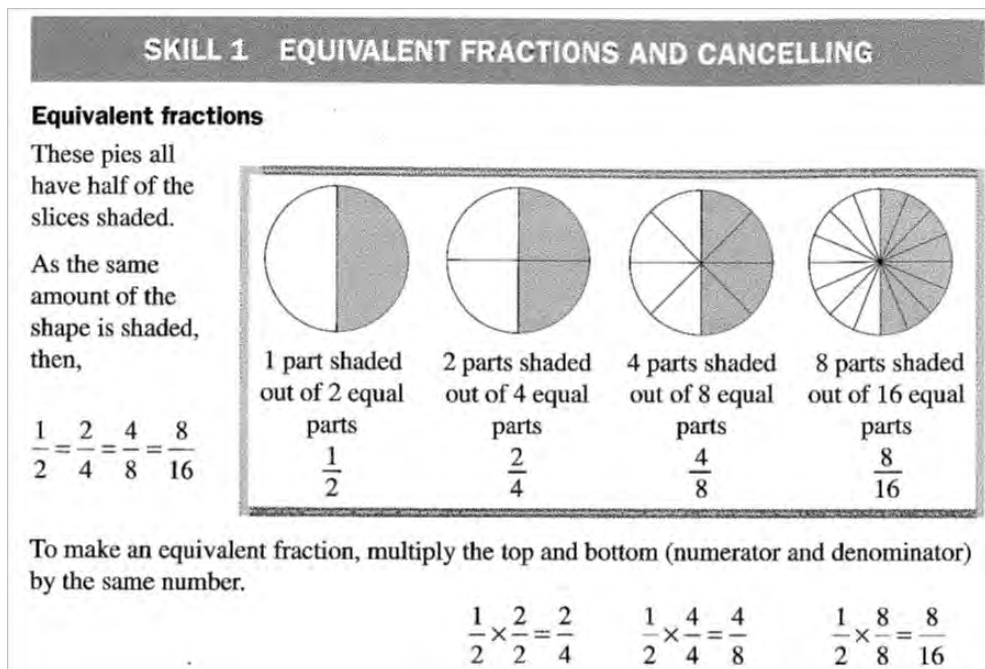


Figure G.12 - Equivalent fractions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 36)

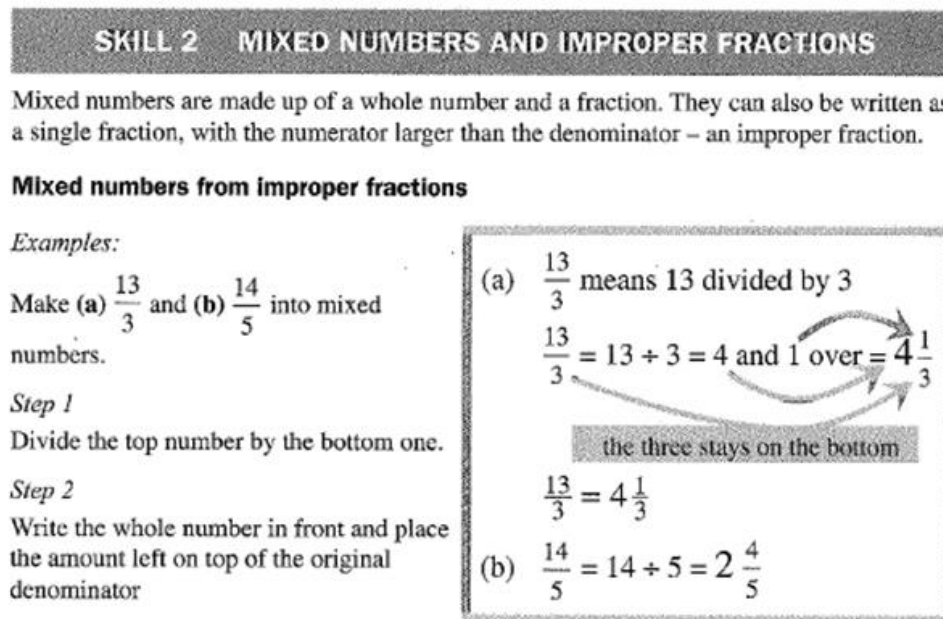


Figure G.13 - Mixed numbers from improper fractions in *Preparing for High School Maths* (Bull & Hepworth, 2008: 37)

Improper fractions from mixed numbers

Example:

Make $6\frac{2}{3}$ into an improper fraction.

Step 1

Put a 'times' sign and an 'add' sign into the mixed number as shown.

Step 2

Calculate clockwise, first multiplying and then adding as in the example

Step 3

Write the result over the original denominator.

$$6\frac{2}{3} = \frac{6 \times 3 + 2}{3} = \frac{20}{3}$$

the three stays on the bottom

$$6\frac{2}{3} = \frac{20}{3}$$

Figure G.14 - Improper fractions from mixed numbers in *Preparing for High School Maths* (Bull & Hepworth, 2008: 38)

A step-by-step procedure coupled with an example is provided in the textbook in Figure G.11, G.13 and G.14 as a means of explicating the topic of fractions.

The mathematics encyclopaedia

Consider the following formal definitions found in the mathematics encyclopaedia of the topics dealt with in this evaluative event:

Equivalent fractions:

[...] two apparently different fractions may have the same value. For example, $\frac{2}{3} = \frac{4}{6}$. This is overcome by allowing the 'cancellation' of common factors, reducing the fraction to 'lowest terms'. (Stewart & Tall, 1977: 14)

Proper fractions (Mathworld definition):

A proper fraction is a fraction p/q with $p, q > 0$ such that $p/q < 1$.

Improper fractions (Mathworld definition):

A fraction $p/q > 1$ is called an improper fraction.

Mixed fractions:

A mixed number is the sum of an integer and a fraction between 0 and 1, typically written with no space between them. For instance, the mixed number $32\frac{4}{5} = 32 + \frac{4}{5}$. In this case 32 is called the integer part of the rational number, and $\frac{4}{5}$ is its fractional part. (Usiskin *et al.*, 2003: 22)

These definitions are very different to the step-by-step procedure provided in the textbook in Figures G.11, G.13 and G.14 for performing operations on different types of fractions. The definitions provided from the encyclopaedia do not illustrate a method for converting mixed numbers into improper fractions.

Regulation of the computational activity

The procedure for working with each of these fractions i.e. equivalent fractions, improper fractions and mixed numbers, shows that the choice of the appropriate algorithm serves as a *computational resource* for producing the required solution. I now present an account of the constitution of equivalent fractions, mixed numbers and improper fractions in Table G.10:

Table G.10: The constitution of equivalent fractions, mixed numbers and improper fractions

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Equivalent fractions, mixed numbers and improper fractions	Multiplication, addition	Computational resource	Multiplication and addition of whole numbers Fractions are treated as pairs of whole numbers	Definition of equivalent fractions, mixed numbers and improper fractions

School P3 Lesson 2 EE2: Addition of fractions

Describing the computational activity

In this lesson, three examples were presented as an exemplification for adding fractions with different denominators:

- $\frac{3}{4} + \frac{1}{2}$
- $\frac{4}{6} + \frac{2}{3}$
- $\frac{5}{7} + \frac{1}{14}$

From the onset, the teacher was explicit that the solution procedure would only be possible if students adhered to the rules:

- Teacher: [...] Ok. Makhe sijonge bethunana kengoku nhe {let us look now} how to add fractions. Yintoni esiyijongayo xa sidibanisa ii-fractions? {What do we look at when we are adding fractions?} okanye uthini umthetho wethu {or what are the rules?} Besitshilo mos sathi {we said that} we must always follow ii-rules. {the rules.} So definitely in adding there is intoni? {what?}
- Learners: ii-rules. {Rules.} (S P3 L2 EE2 lines 26 - 28)

Before the rules were announced by teacher, she provided three learners with an opportunity of presenting their solution attempts to the first example, $\frac{3}{4} + \frac{1}{2}$ (Figures G.15, G.16 and G.17) on the board. It seems that addition of fractions had been covered in primary school⁵⁵ the previous year.

$$\frac{3}{4} + \frac{1}{2} =$$

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$\frac{3}{4} + \frac{2}{4}$$

$$\frac{5}{4}$$

$$1\frac{1}{4}$$

Figure G.15 Learner 1

$$\frac{3}{4} + \frac{1}{2 \times 2} = 4$$

Figure G.16: Learner 2

$$\frac{3}{4} + \frac{1}{2}$$

$$LCM = 4$$

$$LCM = 2$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

Figure G.17: Learner 3

Learner 1 (Figure G.15) provided a correct solution and used the notion of equivalence discussed in EE1 when expressing $\frac{1}{2}$ as $\frac{2}{4}$, allowing him to add two fractions with the same denominator. He converted his answer of $\frac{5}{4}$, which is an improper fraction, to $1\frac{1}{4}$ which is a mixed number. The conversion from an improper fraction to a mixed number in Figure G.15 was done quite easily by Learner 1. Learner 2's solution (Figure G.16) was incorrect – she appeared to use equivalence as well but failed to do so competently. Learner 3's (Figure G.17) approach resembled the algorithm the teacher was about to present, but still yielded an incorrect answer. The teacher provided commentary when the three learners had completed. Her response to Learner 1 and 2's solution was:

Teacher: abantu babalile ba-thimzile apha ikubhalansisa ii-denominators zakhona, nangapha babhalansisa ii-denominators zakhona. Abanye abantu bathini? Nithini kanye? Kutheni ii-denominators zalapha, kufuneka zifanile nje? Niyabonga nalapha ziyafana kutheni? Kutheni bengazidibanisanga $\frac{3}{4} + \frac{1}{2}$ Why? {The learners have done their sums, they have multiplied to

⁵⁵ 'Asiqali mos ukudibana ne-adding ye- fractions, naniyenzile e-primary mos? [...] {it is not the first time we've come across adding of fractions, you have done them in primary, right?}' (S P3 L2 EE2 line 45 - 46)

balance the denominators, they have done that as well on this side, what do others say? Why do the denominators here have to be the same?} Siyakwazi ukudibanisa ii-fractions ze-denominators ezingafaniyo? {Can we add the fractions even though the denominators are not the same?} (S P3 L2 EE2 lines 48 - 53)

Although both Learner 1 and Learner 2 chose to add the fractions using the notion of equivalence dealt with in EE1, the teacher preferred the following series of steps in Figure G.18:

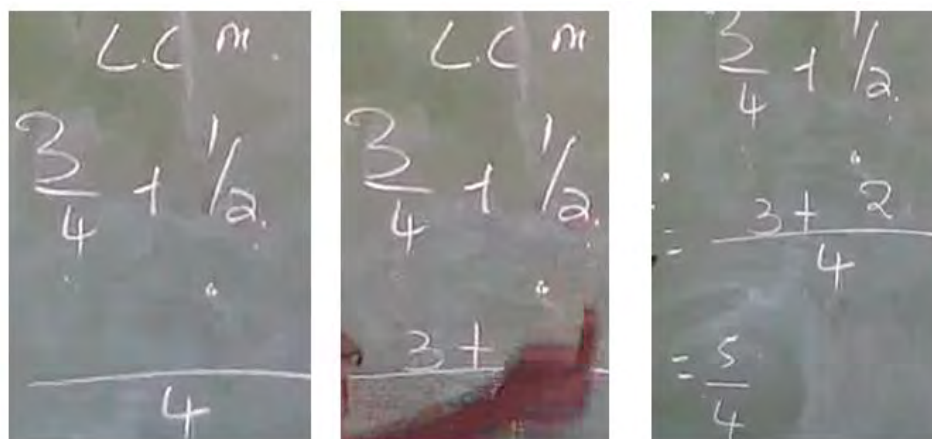


Figure G.18: The algorithm for adding fractions provided by teacher (S P3 L2 EE2)

In the algorithm presented in Figure G.18, the teacher stresses that students need to find the lowest common multiple and immediately thereafter wants to know what the next step is. She emphasises that the denominator has been changed to four and proceeds to use an algorithm which she explains in the following transcript extract:

- Teacher: [...] Masifanise apha. {What are we going to do here?}
 Teacher: Sizakufuna i-LCM, ngubani i-LCM yalapha? {We are going to look for the LCM, what is the LCM here.}
 Learners: Ngu-four {it is four}
 Teacher: Is it?
 Learners: Yes Miss (teacher)
 Teacher: Le division asiyishiyi. {Don't leave division out.} Sithini ke ngoku i-step esilandelayo? {What is the next step?}
 Teacher: Sizakuthini ngoku? {What are we going to do now?} Siyitshintshile i-denominator yethu yangu-four. Then ke ngoku oo-four baya kangaphi ku-four? {We have changed our denominator to four, then how many times does four go into four?}
 Learners: One.
 Teacher: Times three?
 Learners: Three
 Teacher: Plus ... oo-two ku-four? {two goes into four how many times?}
 Learners: Two.
 Teacher: Two, times one?
 Learners: Two
 Teacher: Ngu-two andithi? {it is two, is it not?}
 Learners: Yes
 Teacher: Sigqithe, three plus two? {than three plus two?}
 Learners: Five
 Teacher: Five over four, siyayibona le nto? {do we see this?} (S P3 L2 EE2 lines 59 - 80)

The teacher's procedure does not reference the idea of equivalent fractions explicitly since two rational numbers are discussed and operated on as four distinct whole numbers. Basbozkurt (2010b: 77), in his analysis of Lesson 2 and Lesson 3 at School P3, argues that when adding two rational numbers, the numerators and denominators are treated and operated on as whole numbers and he describes this as a shift from the domain of rational numbers to the domain of whole numbers. He argues that students at School P3 struggle to acquire the rules of the procedure, implicating the phenomenon of domain shifting in the generally poor performance of working class learners in examinations and tests.

The procedure for adding fractions with different denominators can thus be summarised as follows:

1. Find the lowest common multiple of the denominators of the fractions.
2. Divide each denominator of fraction into the lowest common denominator.
3. The quotient of each calculation is then multiplied by the numerator of the fraction.
4. The two obtained in Step 3 are added and constitute the numerator of the answer.
5. The denominator of the answer is the lowest common multiple of both fractions.
6. Simplify the final answer fully.

Consider the following example in Figure G.19 which the teacher completed on the board to illustrate how significant the specific layout of symbols was for the students:

Figure G.19: Addition of fractions using the algorithm – teacher (S P3 L2 EE2)

On closer inspection of Steps 1 and 2 in Figure G.19, the teacher enquired from the class as to how she ought to proceed and the following interaction ensued:

- Teacher: Siphinde sibe nalomzekelo. {Then we have another example.} Four over six plus two over three. Let's add those fractions, four over six plus two over three. We must add those fractions, four over six plus two over three, sizakuthini pha? {What are we going to do?} You must follow the rules, you must know what you must do then you will be able to simplify the fraction. Any questions? What must you do? What is your first step when you get a fraction sum that you must add? What do you look for?
- Learner: Ukhangela i-LCM. {Look for the LCM.}
- Teacher: So ukhangela i-LCM, eyalapha ngubani? {So you look for the LCM. What is the LCM here?}
- Learners: Six.

Teacher: Ngu-six, andithi? Uisix uyangena apha, no three uyangena apha, sithini kengoku? Sithi six, sithini kengoku? Zukisani! Sithini kengoku? {It is six, right? Six goes in here and three goes in here, what then? What do we do now?}

Zukisani: Not responding.

Teacher: Zukisani? Sithini kengoku? {What do we do next?}

Zukisani: Hlisa – u-four. {Lower the four.} (See Figure G.20) (S P3 L2 EE2 lines 84 - 96)

After finding the lowest common multiple of six, a student responds to the teacher's question for how to proceed, by saying 'Hlisa – u-four. {Lower the four}' S P3 L2 EE2 line 118 (see Figure G.18). It appears that this student is comparing the layout of this problem to the one just presented in Figure G.18. ($\frac{3}{4} + \frac{1}{2}$), especially when he says: '(h)lisa – u-four. {(l)ower the four}'. A reliance on what the solution should look like seems to take precedence over the notion of equivalence in this evaluative event.

Figure G.20: 'Lower the four' (S P3 L2 EE2)

Table G.11 describes the computational activity for this evaluative event which commences in the domain of the rational numbers. However, after a few calculations (see steps 2, 3 & 4 in the procedure described earlier) where both the numerator and the denominator are calculated on as if they were natural numbers, the final answer reverts back to the realm of rational numbers.

Table G.11: Computational activity for S P3L2EE2

Lesson 2	EE2 [04:04 – 24:19]	Input (domain)	Output (codomain)	Operation
Addition of fractions		\mathbb{Q}	\mathbb{Q}	<ol style="list-style-type: none"> 1. Find the lowest common multiple of the denominators of the fractions. 2. Divide each denominator of fraction into the lowest common denominator. 3. The quotient of each calculation is then multiplied by the numerator of the fraction.
Example 1: $\frac{3}{4} + \frac{1}{2}$		\mathbb{N}		

Lesson 2	EE2 [04:04 – 24:19]	Input (domain)	Output (codomain)	Operation
Example 2: $\frac{4}{6} + \frac{2}{3}$				<p>4. The two obtained in Step 3 are added and constitute the numerator of the answer.</p> <p>5. The denominator of the answer is the lowest common multiple of both fractions.</p> <p>6. Simplify the final answer fully.</p> <p>Algorithm:</p> <p>$(\mathbb{N}, \div) \rightarrow (\mathbb{N}, \times) \rightarrow (\mathbb{N}, +) \rightarrow (\mathbb{Q}, +)$</p>

Curriculum and textbooks

The curriculum only notes that a student ‘Recognises, describes and uses: [...] algorithms for finding equivalent fractions; [...]’ (DoE, 2002: 69).

The textbook provides a more comprehensive procedure for adding fractions with different denominators than the curriculum and refers to it as the ‘lowest common denominator method’ in Figure G.22 (Bull & Hepworth, 2008: 38). This method stresses the importance of finding the lowest common denominator when adding fractions with unequal denominators, as does the teacher when she presented Example 1 ($\frac{3}{4} + \frac{1}{2}$) and Example 2 ($\frac{4}{6} + \frac{2}{3}$) in this lesson. The textbook, however, stresses the importance of getting equal denominators before adding fractions with unequal denominators in the following instruction in Figure G. 21:

SKILL 7 ADDING FRACTIONS WITH UNEQUAL DENOMINATORS

The denominators must be made the same before the addition can happen.

Figure G.21. – Adding fractions with unequal denominators in *Preparing for High School Maths* (Bull & Hepworth, 2008: 38)

Now consider the lowest common denominator method found in the textbook in Figure G.22:

Lowest common denominator method

This method works for all fraction questions. Finding the lowest common denominator using the lowest common multiple is the most important step.

Examples:

(a) $\frac{2}{3} + \frac{1}{6}$ (b) $\frac{3}{4} + \frac{1}{5} + \frac{7}{10}$

Step 1

Find the lowest common multiple that all the denominators will go into.

Step 2

Make equivalent fractions so that all the denominators are the same.

Step 3

Complete the calculation and simplify where necessary.

<p>(a) $\frac{2}{3} + \frac{1}{6}$</p> <p>Step 1 The lowest common multiple (LCM) of 3 and 6 is 6</p> <p>Step 2 $\frac{2}{3} \times \frac{2}{2} + \frac{1}{6}$</p> <p>Step 3 $= \frac{4}{6} + \frac{1}{6}$ $= \frac{4+1}{6}$ $= \frac{5}{6}$</p>	<p>(b) $\frac{3}{4} + \frac{1}{5} + \frac{7}{10}$</p> <p>The lowest common multiple (LCM) of 4, 5 and 10 is 20</p> <p>$\frac{3}{4} \times \frac{5}{5} + \frac{1}{5} \times \frac{4}{4} + \frac{7}{10} \times \frac{2}{2}$</p> <p>$= \frac{15}{20} + \frac{4}{20} + \frac{14}{20}$</p> <p>$= \frac{15+4+14}{20}$</p> <p>$= \frac{33}{20} = 1\frac{13}{20}$</p>
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Figure G.22. – Adding fractions with unequal denominators (Lowest common denominator method) in *Preparing for High School Maths* (Bull & Hepworth, 2008: 38)

The difference between the teacher's discussion and that of the textbook's is that she uses the algorithm described earlier in Figure G.19 whereas the textbook uses the notion of equivalence in Figure G.22. The textbook presents another method for adding fractions with different denominators in Figure G.23 and refers to it as the 'matching method'. The 'matching method' is described using a series of steps and is accompanied by an example to describe its application. This method is not referenced by the teacher at all during this evaluative event.

Matching method

A quick way of adding two fractions is to use the matching method. This always works, but most times the answer will have to be cancelled.

Example: $\frac{1}{2} + \frac{5}{6}$

Step 1

Take the denominator from each fraction and put it over itself.

Step 2

Multiply them to the opposite fractions.

Step 3

The denominators will now be the same so continue as for SKILL 5.

Step 4

If possible, cancel the result and/or make it into a mixed number.

$\frac{1}{2} + \frac{5}{6}$

$= \frac{1}{2} \times \frac{6}{6} + \frac{5}{6} \times \frac{2}{2}$

$= \frac{6+10}{12}$

$= \frac{16}{12}$

$= \frac{4}{3}$

$= 1\frac{1}{3}$

Figure G.23 – Adding fractions with unequal denominators (Matching method) in *Preparing for High School Maths* (Bull & Hepworth, 2008: 38)

The mathematics encyclopaedia

The formal definitions listed below are not referenced in the curriculum, the textbook or by the teacher as the lesson unfolds. It appears that the procedure for adding fractions or working with equivalent fractions stands in place of its formal definition.

Formal definition of fractions:⁵⁶

Fractions may be represented as ordered pairs of integers $(a; b)$, $b \neq 0$, for which an equivalence relation has been specified (an equality relation of fractions), namely, it is considered that $(a, b) = (c, d)$ if $ad = bc$. The operations of addition, subtractions, multiplications and division are defined in this set of fractions by the following rules:

$$\begin{aligned}(a, b) \pm (c, d) &= (ad \pm bc, bd), \\ (a, b) \cdot (c, d) &= (ac, bd) \\ (a, b) : (c, d) &= (ad, bc)\end{aligned}$$

(thus, division is defined only if $c \neq 0$).

Fractions:

The indicated quotient of a divided by b may be denoted by a slash $\left(\frac{a}{b}\right)$, a bar $\left(\frac{a}{b}\right)$, or a division sign $(a \div b)$. [...] a colon $(a:b)$ [...]. Either a/b or $\frac{a}{b}$ is a fraction. (Usiskin *et al.*, 3003: 20)

Denominator (Mathworld definition):

The number q in a fraction p/q .

Numerator (Mathworld definition):

The number p in a fraction p/q , i.e., the dividend.

Lowest Common Multiple: see L1EE3

Operations on rational numbers:

[...], the sum, difference, product, and quotient of two rational numbers is itself a rational number (provided we do not divide by zero) [...]

- The set \mathbb{Q} of rational numbers is closed under addition, subtraction, and multiplication.
- The set $\mathbb{Q} - \{0\}$ ⁵⁷ of nonzero rational numbers is closed under division. (Usiskin *et al.*, 2003: 21)

Regulation of the computational activity

The announced topic, i.e. addition of rational numbers, is reduced to a series of steps in a procedure announced by the teacher and it appears that the procedure regulates the outcome of the solution. As a result, the algorithm for addition fractions is the *computational resource* that substitutes for a series of propositions for how to operate on rational numbers. The distinct layout of this procedure in Figure G.19 relates to the *reliance on what the solution should look like* as being another means of regulation in this evaluative event. Johnson and Davis (2010) refer to the production and spatial arrangement of characters as a *character distribution matrix*⁵⁸ where

⁵⁶ <http://www.encyclopediaofmath.org/index.php?title=Fraction&oldid=24256>

⁵⁷ $\mathbb{Q} - \{0\}$ means the set of rational numbers with 0 removed.

⁵⁸ ‘We call such a type of regulatory text a *character distribution matrix* and define it as a resource for the regulation of the mathematical activity demanding the use of very particular spatial distribution of symbols in the organization and presentation of transformations from one mathematical expression to another as a solution is generated according to a procedure. The presentation of mathematics is, of course, always subject to conventions for the display and regulation of mathematical expressions. That is, however, not the same as attempting to regulate mathematical activity by prioritizing

there is a reliance on the spatial distribution of characters in a particular problem. Consider how this topic is constituted in Table G.12:

Table G.12: How the addition of fractions with the different denominators is constituted

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Addition of fractions with different denominators	Algorithm	Computational resource Reliance on what solution looks like	Multiplication, division and addition of whole numbers	Fractions may be represented as ordered pairs of integers $(a; b), b \neq 0$, for which an equivalence relation has been specified (an equality relation of fractions), namely, it is considered that $(a, b) = (c, d)$ if $ad = bc$. The operations of addition, subtractions, multiplications and division are defined in this set of fractions by the following rules: $(a, b) \pm (c, d) = (ad \pm bc, bd),$ $(a, b) \cdot (c, d) = (ac, bd)$ $(a, b) : (c, d) = (ad, bc)$ (thus, division is defined only if $c \neq 0$).

School P3 Lesson 2 EE3: Algorithm for addition of fractions

Describing the computational activity

This evaluative event focuses on providing a detailed explanation of the algorithm for adding fractions with different denominators as described in EE2. Consider the following instructions issued by the teacher for

calculating $\frac{5}{7} + \frac{1}{14}$ after a learner went to the board and completed it unsuccessfully in Figure G.24:

Teacher: If you just follow your rules or procedure, ubuyayazi i-first step yakho kukukhangela i-LCM, then I denominator, then i-step esilandelayo uyabuzela kengoku andithi? Ubuze ke ngoku u-seven uya kangaphi ku-fourteen, uya kavi one, la one simthini siya-multiplaya nge-numerator, ibe ngubani ke ngoku, ibe ngu-ten and then from then ten plus one is eleven over fourteen. Nivile nina bantu abame ngenyawo? {If you know your first step is to look for the LCM, than your second step is to take that number that you have identified and make it the denominator, then the following step is to ask yourself how many times does seven go into fourteen, one time then we multiply that one by the numerator which is ten and then add ten plus one which is eleven then

very specific spatial distributions of mathematical symbols and terms, perhaps hoping that the intended mathematics might be reproduced despite the presence of the learner.' (Johnson & Davis, 2010: 135)

the answer will be eleven over fourteen. Those that are standing, did you hear/get that?} (S P3 L2 EE2 lines 136 - 142)

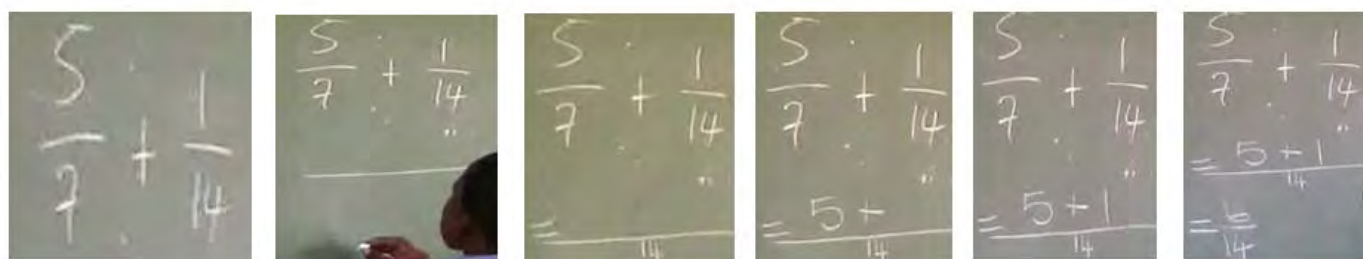


Figure G.24: Learner 4's incorrect solution attempt using the algorithm for addition of fractions (S P3 L2 EE3)

Before Learner 4 even begins the problem in Figure G.24, she draws a horizontal line in Step 2 and then proceeds to find the lowest common denominator. Once the lowest common denominator has been found in Step 3 it seems as if Learner 4 also ‘lowers the five’ and ‘lowers the one’, similar to what transpired in EE2 (Figure G.20), rather than using the algorithm or any notion of equivalence. The final solution was incorrect and as a result the teacher explained the algorithm once again.

Table G.13 presents the computational activity for this evaluative event which displays that \mathbb{Q} are the initial input and output objects, but the intermediary objects are whole numbers:

Table G.13: Computational activity for S P3L2EE3

Lesson 2	EE3 [25:58 – 30:03]	Input (domain)	Output (codomain)	Operation
Algorithm for addition of fractions		\mathbb{Q}	\mathbb{Q}	<ol style="list-style-type: none"> 1. Find the lowest common multiple of the denominators of the fractions. 2. Divide each denominator of fraction into the lowest common denominator. 3. The quotient of each calculation is then multiplied by the numerator of the fraction. 4. The two answers obtained in Step 3 are added and constitute the numerator of the answer. 5. The denominator of the answer is the lowest common multiple of both fractions. 6. Simplify the final answer fully. Algorithm: $(\mathbb{N}, \div) \rightarrow (\mathbb{N}, \times) \rightarrow (\mathbb{N}, +) \rightarrow (\mathbb{Q}, +)$
Example : $\frac{5}{7} + \frac{1}{14}$		\mathbb{N}		

Curriculum and textbooks

See L2 EE2 for a discussion on adding fractions with unequal denominators.

The mathematics encyclopaedia

Lowest Common Multiple: see L1 EE3

Fractions: see L2 EE2

Denominator: see L2 EE2

Numerator: see L2 EE2

Regulation of the computational activity

Students resort to using the algorithm provided by the teacher in L2 EE2 (Figure G.18 and Figure G.19) even though it appears that some students understand the notion of equivalence when adding fractions. However, the need to present their solution in the same form as what the teacher does over-determines the more logical explanation provided by equivalence. As a result the algorithm for addition of fractions with different denominators regulates this evaluative event and serves as a *computational resource* that stands in place of formal propositions for how to operate on rational numbers. Consider the formal definition of fractions as well as the rule related to addition and subtraction:

Fractions may be represented as ordered pairs of integers $(a; b)$, $b \neq 0$, for which an equivalence relation has been specified (an equality relation of fractions), namely, it is considered that $(a; b) = (c; d)$ if $ad = bc$. The operations of addition, subtractions, multiplications and division are defined in this set of fractions by the following rules: $(a; b) \pm (c; d) = (ad \pm bc, bd)$, [...]

In addition to the algorithm being privileged in this evaluative event, a *reliance on what the final solution should look like* is another important means of regulation and both the students and the teacher adhere to the format of the solution very strictly.

School P3 Lesson 3 EE1: Addition of fractions with the same and different denominators

Table G.14: Evaluative events spanning Lesson 3 at School P3 (S P3 L3 EE1)

Evaluative Event	Type	Length
01 Addition of fractions with same and different denominators [00:00 – Exercise 41:41]	Exercise	41:41
Entire lesson is spent checking homework on the board		41:41

Describing the computational activity

The entire lesson is spent checking homework based on addition of fractions with equal denominators and addition of fractions with different denominators. The homework was given from the textbook, *Preparing for High School Maths*, and individual students came up to the board to display their various solutions. I have provided the homework exercises, i.e. Skill 5 and Skill 7, in Figure G.25 and Figure G.29 respectively:

SKILL 5 Adding fractions with equal denominators

1. $\frac{5}{9} + \frac{7}{9}$

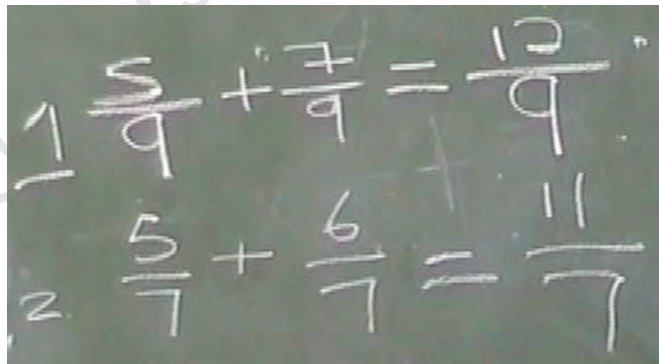
2. $\frac{5}{7} + \frac{6}{7}$

3. $\frac{11}{13} + \frac{5}{13}$

4. $\frac{1}{7} + \frac{6}{7}$

Figure G.25 – Adding fractions with equal denominators (Skill 5) in *Preparing for High School Maths* (Bull & Hepworth, 2008: 34)

The first homework exercise in Figure G.25 was completed successfully by most students on the board. The teacher summarised the procedure for recognising and reproducing the lowest common multiple when adding fractions with equal denominators as follows: If denominators of two fractions are the same, add numerators and write down the denominator. A minor addition error was made by a student in Figure G.27 when adding the numerators of $\frac{11}{13} + \frac{5}{13}$. I have provided snapshots of students' attempts of these problems as a means of examining what is constituted as mathematics in this evaluative event.



1. $\frac{5}{9} + \frac{7}{9} = \frac{12}{9}$

2. $\frac{5}{7} + \frac{6}{7} = \frac{11}{7}$

Figure G.26: Students' correct attempt of Number 1 and Number 2 of Skill 5 (S P3 L3 EE1)

$$(3) \frac{41}{13} + \frac{5}{13} = \frac{6}{13}$$

$$(3) \frac{41}{13} + \frac{5}{13} = \frac{16}{13}$$

Figure G.27: An incorrect and a correct attempt of Number 3 of Skill 5 (S P3 L3 EE1)

$$\frac{1}{7} + \frac{6}{7} = \frac{7}{7}$$

1 + 6

Figure G.28: A correct attempt of Number 4 of Skill 5 (S P3 L3 EE1)

In Figure G.28, it is clear that rational numbers i.e. $\frac{1}{7} + \frac{6}{7}$ are operated on in a manner that transforms the original rational numbers presented in the problem to whole numbers i.e. the student explicitly adds 1 + 6 in order to calculate the numerator of the fraction in the final answer. These snapshots show that students adhere to the procedure provided by the teacher for adding fractions with the same denominators very rigidly. Consider the constitution of this topic in Table G.15:

Table G.15: How the addition of fractions with the same denominators is constituted

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Addition of fractions with the same denominator	Algorithm	Computational resource Reliance on what solution looks like	Multiplication, division and addition of whole numbers	Fractions may be represented as ordered pairs of integers $(a; b)$, $b \neq 0$, for which an equivalence relation has been specified (an equality relation of fractions), namely, it is considered that $(a, b) = (c, d)$ if $ad = bc$. The operations of addition, subtractions, multiplications and division are defined in this set of fractions by the following rules: $(a, b) \pm (c, d) = (ad \pm bc, bd)$, $(a, b) \cdot (c, d) = (ac, bd)$ $(a, b) : (c, d) = (ad, bc)$ (thus, division is defined only if $c \neq 0$).

When students attempted the exercise for adding fractions with different denominators in Figure G.29, of the individual students who came to the board to do them, two of them experienced some difficulty.

SKILL 7 Adding fractions with unequal denominators

1. $\frac{5}{9} + \frac{7}{8}$

2. $\frac{5}{9} + \frac{6}{7}$

3. $\frac{1}{3} + \frac{5}{7}$

4. $\frac{3}{7} + \frac{2}{5}$

Figure G.29 – Adding fractions with unequal denominators (Skill 7) in *Preparing for High School Maths* (Bull & Hepworth, 2008: 35)

Now consider snapshots of students' attempts of this exercise in Figure G.30, G.31 and G.32. In Figure G.30, when the first student attempted Number 1, it appeared as if she was extremely unsure of herself, even though she used her notebook to complete the problem. Finding the lowest common multiple of the denominators was problematic for her - as was applying the algorithm prescribed by the teacher. The final step in the first attempt

where the numerators, 3 and 7 had to be added did not pose a problem even though the final solution was incorrect. The second student that attempted Number 1, erased the first student's attempt and applied the teacher's algorithm successfully. She appeared confident that her answer was correct.

Figure G.30 shows two chalkboard attempts at adding the fractions $\frac{1}{9} + \frac{7}{8}$. The left board shows an incorrect attempt where the student incorrectly identified the LCM as 14 and performed calculations that led to an incorrect result of $\frac{10}{103}$. The right board shows a correct attempt where the student correctly identified the LCM as 72, converted the fractions to $\frac{8}{72} + \frac{63}{72}$, and correctly added them to get $\frac{71}{72}$.

Figure G.30: An incorrect and a correct attempt of Number 1 of Skill 7 (S P3 L3 EE1)

The teacher provided a procedure for recognising and reproducing the lowest common multiple when adding fractions with unequal denominators in the following transcript extract:

Teacher: Akekho omnye umntu onokulungisa pha? {Is there someone who can fix this?} He bethunana besingathanga xa idenominators zingafani sifuna inani {People did we not say when our denominators are different we look for a number} ... Sijonga inani eza denominators zethu zingena kuzo zombini zingashiyi iremainder? {We look for a number that both our denominators can go into and that will not leave remainders} (S P3 L3 EE1 lines lines 63 -66)

In essence then, her procedure can be stated as follows: The lowest common multiple is the smallest value into which both denominators of the two fractions can divide into without leaving a remainder and students adhere to this procedure in Number 2, 3 and 4 of this exercise:

$$\textcircled{2} \quad \frac{35}{9} + \frac{54}{7}$$

$$\frac{35 \times 7}{9 \times 7} + \frac{54 \times 9}{7 \times 9}$$

$$\frac{245}{63} + \frac{504}{63}$$

$$= \frac{749}{63}$$

$$= \frac{107}{9}$$

$$\textcircled{3} \quad \frac{1}{3} + \frac{5}{7}$$

$$\frac{1 \times 7}{3 \times 7} + \frac{5 \times 3}{7 \times 3}$$

$$\frac{7}{21} + \frac{15}{21}$$

$$= \frac{22}{21}$$

Figure G.31: A correct attempt of Number 2 and Number 3 of Skill 7 (S P3 L3 EE1)

Number 2 and Number 3 in Figure G.31 were completed successfully by two different students. When the teacher described finding the lowest common multiple of the denominators in Number 2 the following interaction ensued:

- Teacher: Yhe wethu, {hey you} Ufuna inani elingena ku 9 no 7 andithi? {You are looking for a number that goes into 9 and 7 right?}
- Learners: Yes miss
- Teacher: Inani elingena ku 9 no 7 ngubani? {which number goes into 9 and 7?}
- Learners: Ngu {It's} 63
- Teacher: Akukho elinye elincinci? {Is there no smaller one?}
- Learners: No
- Teacher: Mmm.
- Learners: No. (S P3 L3 EE1 lines 105-112)

Instead of saying to students that they should look for the smallest whole number that is divisible by both 9 and 7, she states the following: 'Ufuna inani elingena ku 9 no 7 andithi? {You are looking for a number that goes into 9 and 7 right?}' (S P3 L3 EE1 line 107). It appears that students, however, calculate the lowest common multiple correctly in most instances. Applying the algorithm for adding fractions with different denominators, as described in Lesson 2, proceeds as follows for Number 2 in Figure G.31:

- Teacher: So, siyabuza o 9 ku 63 bangaphi? {So we ask how many times does 9 go into 63?}
- Learner: Bayi {there are} 7.
- Teacher: o 9 ku {in} 63 ?
- Learner: 7.
- Teacher: Bayi {there are} 7, times 5?
- Learners: 35.
- Teacher: Bayi {there are} 35, simbeke apha anditsho? {we put it here, right?}
- Learners: Yes.

Teacher: Faka iaddition sibuze {you put in addition and ask} o7 bangaphi ku 63? {how many times does 7 go into 63?}

Learners: Bayi {they are} 9.

Teacher: Bayi {they are} 9 times 6?

Learners: 54.

Teacher: It is 54 andithi? {right?}

Learners: Yes miss.

Teacher: 36 plus 54?

Learners: ibengu {it will be} 89.

Teacher: 89 divided by bani? {what?}

Learners: 63. (S P3 L3 EE1 lines 113-130)

In Number 4 (see Figure G.32), the final step of the algorithm which entailed adding the numerators, 15 and 14, of the fraction $\frac{15+14}{35}$ yielded an incorrect answer of $\frac{19}{35}$. The teacher corrected this answer to $\frac{29}{35}$.

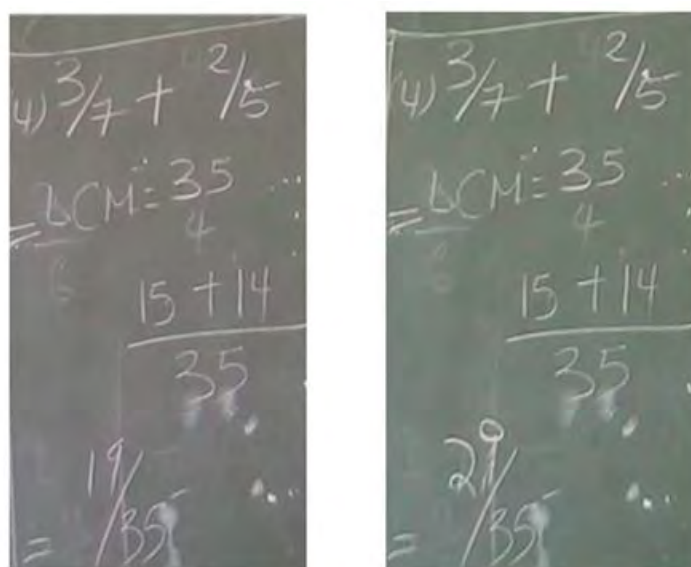


Figure G.32: An incorrect and a correct attempt of Number 4 of Skill 7 (S P3 L3 EE1)

As in Lesson 2, specifically in EE2, the algorithm for addition of fractions with different denominators appeared to regulate this particular lesson as well. As described earlier, this procedure can be described as follows:

Procedure for addition of fractions:

1. Find the lowest common multiple of the denominators of the fractions.
2. Divide each denominator of fraction into the lowest common denominator.
3. The quotient of each calculation is then multiplied by the numerator of the fraction.
4. These two answers are added and constitute the numerator of the answer.
5. The denominator of the answer is the lowest common multiple of both fractions.
6. Simplify final answer fully.

Both the transcript extracts as well as the snapshots of students' attempts of these eight problems on the board suggest that when the denominators of the fractions are equal, one simply adds the numerators of the two fractions. However, when the denominators are not the same the lowest common denominator of the two fractions must be calculated. Fractions and their fundamental constituent parts, i.e. numerators and denominators, are articulated by the teacher and students in a manner that does not relate to what is found in the mathematics encyclopaedia i.e. no formal definition of a fraction is discussed at any point. The answers that result from the series of problems from Skill 5 in the textbook, *Preparing for High School Maths*, transforms addition over the rational numbers to addition over the natural numbers when the numerators are added independently and the problem then reverts back to addition over the rational numbers when the final answer is presented as a fraction. For Skill 7, when the denominators are not the same – the denominators are first operated on independently in order to find the lowest common multiple. It appears that addition over the rational numbers is first transformed to multiplication over the natural numbers in this particular step for finding the lowest common multiple of the denominators. Once the lowest common multiple has been found, the algorithm directs the student to carry out both division and multiplication of natural numbers, followed by addition of natural numbers to arrive at the final answer for the numerator of the fraction. The entire problem then reverts back to the realm of rational numbers when the final answer is expressed as a fraction. Now consider the computational activity just described for this evaluative event in Table G.16 which displays that \mathbb{Q} are the initial input and output objects, but the intermediary objects are whole numbers:..

Table G.16: Computational activity at S P3L3EE1

Lesson 3	EE1 [00:00 – 41:41]	Input (domain)	Output (codomain)	Operation
Addition of fractions		\mathbb{Q}	\mathbb{Q}	<ol style="list-style-type: none"> 1. Find the lowest common multiple of the denominators of the fractions. 2. Divide each denominator of fraction into the lowest common denominator. 3. The quotient of each calculation is then multiplied by the numerator of the fraction. 4. The two obtained in Step 3 are added and constitute the numerator of the answer. 5. The denominator of the answer is the lowest common multiple of both fractions. 6. Simplify the final answer fully. <p>Algorithm: $(\mathbb{N}, \div) \rightarrow (\mathbb{N}, \times) \rightarrow (\mathbb{N}, +) \rightarrow (\mathbb{Q}, +)$</p>
Addition of fractions with equal denominators: $\frac{5}{9} + \frac{7}{9}, \quad \frac{5}{7} + \frac{6}{7}, \quad \frac{11}{13} + \frac{5}{13}, \quad \frac{1}{7} + \frac{6}{7}$ Addition of fractions with different denominators $\frac{5}{9} + \frac{7}{8}, \quad \frac{5}{9} + \frac{6}{7}, \quad \frac{1}{3} + \frac{5}{7}, \quad \frac{3}{7} + \frac{2}{5}$		\mathbb{Q}	\mathbb{Q}	

Curriculum and textbooks

The textbook prescribes two methods for adding fractions with equal denominators – the ‘diagram method’ (Figure G.34) or the ‘arithmetic method’ (Figure G.35) in the textbook extract in Figure G.33:

SKILL 5 ADDING FRACTIONS WITH EQUAL DENOMINATORS

When fractions have the same denominators they can be combined straight away. The addition can be completed using either a diagram or an arithmetic method.

Figure G.33 – Adding fractions with equal denominators in *Preparing for High School Maths* (Bull & Hepworth, 2008: 40)

When reflecting on the problems that students displayed on the board in Figure G.24, G.26 and G.27, it is clear that the arithmetic method is preferred by the teacher as her choice of procedure rather than the diagram method. Consider the two methods presented by the textbook in Figure G.34 and G.35:

Diagram method

Example:

Use a diagram to show the

addition: $\frac{5}{7} + \frac{4}{7}$

Step 1

Draw two shapes both divided into sevenths.

Step 2

Shade in the number of parts to stand for each fraction.

Step 3

Select the shaded parts.

Step 4

Add up the total number of shaded parts.

Step 5

Combine these shapes into whole groups of sevenths and leftovers.

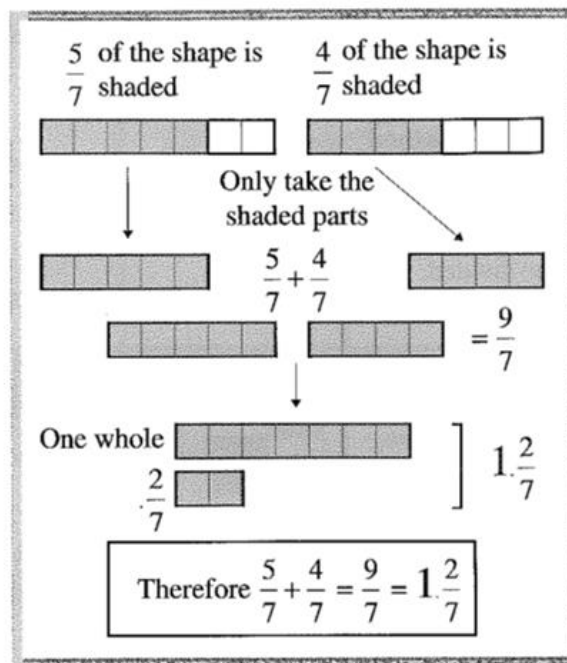


Figure G.34 – The diagram method for adding fractions with equal denominators in *Preparing for High School Maths* (Bull & Hepworth, 2008: 40)

The diagram method is not referenced or discussed at all during this lesson, neither is the arithmetic method named as such. However, the layout of students' presentation of solutions in the board snapshots presented earlier resembles the arithmetic method to a large degree – especially when adding fractions with unequal

denominators. Consider the arithmetic method in Figure G.35 as a comparison of what was presented during the both lessons that dealt with addition of fractions:

Arithmetic method

When the denominators are equal, add the numerators together and keep the denominator the same.

Examples:

Use the arithmetic method to work out

the additions: (a) $\frac{5}{7} + \frac{4}{7}$ (b) $\frac{3}{8} + \frac{7}{8}$

Step 1

Add the top numbers together – don't change the denominator.

Step 2

Cancel the result and/or make it into a mixed number if possible.

(a) $\frac{5}{7} + \frac{4}{7}$	(b) $\frac{3}{8} + \frac{7}{8}$
$= \frac{5+4}{7}$	$= \frac{3+7}{8}$
$= \frac{9}{7}$	$= \frac{10}{8}$
$= 1\frac{2}{7}$	$= \frac{5}{4} = 1\frac{1}{4}$

Figure G.35 – The arithmetic method for adding fractions with equal denominators in *Preparing for High School Maths* (Bull & Hepworth, 2008: 40)

See L2EE2 for discussion on adding fractions with unequal denominators.

The mathematics encyclopaedia

Fractions: see L2EE2

Denominator: see L2EE2

Numerator: see L2EE2

Lowest common multiple: see L1EE3

Operations on rational numbers:

[...], the sum, difference, product, and quotient of two rational numbers is itself a rational number (provided we do not divide by zero) [...]

- The set \mathbb{Q} of rational numbers is closed under addition, subtraction, and multiplication.
- The set $\mathbb{Q} - \{0\}$ ⁵⁹ of nonzero rational numbers is closed under division. (Usiskin *et al.*, 2003: 21)

⁵⁹ $\mathbb{Q} - \{0\}$ means the set of rational numbers with 0 removed.

Regulation of the computational activity

As discussed at length in Lesson 2 (EE2) this evaluative event is regulated primarily by the use of the algorithm or procedure for addition of fractions provided by the teacher. She insists that students use this algorithm for every one of the eight problems attempted on the board. As a result, the algorithm serves as a *computational resource* that substitutes for formal propositions that relate to how one operates on rational numbers. There certainly is a *reliance on what the final solution should look like* and the specific spatial distribution of symbols is another important means of regulation in this evaluative event. This is particularly evident when the teacher specifies exactly where certain numbers need to be positioned as she describes the algorithm for adding $\frac{5}{9} + \frac{6}{7}$ in Number 2 of Skill 7: ‘Bayi {there are} 35, simbeke apha anditsho? {We put it here, right?}[...] Faka iaddition sibuze {you put in addition and ask} o7 bangaphi ku 63? {how many times does 7 go into 63?}.

University of Cape Town

Appendix H: Analysis for School P6

School P6 Lesson 1

Generating evaluative events

Table H.1: Evaluative events spanning Lesson 1 at School P6 (S P6 L1 EE1-EE3)

Evaluative Event	Type	Length
01 Recap of rules for addition and subtraction of integers [00:00 – 03:46]	Expository	03:46
02 Operations (addition and subtraction) on integers [03:46 – 21:03]	Exercise	12:17
03 Recap of rules for multiplication and division of integers [21:03– 40:36]	Expository	19:53
		40:36

School P6 Lesson 1 EE1: Recap of rules for addition and subtraction of integers

Describing the computational activity

Lesson 1 commenced with a recap of the rules for addition and subtraction of integers. The rule for integer addition when the signs were the same was different to the rule presented for when signs were not the same. Whenever the teacher presented a rule for integer addition or integer subtraction, it was usually accompanied by one or two examples to demonstrate the procedure.

Integer addition

Integer addition when signs are the same

The board snapshot in Figure H.1 as well as the transcript extract which follows describes an explicit procedure provided by the teacher for adding integers having the same sign. The rule applied when adding integers that were either both positive or for integers that were both negative.

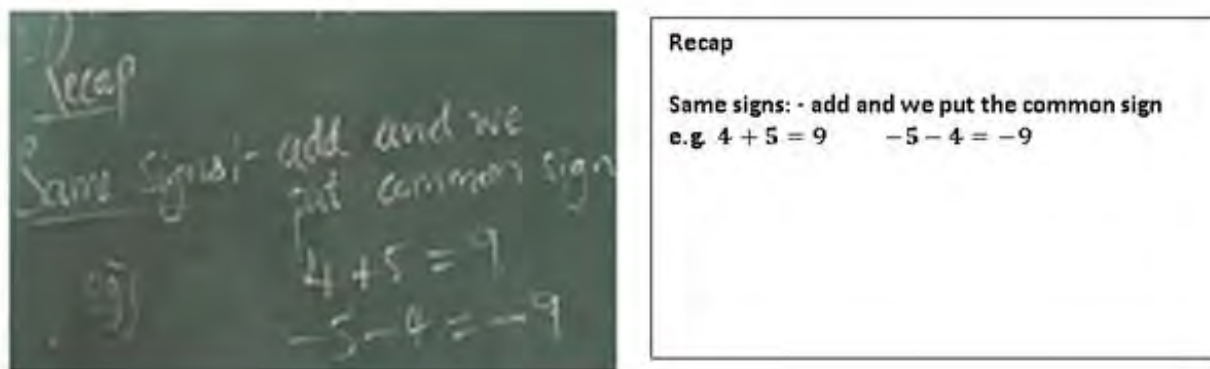


Figure H.1: Rule for integer addition when the signs are the same⁶⁰ (S P6 L1 EE1)

⁶⁰ I have re-typed the board writing which is unclear at times.

- Teacher: [...] the rules of adding and subtracting integers. So let's quickly reflect what are those rules. When adding one take the rule, when adding integers of the same signs what do you do, kanene? {again} Yes, bhuti {boy}, add like normal and put the sign over. Huh? Same sign, you add them you put the common sign for example... let's start with the normal four plus five. (See Figure H.1) So you add them. Which is?
- Learners: Nine.
- Teacher: Positive?
- Learners: Nine.
- Teacher: Minus five and minus four? (See Figure H.1) Yes, bhuti {boy}? Negative nine. Remember we add the common sign which is negative, [...] (S P6 L1 EE1 lines 1-8)

It is worth noting that positive integers were referred to as 'normal four plus five' (S P6 L1 EE1 line 3) and negative integers were referred to as 'minus five and minus four' (S P6 L1 EE1 line 8) (See Figure H.1). The criteria that students may have picked up from the way integers were referenced could possibly be that positive whole numbers were the preferred objects to be working with – seeing as they were referred to as being 'normal'. Students may also have assumed that whenever two negative signs co-occurred the answer would always be negative, irrespective of the operation performed. Later, however, the teacher read the following rule from the textbook to clarify the procedure for adding negative integers:

- Teacher: Okay, so ithini pha iexplanation ecaleni? Ithi {What is the explanation on the side? It says:} "when you add negative integers the answer will always be?"
- Learners: Negative
- Teacher: Niyabona lonto? {Do you all see?}
- Learners: Yes (S P6 L1 EE1 lines 50-53)

An analysis of the computational activity in Table H.2 (which follows later) shows that the objects that were announced at the start of the lesson were integers. However, an imposition of a procedure presented by the teacher for producing a solution transformed integers to whole numbers and signs. The whole numbers were then operated on and the signs were re-attached to the answer. The final solution returned the whole numbers to the realm of integers when 'we add(ed) the common sign which is negative' (S P6 L1 EE1 line 8). Jaffer (2009: 50), in *Breaking up and making up: a feature of school mathematics pedagogy*, describes a 'splitting apart or *sundering*⁶¹ of the sign and number' and 'a putting together or the *concatenation*⁶² of the result of 'operating' on the sign and the result of operating on the whole number' (italics in original).

⁶¹ '*Sundering* is understood in general as a pseudo-operation that separates a mathematical expression into a series of two or more expressions. The component expressions into which an expression is sundered are not unique and the production of the component expressions is based on the decision of the individual performing the sundering' (Jaffer: 2009: 50) in Davis (2010a). (italics in original). Davis later referred to a pseudo-operation as an operation-like manipulation.

⁶² *Concatenation* is understood in general as a pseudo-operation which links two or more expressions into a composite expression. The order of the concatenated elements is decided by the individual performing the concatenation (Jaffer: 2009: 50) in Davis (2010a). (italics in original)

Consider the procedure for adding integers with the same sign:

1. Separate the sign from the whole number.
2. Add the whole numbers.
3. Re-attach the common sign to the final whole number answer.

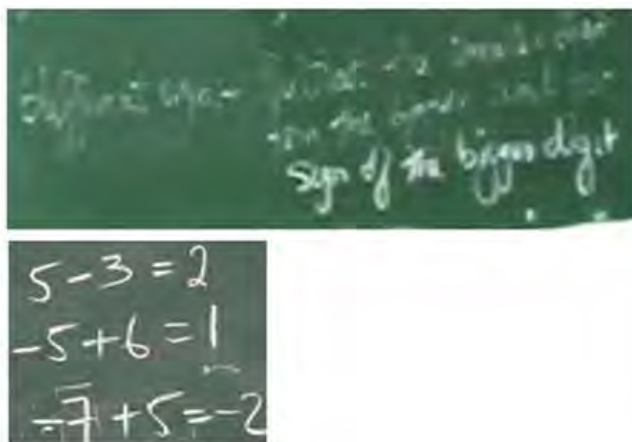
Integer addition when signs are different

When the signs of the integers were not the same, a different rule was applicable. The teacher offered the following procedure presented in both the transcript extract and the board snapshot in Figure H.2:

Teacher: [...] and then for different signs ... and then izandla ziphakanyisiwe {put up your hands}, yes? You subtract the smaller digit from the bigger digit, okay? We subtract the smaller digit from the bigger digit and put the sign of the bigger digit. (See Figure H.2) [...] (S P6 L1 EE1 line 8-10)

For this particular procedure, objects once again entered the pedagogic space as integers. They were then transformed into whole numbers and signs. The whole numbers were operated upon and they exited the procedure as integers once more. Consider the elaborate procedure suggested for addition of integers when the signs were different:

1. Separate the sign from the whole number.
2. Subtract the smaller whole number from the bigger whole number.
3. Re-attach the sign of the bigger whole number to the final whole number answer.



Different signs: Subtract the smaller digit from the bigger digit and put the sign of the bigger digit.

$$5 - 3 = 2$$

$$-5 + 6 = 1$$

$$-7 + 5 = -2$$

Figure H.2: Rule for integer addition when the signs are different ⁶³(S P6 L1 EE1)

Students adhered to these rules in Figure H.3 for all of their calculations:

⁶³ I have re-typed the board writing which is unclear at times.

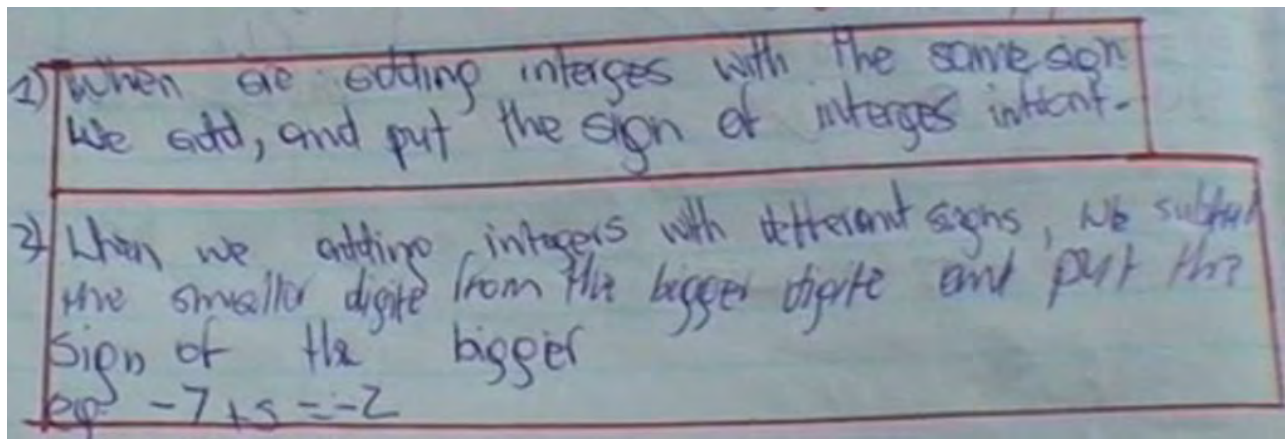


Figure H.3: Summary of the rules for integer addition in a student's notebook

In this evaluative event, the initial and final input and outputs were integers but the intermediary objects were natural numbers. Consider the computational activity for integer addition in Table H.2:

Table H.2: The computational activity for S P6L1EE1

Lesson 1	EE1 [00:00 – 03:46]	Input (domain)	Output (codomain)	Operation
Recap of rules for addition of integers		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<p><u>Adding integers with the same sign:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Add the whole numbers. 3. Re-attach the common sign to the final whole number answer. <p><u>Adding integers with different signs:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Subtract smaller whole number from the bigger whole number. 3. Re-attach the sign of the bigger whole number to the final whole number answer <p>$(\mathbb{Z}, +) \rightarrow (\mathbb{N}, -) \rightarrow (\mathbb{Z}, +)$ Sundering Concatenation Character strings</p>

Curriculum and textbooks

When examining the guidelines offered by RNCS with regards to integer addition, very little information is provided. It only states that integers need to be recognised, classified and represented:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them: [...]

- integers; [...] (DoE, 2002: 69)

The recommendations offered by the RNCS relating to integers do not specify procedures for integer arithmetic and so the textbook in use, *My Clever*, was relied on to a great extent by the teacher. The textbook provided the following definition of integers: ‘Integers are made up of 0 and all the natural numbers to the right of 0 on the number line, which we call the positive integers, as well as all the numbers to the left of 0 on the number line’ (Nel *et al.*, 2006: 28). It also presented a clear account of the notation and representation of integers as presented in Figure H.4. This amount of detail is not commonly found in the RNCS.

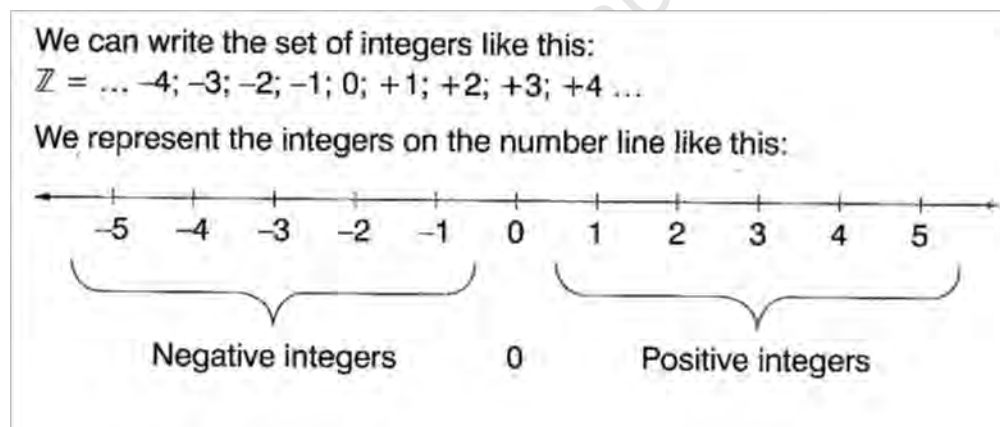


Figure H.4: Integer representation in *My Clever* (Nel *et al.*, 2006: 28)

The textbook introduced the topic of Integers as, *Investigating integers*, and one of the aims of this chapter promised that students would acquire knowledge regarding the properties of integers as described in Figure H.5:

At the end of this unit you will know

- what the properties of integers are.
- what the additive inverse of integers are.
- how deposits and withdrawals on an account are calculated.
- a basic understanding of shares and the stock market.

At the end of this unit you will be able to

- recognise and compare integers.
- add, subtract, multiply and divide integers.
- solve problems with negative numbers.

Figure H.5: Aim of this chapter in *My Clever* (Nel *et al.*, 2006: 27)

The textbook also presented a simplified notation for writing integers in the Figure H.6:

1. $+5$ can be written simply as 5
 $(+5) - (+3)$ can be written as $5 - 3$
A positive integer can be written without the $+$ sign
2. $5 + (-3)$ can be written as $5 - 3$
When adding a negative integer, the $+$ sign can be left out.

Figure H.6: How integers should be treated in *My Clever* (Nel *et al.*, 2006: 31)

The textbook extract presented in Figure H.6 suggests a conflation of $' + '$ as positive and $' + '$ as addition.

Now consider how the textbook suggested that students acquire the properties of integers in Figure H.7 where they were presented with five examples of integer addition for integers with the same and different signs. They were then challenged to explain why the final answers may be positive or negative for these five examples. No reference was made to the properties of integer addition in this extract – rather, students were given a list of instructions for working with integers.

2. Let us look at some examples when adding integers:

- | | |
|-----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| a) $(-2) + (-4) = -6$ | When you add negative integers the answers will always be negative. Can you explain why? |
| b) $(-1) + (-8) = -9$ | |
| c) $(-2) + (+4) = +2$ | When adding a negative and a positive integer, ignore the signs, find the difference between the numbers, the answer will have the sign of the larger number. Can you explain why? |
| d) $(-1) + (+8) = +7$ | |
| e) $(+1) + (-8) = -7$ | |

Figure H.7: Addition of integers in *My Clever* (Nel *et al.*, 2006: 31)

The teacher read the rules for integer addition (see Figure H.7) from the textbook during the course of the lesson as he applied this rule to do a range of problems. As already mentioned the textbook did not present an account of any of the operatory properties of integers but rather presented a procedure which was also employed by the teacher in all of the problems that he presented to the class. Both the RNCS and the textbook's account of an integer are very different to definitions encountered in the mathematics encyclopaedia.

The mathematics encyclopaedia

The formal definitions of integers found in the mathematics encyclopaedia provide clarity in terms of its operatory properties:

Integers:

The set on *integers* comprises the natural numbers, their additive inverses (opposites), and zero. Addition, subtraction, and multiplication are possible within the set of integers, but division is not (for example, $2 \div 5$ is not an integer). (Usiskin *et al*, 2003: 179, italics in original)

An integer is either a natural number n , or a symbol $-n$ where n is a natural number or 0. We use \mathbb{Z} to denote the set of integers. (Stewart & Tall, 1977: 14)

Addition over the integers and its properties has some convenient operatory possibilities:

1. $\forall a, b \in \mathbb{Z}, (a + b) \in \mathbb{Z}$;
2. $\forall a, b, c \in \mathbb{Z}, (a + b) + c = a + (b + c)$;
3. $0 \in \mathbb{Z}$, and for $\forall a \in \mathbb{Z}, a + 0 = a = 0 + a$;
4. $\forall a \in \mathbb{Z}, \exists a^{-1} \in \mathbb{Z}$ such that $a + a^{-1} = 0$;
5. for $\forall a, b \in \mathbb{Z}, a + b = b + a$.

Natural numbers:

The natural numbers are the familiar counting numbers 1, 2, 3, 4, 5, ... [...] We write \mathbb{N} for the set of all natural numbers. (Stewart & Tall, 1977: 12-13)

If students were aware of the operatory properties that exist for integers, it is possible that they may have been better equipped to deal with a wider range of problems relating to integer arithmetic, irrespective of the context.

Regulation of the computational activity

In this evaluative event, the selection of the correct series of computations was dependent on the signs of the integers that were to be added. The *computational procedure* therefore regulated this evaluative event rather than any of the properties associated with integers. Integers were never referenced as such but were rather referred to as whole numbers with a sign. For all of the problems presented, the signs were disassociated from the numbers so that the appropriate computational procedure could be applied.

Table H.3 summarises the procedure used, the content realised through the procedure, the content substituted by the procedure as well as the form of regulation for the constitution of integer addition:

Table H.3: The constitution of integer addition

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Integer addition when signs are the same	Add whole numbers and attach common sign to answer	Computational resource	Whole number arithmetic	Operatory properties of integer addition
Integer addition when signs are different	Subtract whole numbers and attach sign of the bigger whole number to answer	Computational resource	Whole number arithmetic	Operatory properties of integer addition

School P6 Lesson 1 EE2: Operations (addition and subtraction) of integers

Describing the computational activity

Models that represent integer addition

Integer addition was the intended topic to be acquired in this evaluative event and it is interesting how the teacher and the textbook presented this topic using a model. Consider the following excerpt from a lesson where the teacher made use of the poker chip model to further discuss integer addition for $4 + (-3)$:

- Teacher: [...] we use models and theories to help us, you can do the same in mathematics. One way to illustrate the addition and subtraction of integers is to use the poker chip model. This may help you to understand. Let's look at that example kuthiwa pha {they say here} for all those positive ones use an open circle and then a solid circle represents a? Negative one. [...] how many circles represent positive 4? {Teacher reads from the textbook.} (See Figure H.8 and Figure H.9)
- Learners: 4
- Teacher: Then how many solid circles represent negative 3?
- Learners: 3
- Teacher: [...] minus 4 plus 3 so niyabona ukuba {you see that} every case ezi ziya perisha then kengoku le ishiyekileyo {those that pair up and what is left} is your answer. Then kengoku ngubani {What is it?}
- Learners: Negative 1. (S P6 L1 EE2 lines 70-80)

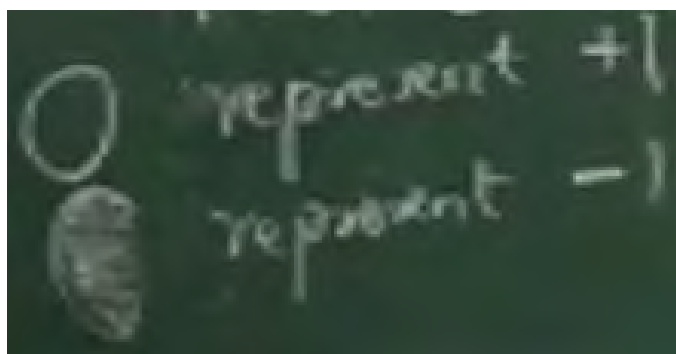


Figure H.8: A representation of positive and negative integers (S P6 L1 EE2)

The method of co-pairing prescribed by the teacher in the transcript extract that follows entailed pairing an open circle and a solid circle to obtain an answer in Figure H.9:

- Teacher: Okay class, a quick question from the last one. Xa une (When you have) pairing, what are you going to do? From the last one. Xa une (When you have) co-pairing ngolohlobo (like that). What are you going to do to the last one? Yes, bhuti (boy)? (S P6 L1 EE2 lines 93-94)
- Teacher: No, funeka sithini? Sibhale le ine amount enintsi si cancelishe kweziya sibale le ishiyekileyo, sihamba sonke? {What should we do? We write the one with the most amount and cancel it from the remaining ones, we're clear?} So ishiyeka phi le ishiyekayo? {Where does the remainder go?} (S P6 L1 EE2 lines 111-112)

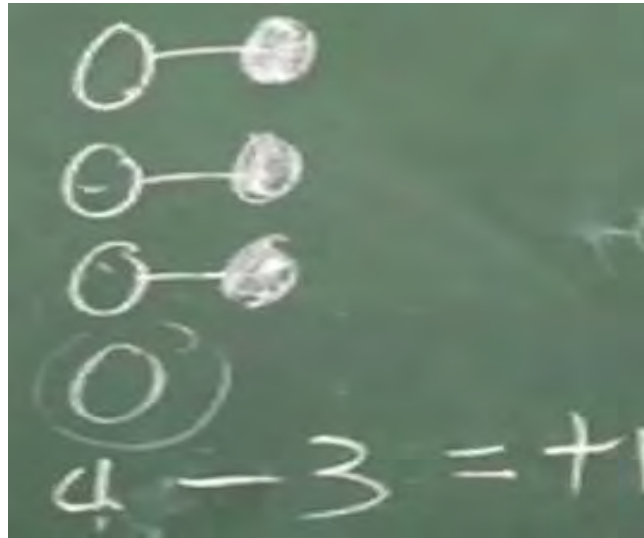


Figure H.9: Co-pairing as a means of integer addition (S P6 L1 EE2)

The teacher presented the problem with addition over the \mathbf{R} , i.e. $4 + (-3)$, and used ‘pairing’ or ‘cancelling’ as a means for solving the problem. Pairing or cancelling produced a ‘void’ and the notion of additive inverses was a key feature used in this exercise. Once cancelling had been completed, the student counted the number of open or closed circles remaining to arrive at the correct solution.

In an attempt to define the procedure described by the teacher in Figure H.9 in computational terms, one ideally requires an account of the operation, ‘pairing’. Identifying elements of the input, the domain and the codomain will provide some indication of the criteria employed in this evaluative event.

The ‘open’ and ‘closed’ counters used in pairing required that students pair and count recursively, where open counters represented positive integers and solid counters represented negative integers. It is interesting to note that the function, pairing, as presented in Figure H.9 is in fact a substitution for the additive inverse axiom; $(\forall x \in \mathbf{Z}, \exists -x \in \mathbf{Z} | x + (-x) = 0)$. It is evident that this axiom had been replaced by a technique that still enabled one to generate the correct outcome.

The object, integers, was not in sight when using the model in Figure H.8 as means of obtaining a solution. Co-pairing and counting the remainder of unpaired circles were the criteria that were employed in integer addition. Rather than paying explicit attention to complex objects i.e. addition over the integers, a sub-set of the \mathbf{R} and their properties, mathematics was constituted as a repertoire of procedures in the rules presented. Addition over the integers has some very convenient operatory properties, namely, associativity, commutativity, closure, identity and nullity. Hence $4 + (-3)$, could also have been calculated using the symmetry of \mathbf{R} with respect to addition and multiplicative inverses i.e. $4 + (-3) = 1 + 3 + (-3) = 1$. Performing the desired calculations in integer addition really does not require such elaborate procedures as presented by the teacher and texts for teaching.

If knowledge of integer addition was a prerequisite for performing these calculations, the examples used in Figure H.2 at the commencement of the lesson in EE1 could have been described a lot simpler:

- $-5 + 6 = 6 - 5$ (Both the teacher and students fail to recognize that $(\mathbb{Z}, +)$ is commutative)
- $-7 + 5 = -2 + -5 + 5$ (Using the property of the identity element i.e. $a + (-a) = 0$)

Integer subtraction

Subtraction of integers was not dealt with explicitly; instead a rule was formulated as an alternative to cope with subtraction problems of the type: $-8 - (-2)$. Rule 2 (Jaffer, 2009: 51) dealt with these types of problems using multiplication (because of the brackets) and addition to generate the required solution, rather than subtraction. Consider the following example (Figure H.10) presented in Lesson 1 which makes no mention of subtraction of integers:



$$4 - (-3) \Rightarrow 4 - (-3) = 4 + 3 = 7$$

Figure H.10: Subtraction of integers using Rule 2 (S P6 L1 EE3)

The presence of brackets regulated both the students and the teacher to multiply and then apply the rule for integer addition when the signs were the same.

Now consider the computational activity for this evaluative event in Table H.4:

Table H.4: The computational activity for S P6L1EE2

Lesson 1	EE2 [03:46 – 21:03]	Input (domain)	Output (codomain)	Operation
Operations (addition and subtraction) on integers		\mathbb{Z}	\mathbb{N} \mathbb{Z}	$(\mathbb{Z}, +) \rightarrow (\mathbb{N}, -) \rightarrow (\mathbb{Z}, +)$ Sundering Concatenation Character strings

Curriculum and textbooks

The teacher used the ‘poker chip model’ (Figure H.11) presented by the textbook as a means for illustrating addition and subtraction of integers. As mentioned earlier, co-pairing and counting the remainder of unpaired circles are the criteria that were employed in integer addition when using this model.

- When we try to understand science phenomena we often use models and theories to help us. We can do the same in mathematics. One way to illustrate the addition and subtraction of integers is to use a poker chip model. This may help you to understand.

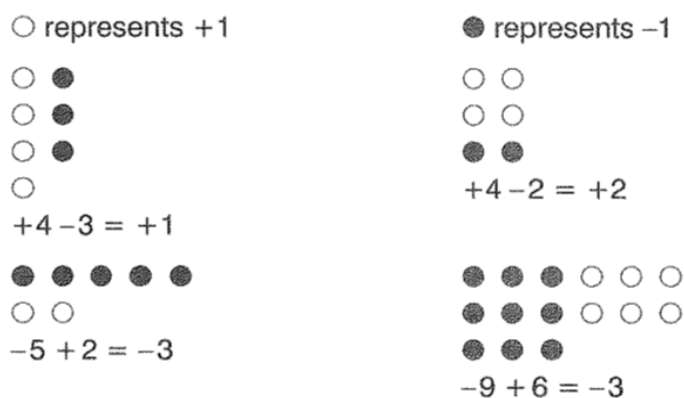


Figure H.11: The poker chip model in *My Clever* (Nel *et al.*, 2006:30)

After presenting four examples of how to apply this model in Figure H.11, students were instructed to draw poker chip models to solve three problems in the exercise presented in Figure H.12:

On your own, in your activity book, draw poker chip models to solve the following:

- $-4 + 3$
- $6 + -4$
- $-3 + -2$

Figure H.12: An exercise on integers using the poker chip model in *My Clever* (Nel *et al.*, 2006:30)

Now consider a student's correct responses to (a) and (b) in Figure H.13, yet he experienced a problem with (c) which he later corrected when the teacher came to his desk to check his solutions. When observing the video footage of this lesson, this particular student did not use the concept of pairing and cancelling as prescribed by the textbook. He referred to a number line on the opposite page in his notebook as he calculated the answers.

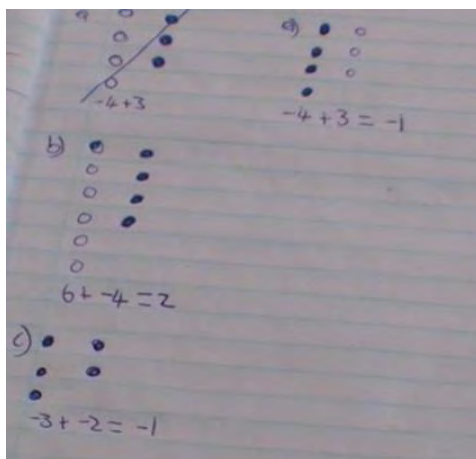


Figure H.13: Solutions to the exercise on the poker chip model

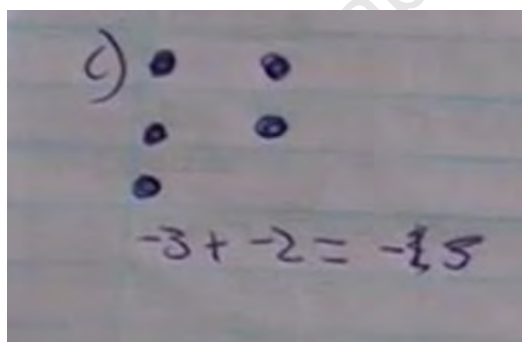


Figure H.14: A corrected version of (c)

So, rather than discussing the operatory possibilities that exist over integer addition, the textbook presented this topic using a model stating: 'This may help you understand' (Nel *et al.*, 2006:30).

The mathematics encyclopaedia

The mathematics encyclopaedia provides an extensive account of the convenient operatory properties that hold for both addition and subtraction over the integers. Rather than pay attention to properties such as associativity, commutativity, closure, identity and nullity, mathematics is constituted as a selection of procedures in the rules presented at the start of Lesson 1. Consider these operatory properties for addition and subtraction of integers:

Addition over the integers and its properties has some convenient operatory possibilities:

1. $\forall a, b \in \mathbb{Z}, (a + b) \in \mathbb{Z};$
2. $\forall a, b, c \in \mathbb{Z}, (a + b) + c = a + (b + c);$
3. $0 \in \mathbb{Z}$, and for $\forall a \in \mathbb{Z}, a + 0 = a = 0 + a;$
4. $\forall a \in \mathbb{Z}, \exists a^{-1} \in \mathbb{Z}$ such that $a + a^{-1} = 0;$
5. for $\forall a, b \in \mathbb{Z}, a + b = b + a.$

$(\mathbb{Z}, -)$ has the following operatory properties:

1. $\forall a, b \in \mathbb{Z}, (a - b) \in \mathbb{Z};$
2. $\forall a, b, c \in \mathbb{Z}, (a - b) - c \neq a - (b - c);$
3. $\exists 0 \in \mathbb{Z}$ such that, $\forall a \in \mathbb{Z}, a - 0 = a$, but $0 - a = -a.$
4. $\forall a \in \mathbb{Z}, a - a = 0$ [every element of \mathbb{Z} is its own inverse];
5. $\forall a, b \in \mathbb{Z}, a - b \neq b - a$, unless $a = b.$

Integers: see L1 EE1

Natural numbers: see L1 EE1

Regulation of the computational activity

Although integers were named in this evaluative event, operations were not performed on integers but on whole numbers. The signs of the integers served as a marker for which calculation procedure to follow and as a result the content to be acquired was a selection of the correct *computational procedure* (for addition of integers). All the calculations involving integers were transformed into whole number calculations with additional rules for dealing with signs. In the absence of teaching students the fundamental axioms and propositions relating to integers, schooling has to account for propositions in some kind of way by devising a range of procedures and operations for producing the same outcome. Table H.3 provided one account of the constitution of integer addition. Now consider another account of how this topic is constituted in Figure H.5:

Table H.5: The constitution of integer addition

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Integer addition when signs are different	Poker chip model involving pairing and cancelling	Computational resource	Counting	Additive inverse theorem ($\forall x \in \mathbb{Z}, \exists -x \in \mathbb{Z} x + (-x) = 0$)

School P6 Lesson 1 EE3: Recap of rules for multiplication and division of integers

Describing the computational activity

Multiplication and division of integers

Multiplication and division of integers was announced, but as with addition and subtraction of integers, the rules remained an important regulative resource for producing the required answers. Consider the following extracts from the transcript in Lesson 1 which presented an introduction of the topic followed by the criteria made available to regulate the solution procedure:

Announcing the topic

Teacher: [...] let's talk about imultiplication {multiplication} as well as division because the rules are more or less the same. Multiplication and division of integers. [...] (S P6 L1 EE3 lines 130-131)

Multiplication of integers

Teacher: So ngamanye amazwi {in other words} it's a rule, ithi irule yethu {our rule says}, when you multiply a negative multiplied by negative we get a positive. When you multiply a positive by a positive you get a?

Learners: Negative.

Learners: Positive. (S P6 L1 EE3 lines 149-152)

Teacher: So when signs are the same, that is very important. Negative negative or positive positive, you get a?

Learners: Positive answer.

Teacher: But when the signs are different when you are multiplying you get a?

Learners: Negative. (S P6 L1 EE3 lines 204-207)

Division of integers

Teacher: So irules ze division ne {the rules of division and} multiplication are the same. Sihamba sonke {Are we still following?}?

Learners: Yes.

Teacher: Are the same, so when we are dividing a like sign, we get a?

Learners: Negative.

Teacher: iSign zethu {Our sign} are not the same so we get a?

Learners: Negative.

Teacher: But when isigns zethu zifana {Our signs are the same} we get a?

Learners: Positive. (S P6 L1 EE3 lines 222-229)

Often when the teacher asked for the answer when multiplying or dividing integers, questions were phrased in terms of the signs of the integers only (See Figure H.15). Understandably, students then responded in terms of the signs but were often undecided as to whether the final answer was positive or negative and answered incorrectly.

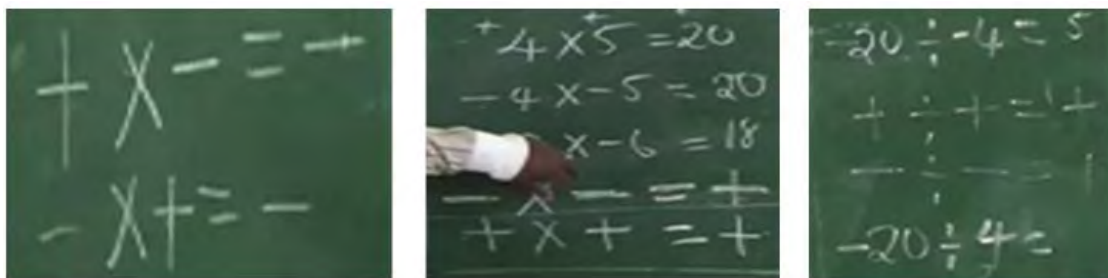


Figure H.15: The rules for multiplication and division of integers (S P6 L1 EE3)

Multiplication of integers and the presence of brackets

Now consider how to proceed in Figure H.16 when there were brackets instead of multiplication signs:

- Teacher: [...] remember mos sasithe ibrackets {we spoke about the brackets}, what did ibrackets {brackets}, when we see ibrackets {brackets} what do we do? Okay umzekelo {an example} what is the answer there? (See Figure H.12)
- Learner: Twenty
- Teacher: So ibrackets ziminisha {brackets mean} multiply.
- Learners: Multiply. (S P6 L1 EE3 lines 240-244)

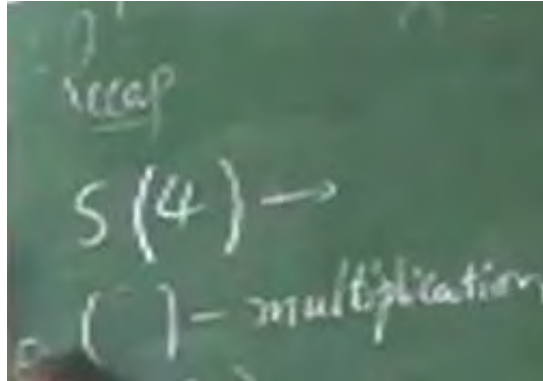


Figure H.16: Brackets and multiplication (S P6 L1 EE3)

There was a reliance on the iconic feature of the brackets in Figure H.16 and Figure H.18 which signalled the operation of multiplication and hence the rules for multiplying integers also applied. The topic, multiplication of integers, did not necessarily regulate the activity but rather the new object appeared to be a selection of the appropriate rules for generating the correct answer. It is clear that the presence of brackets indicated multiplication, e.g. $5(4)$, $-5(-4)$, $5(-4)$, and that memorising the rules for multiplication and division of integers were extremely important. Students were constantly asked to recite these rules as displayed in Figure H.17:

Multiplication of integers					Division of integers				
+	×	+	=	+	+	÷	+	=	+
-	×	-	=	-	-	÷	-	=	+
+	×	-	=	-	+	÷	-	=	-
-	×	+	=	-	-	÷	+	=	-

Figure H.17: Rules for multiplication and division of integers

Consider the following two calculations in Figure H.18 and Figure H.19 where brackets occurred and students also needed to apply the appropriate rule for multiplying positive and negative integers:

Figure H.18: A calculation with brackets (S P6 L1 EE3)

$$-(-2) = 2$$

Figure H.19: Multiplying $-(-2)$

Students were constantly quizzed on the rules that applied when multiplying integers and were encouraged to memorise these rules as they were the same for multiplication and for division as displayed in Figure H.20:

$$\begin{array}{l} +4 \times 5 = 20 \\ -4 \times -5 = 20 \\ + \times -6 = 18 \\ + \times + = + \\ - \times - = + \end{array}$$

$$\begin{array}{l} -20 \div -4 = 5 \\ + \div + = + \\ - \div - = + \\ -20 \div 4 = \end{array}$$

Figure H.20: The rules for division and multiplication are the same.

An effect of a dependency on the rules for integer arithmetic

The calculation that follows in Figure H.21 is a classic example of how the emphasis and reliance on a series of rules as the means of regulation for producing a legitimate solution often produced outcomes that were uncertain. The problem involved integer addition and the presence of a set of brackets in Step 1 was a cue for both the teacher and the students to multiply.

$$\begin{array}{l} -5 + (-2) \rightarrow \text{STEP 1} \\ = -5 - 2 \rightarrow \text{STEP 2} \\ = -7 \rightarrow \text{STEP 3} \end{array}$$

Figure H.21: Confusion in integer addition (S P6 L1 EE3)

In Step 2 of Figure H.21 students were supposed to rely on the rule for integer addition for when the signs were the same. Instead, they relied on the rule for integer multiplication when the signs were the same. When the teacher requested an answer in Step 3 the following dialogue ensued:

- Teacher: Ngubani kengoku ianswer pha? {What's the answer there?} Uzoqala ngantoni qala? {Where will you begin?}
- Learners: Brackets.
- Teacher: So positive times la {that} negative.
- Learners: Negative.
- Teacher: Lena yona sizoyithoba yona {That one we will leave}, it's not part of the ...?
- Learners: Multiplication
- Teacher: Multiplication, so we are going to go with it as it is. So negative five minus two?
- Learners: Positive seven.
- Teacher: Huh? Minus five minus two?
- Learners: Negative seven. (S P6 L1 EE3 lines 308- 317)

Finding the answer in Step 3 in Figure H.20 was problematic for learners; they observed the co-occurrence of two negative signs in Step 2 and expected a particular outcome. They possibly confused this with the rule for multiplication of integers with the same sign when they responded with an answer of 'positive seven'. They needed to be questioned again, after which they simply provided the alternative to 'positive seven', namely, 'negative seven'. Table H.6 presents an account of the computational activity for this evaluative event.

Table H.6: The computational activity for S P6L1EE3

Lesson 1	EE3 [21:03 – 40:36]	Input (domain)	Output (codomain)	Operation
Recap of rules for multiplication and division of integers		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<u>Multiplication and division of integers:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply or divide signs using the rules of multiplication and division of signs. 3. Multiply or divide whole numbers. 4. Re-attach sign to the final whole number answer $(\mathbb{Z}, \times) \rightarrow (\mathbb{N}, \times) \rightarrow (\mathbb{Z}, \times)$ Sundering Concatenation Character strings

Curriculum and textbooks

As described earlier, the RNCS' commentary on the topic of integer multiplication is rather inexplicit. It requires that a student simply:

- (r)ecognises, classifies and represents the following numbers in order to describe and compare them:
 - integers; [...]
 - additive and multiplicative inverses; [...] (DoE, 2002: 69)

The textbook's account of this topic was more comprehensive and closely resembled the teacher's presentation of this topic. A similar summary for the rules for multiplication and division as presented by the teacher in Figure H.20 is also found in the textbook (in Figure H.22):

An important summary:

$$(+)\times(+)=+$$

$$(+)\times(-)=-$$

$$(-)\times(-)=+$$

$$(-)\times(+)= -$$

The same applies to division:

$$\frac{+}{+}=+$$

$$\frac{-}{-}=+$$

$$\frac{+}{-}= -$$

$$\frac{-}{+}= -$$

Figure H.22: A summary of rules for multiplication and division in *My Clever* (Nel *et al.*, 2006: 32)

Both the teacher and the textbook, in Figure H.22, highlighted the fact that the rules for multiplication and division were the same:

Teacher: So irules ze division ne {the rules of division and } multiplication are the same. Sihamba sonke?
 {Are we still following}

Learners: Yes (S P6 L1 EE3 lines 222-223)

The textbook also presented students with an exercise in Figure H.23 where, from a series of calculations, they needed to draw conclusions regarding the rules for integer multiplication in Figure H.24:

Activity 3.5: Multiplication and division of integers (group and individual activity)

- Copy and complete the following in your activity book:

Factors	→	Product	Factors	→	Product
$(+2) \times (+4)$	=	+8	$(-2) \times (+4)$	=	-8
$(+2) \times (+3)$	=	+?	$(-2) \times (+3)$	=	-?
$(+2) \times (+2)$	=	+?	$(-2) \times (+2)$	=	-?
$(+2) \times (+1)$	=	+?	$(-2) \times (+1)$	=	-?
$(+2) \times 0$	=	0	$(-2) \times 0$	=	0
$(+2) \times (-1)$	=	-2	$(-2) \times (-1)$	=	+2
$(+2) \times (-2)$	=	-?	$(-2) \times (-2)$	=	+?
$(+2) \times (-3)$	=	-?	$(-2) \times (-3)$	=	+?
$(+2) \times (-4)$	=	-?	$(-2) \times (-4)$	=	+?

Figure H.23: Multiplication and division of integers in *My Clever* (Nel *et al.*, 2006: 31)

2. In your groups, compare your answers. What do you notice? Discuss the multiplication of integers until you reach a consensus (an agreement) about the signs. Now complete the following together:

When multiplying a positive integer and a positive integer, the product is always

When multiplying a positive integer and a negative integer, the product is always

When multiplying a negative integer and a negative integer, the product is always

Figure H.24: General pattern for the rules for multiplication and division in *My Clever* (Nel *et al.*, 2006: 32)

As with integer addition, the rules for integer multiplication and division took precedence over any of the operatory properties that exist for integers. The range of different rules that students had at their disposal could only be used in specific instances and this created an added burden of having to remember when to apply which rule. The textbook presented a summary of the rules for multiplication and division of integers in Figure H.25 after the investigative exercise in Figure H.24:

When multiplying or dividing integers with the same sign, the answers will be positive. When multiplying or dividing integers with different signs, the answers will be negative.

Figure H.25: A summary of the rules for multiplication and division of integers in *My Clever* (Nel *et al.*, 2006:36)

What follows now is a description of integer multiplication and integer division in the mathematics encyclopaedia.

The mathematics encyclopaedia

The mathematics encyclopaedia provides a complete account of the operatory properties for both integer multiplication and integer division:

(\mathbb{Z}, \times) has the following operatory properties:

1. $\forall a, b \in \mathbb{Z}, (a \times b) \in \mathbb{Z}$;
2. $\forall a, b, c \in \mathbb{Z}, (a \times b) \times c = a \times (b \times c)$;
3. $1 \in \mathbb{Z}$, and for $\forall a \in \mathbb{Z}, a \times 1 = x = 1 \times a$;
4. $\forall a \in \mathbb{Z}, a \neq 1, \exists a^{-1} \in \mathbb{Z}$ such that $a \times a^{-1} = 1$;
5. for $\forall a, b \in \mathbb{Z}, a \times b = b \times a$.

(\mathbb{Z}, \div) has the following operatory properties:

1. $\forall a, b \in \mathbb{Z}, (a \div b) \in \mathbb{Z}$ only if $a|b$;
2. $\forall a, b, c \in \mathbb{Z}, (a \div b) \div c \neq a \div (b \div c)$;
3. $1 \in \mathbb{Z}$, and for $a \in \mathbb{Z}, a \div 1 = a$, but $1 \div a$ is undefined if $a \neq 1$;
4. $\forall a \in \mathbb{Z}, a \div a = 1$ [every element of \mathbb{Z} is its own inverse]; for $a, b \in \mathbb{Z}, a \div b \neq b \div a$, unless $a = b$.

Integers: see L1 EE1

Natural numbers: see L1 EE1

These operatory properties found in the encyclopaedia do not require memorisation of specific rules for operating on integers, neither does it force one to sunder or concatenate integers into whole numbers and signs as described by Jaffer (2009).

Regulation of the computational activity

The rules for multiplication and division of integers regulated this evaluative event and served as *computational resources* for achieving the required solution. I have provided an account of the constitution of the integer multiplication and division in Table H.7:

Table H.7: The constitution of integer multiplication and addition

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Integer multiplication	Separate signs and numbers and multiply separately	Computational resource	Whole number arithmetic	Operatory properties of integer multiplication
Integer division	Separate signs and numbers and divide separately	Computational resource	Whole number arithmetic	Operatory properties of integer division

School P6 Lesson 2

Generating the evaluative events

Table H.8: Evaluative events spanning Lesson 2 at School P6 (S P6 L2 EE1)

Evaluative Event	Type	Length
01 Recap of the rules for multiplying and dividing integers [00:00 – 02:12]	Expository	02:12
02 Integer arithmetic (all four operations) [02:49 – 36:09]	Exercise	04:04
		36:09

School P6 Lesson 2 EE1: Review of the rules for multiplying and dividing integers

Describing the computational activity

This lesson commenced with a review of the rules for multiplication and division of integers, similar to what was presented in L1 EE3. Memorising these rules was a pre-requisite for achieving a successful outcome and the teacher drilled students in the transcript extract that follows:

Teacher: Positive times positive equals?
 Learners: Negative
 Teacher: Negative times negative equals?
 Learners: Positive
 Teacher: Remember if the signs are positive, we get a positive sign, sihamba sonke? {are we altogether?}
 Learners: Yes
 Teacher: Positive times negative equals?
 Learners: Negative
 Teacher: Negative times positive equals?
 Learners: Negative
 Teacher: Remember sithe {we said} if the signs are not the same we get a?
 Learners: Negative answer
 Teacher: That is for multiplication. Than for i-division {division} sithe {we said} positive divide by positive equals?
 Learners: Positive
 Teacher: Negative divide by negative?
 Learners: Positive
 Teacher: Remember sithe {we said} if the signs are the same kwi-division {in division} it's a positive. A negative divide by positive?
 Learners: Negative
 Teacher: Then positive divide by negative?
 Learners: Negative (S P6 L2 EE1 lines 1-20)

Table H.9 presents an account of the computational activity for this evaluative event.

Table H.9: The computational activity for S P6L2EE1

Lesson 2	EE1 [00:00 – 02:12]	Input (domain)	Output (codomain)	Operation
Review of rules for multiplication and division of integers		Character strings	Character strings	(SIGN, \times) (SIGN, \div)

Curriculum and textbooks

See L1 EE3

The mathematics encyclopaedia

See L1 EE3

Integers: see L1 EE1

Regulation of the computational activity

See L1 EE3

School P6 Lesson 2 EE2: Integer arithmetic (all 4 operations)

Describing the computational activity

Summary of the rules for integer addition

Although the announced topic, at the commencement of this evaluative event, was addition of integers, the true nature of an integer and its properties were never described or engaged with. Jaffer (2009: 51), in her analysis of the series of lessons at School P6 provides a review of the three rules and associated problems generated by the teacher for integer arithmetic:

Type 1: $-10 + 2$: Rule 1: If the signs are different, subtract the digits and take the sign of the bigger number. If the signs are the same, add the digits and use the common sign.

Teacher: Subtract the bigger digit from the smaller digit and put the sign of the bigger digit. (S P6 L1 EE1 line 10)

Type 2: $-8 + (-2)$: Rule 2: If there are brackets, multiply the sign outside the bracket with the sign inside the bracket and then apply Rule 1.

Teacher: When we see brackets that means what? Multiplication. But when there is a sign before the bracket we need to multiply whatever's inside the bracket by the sign before we can add. (S P6 L2 EE2 lines 78-80)

Type 3: $20 + \Delta = 12$: Rule 3: When adding, we expect the answer to get bigger. If the answer is smaller, then one must add a negative number. If the answer is bigger, then add a positive number.

Teacher: When we are adding to a positive number that number is going to increase, but xa- u-addisha kwi positive number then i-answer yakho iya dece-reasea {when you are adding to a positive number then your answer will decrease} that means u-addisha what? {What are you adding?} What did you add? A negative number. (S P6 L2 EE2 lines 56-58)

The three rules presented certainly displayed that the criteria made available for obtaining a solution entailed operations that separated signs *and* numbers and signs were also treated separately.

Besides integer addition, this lesson also covered a series of problems relating to multiplication and division of integers where the emphasis remained focused on the rules:

Teacher: Negative eight times negative two gives us sixteen. Remember kaloku sithe {we said} to get a positive answer the signs must be the same, that is very, very important to get a positive answer, the signs must be the same. It can be positive-positive or it can be negative-negative. (S P6 L2 EE2 lines 180-181)

The teacher then referred to a list of rules in Figure H.26 and classified them as 'tools' for doing exercises. Consider the 'tools' snapshot in Figure H.26 as well as the transcript extract which emphasised the utility of these rules:

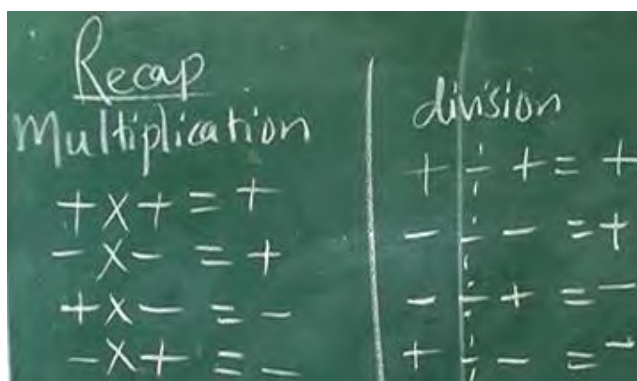


Figure H.26: The rules for multiplication and division of integers (S P6 L2 EE2)

Teacher: Kengoku {Now} in future when we doing i-exercise zethu {our exercises} I want to see this (pointing to the signs on the board in Figure H.21) because zi-tools zethu, umzekelo {they are our tools}. For example, (Opening one of the learners exercise book showing to the class how he wants it to be). Whenever you going to do your calculations I want to see your tools on this side (left hand side) and then your calculations on this side (right hand side). Why ndifuna nibe nazo ezi-tools? {Why do I want you to have these tools?} I want you to remember these signs, this is very important. (Pointing to the signs on the board.) Sihamba sonke, class? (Are we following, class?)

Learners: Yes. (S P6 L2 EE2 lines 188-193)

Now consider the computational activity for this evaluative event in Table H.10:

Table H.10: The computational activity for S P6L2EE2

Lesson 2	EE2 [02:12 – 36:09]	Input (domain)	Output (codomain)	Operation
Integer arithmetic (all 4 operations)		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<p><u>Adding integers with the same sign:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Add the whole numbers. 3. Re-attach the common sign to the final whole number answer. <p><u>Adding integers with different signs:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Subtract smaller whole number from the bigger whole number. 3. Re-attach the sign of the bigger whole number to the final whole number answer

Lesson 2	EE2 [02:12 – 36:09]	Input (domain)	Output (codomain)	Operation
				<p><u>Multiplication and division of integers:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply or divide signs using the rules of multiplication and division of signs. 3. Multiply or divide whole numbers. 4. Re-attach sign to the final whole number answer <p> $(\mathbb{Z}, +) \rightarrow (\mathbb{N}, +) \rightarrow (\mathbb{Z}, +)$ (\mathbb{Z}, \times) (\mathbb{Z}, \div) Sundering Concatenation Character strings </p>

Curriculum and textbooks

See L1 EE1, EE2 and EE3

The mathematics encyclopaedia

Integers: see L1 EE1

See L1 EE1, EE2 and EE3

Regulation of the computational activity

A reliance on the rules for integer arithmetic, as presented in Lesson 1 and Lesson 2 was the primary means of regulation in this evaluative event. Students were faced with having to select the correct series of computations that related to addition, subtraction and multiplication of integers. These rules therefore served as *computational procedures* that were heavily dependent on the signs of the integers and they acted as substitutes for providing students with an account of the operatory properties of integer addition, subtraction, multiplication and division.

School P6 Lesson 3

Lesson 3 was a double period lesson and the teacher did not refer to the textbook, *My Clever*, at all. He explained problems from a worksheet which was handed out to students. The researcher who video-recorded this particular lesson did not provide a copy of this worksheet.

Generating the evaluative events

Table H.11: Evaluative events spanning Lesson 3 at School P6 for Periods 1 and 2 (S P6 L3 EE1-EE6)

Evaluative Event (Period 1)	Type	Length
01 Recap of the rules for multiplying and dividing integers [00:00 – 08:13]	Expository	08:13
02 Order of operations [08:30 – 45:38]	Expository	01:24
		36:52
Evaluative Event (Period 2)	Type	Length
03 Description of the commutative property and exercise[45:38 – 47:20]	Expository	01:42
04 Description of distributive property [47:20 – 51:10]	Expository	03:50
05 Distributive rule over division [51:10 – 59:15]	Expository	08:05
06 A description of the utility of the associative, commutative and distributive properties [59:15 – 66:45]	Expository	07:30
		21:07

School P6 Lesson 3 EE1: Review of the rules for multiplying and dividing integers

Describing the computational activity

This particular evaluative event focused on multiplication and division of integers once again and the lesson commenced with a recap of the rules as well as the teacher doing problems relating to this topic on the board. The rules, as described in Figure H.26, remained an important means of regulation for producing the required answers. A homework exercise given to students, based on multiplication of integers, was checked during this lesson.

When the problems in Figure H.27 were explained on the board, integers were seldom referenced as such – but were rather treated in a manner that either considered multiplication of signs or multiplication of whole numbers.

Handwritten mathematical problems on a chalkboard:

- 19) $(-3) \times (-3) = 9$
- 20) $-1 \times 8 = -8$
- 21) $-8 \div \boxed{2} = \underline{-4}$
- 22) $\underline{-5} \times (-2) \times (-2) = \underline{-20}$

Figure H.27: Multiplication of integers

Consider the transcript extract that follows for multiplying three signs as well as Figure H.28 where no reference was made to indicate that these signs were actually related to an integer:

- Teacher: Sihamba sonke? {We agreed} [...]Remember kaloku sithe {we said that} a positive times a negative times a positive gives a? Positive times a negative?
- Learners: Negative
- Teacher: Negative, ezi yi {there are.} two. Negative times positive?
- Learners: Negative
- Teacher: Negative, siyabona? {Do we all see?}
- Learners: Yes
- Teacher: Negative times a positive times a negative? Negative times a positive?
- Learners: Negative
- Teacher: Negative. Negative times a negative?
- Learners: Positive
- Teacher: Positive, siyayibona lonto? Siyayibona lonto? {Do we all see?}
- Learners: Yes (S P6 L3 EE1 lines 74-86)

Handwritten sign multiplication rules on a chalkboard:

- $+\times - \times + = -$
- $- \times + \times - = +$
- $- \times - \times - = -$

Figure H.28: Multiplication of signs

Now consider the computational activity for this evaluative event in Table H.12:

Table H.12: The computational activity for S P6L3EE1

Lesson 3	EE1 [00:00 – 08:13]	Input (domain)	Output (codomain)	Operation
Review of the rules for multiplying and dividing integers		Character strings	Character strings	(SIGN, \times) (SIGN, \div)

Curriculum and textbooks

See L1 EE3

The mathematics encyclopaedia

See L1 EE3

Integers: see L1 EE1

Regulation of the computational activity

See L1 EE3

School P6 Lesson 3 EE2: Order of operations

Describing the computational activity

This evaluative event dealt primarily with processing problems that had more than one operation. It presented yet another set of rules for doing this:

Teacher: Yimani, yimani, yimani, yiyekeni ndizofunda ngokwam {Wait, wait, wait, I'll read it myself}, Nantsiya, when a calculating instruction, that is i-instruction {an instruction}, a calculating instruction includes more than one operation. You need to use the following rules to calculate the answer. Nantsi i-ruleyokuqala {Here, the first rule}, went from left to right. Do multiplication and division before addition and subtraction. You may find it helpful to put in brackets to show what to do first, sihamba sonke? {are we together?} (S P6 L3 EE2 lines 105-108)

This rule entailed that students always worked from left to right, that they did multiplication and division before addition and subtraction and that brackets were often helpful in indicating what operation to perform first. I have summarised three main rules from the transcript records for this evaluative event for processing more than one operation in a given problem. These rules were really an application of BODMAS⁶⁴ when presented with integer arithmetic. The criteria that students picked up from the following rules related to counting the number of operations, identifying them and then applying the correct procedure in terms of which operation needed to be performed first. The instruction to count the operations and identify them in the problem, $12 - 3 + 4$, was an explicit instruction given by the teacher and he referred to it as the first rule:

Teacher: Example efresh nantsiya {Another example}. 12 minus 3 plus 4. now let's look at those operations, how many operations are we having there?
Learners: Yes
Teacher: Iyintoni i-operation phofu? {What is the operation actually?} Give me an example of an operation.
Learner: Subtraction.
Teacher: Subtraction.
Learners: Division.
Teacher: Division.

⁶⁴ BODMAS is a mnemonic for helping remember the order for dealing with mathematical operators in a mathematics statement i.e. B – brackets, O – of, D – division, M – multiplication, A – addition and S – subtraction. (O-also stands for order (powers/exponents)).

Learners: Multiplication.
 Teacher: Multiplication.
 Learners: Addition.
 Teacher: Addition, okay so how many operations are we having here?
 Learners: 2
 Teacher: It's subtraction plus?
 Learners: Addition.
 Teacher: Now kengoku kuthiwani pha? {Then what is said there?} Kuthiwa {It says} let's work through the left going to the right.
 Learners: Right
 Teacher: Now 12 minus 3?
 Learners: 9
 Teacher: 9 plus 4?
 Learners: 13
 Teacher: Sihamba sonke? {Do we all agree?} So that was the first rule, ithini kengoku icomment? {So what does the comment say?} It's only a positive and a subtraction so went from left to right. (S P6 L3 EE3 lines 110-132)

Consider the procedure for Rule 1:

Rule 1: Processing more than one operation: (+ and -):

1. Count the number of operations.
2. Identify the operations.
3. If there are only two operations (+ and -), proceed from left to right.

Rule 1 was later revised and students were instructed to proceed in any order since integer addition, for when signs were the same and when they were different, had already been covered previously in Lesson 1. Another set of rules was presented for dealing with processing operations such as (+, -, ×, ÷). Consider how the problem $2 \times 4 - 1$ was dealt with in the following transcript extract:

Teacher: Kuthweni kengoku xa iexplainwa pha? {What is said when it is explained there?} Kuthiwa, {It is said} do division before addition. Let's do the next one. 2 times 4 minus 1? How many operations are we having there?
 Learners: 2
 Teacher: Which ones?
 Learners: Multiplication and subtraction
 Teacher: Now which one are we going to start with?
 Learners: Multiplication.
 Teacher: So we put in our brackets.
 Learners: Brackets.
 Teacher: To show that we are going to start with multiplication.
 Learners: Multiplication
 Teacher: So 2 times 4?
 Learners: 8
 Teacher: Siyabhala, {We are all writing}, you write it down because we have not subtracted yet. So kengoku {then} 8 minus 1?
 Learners: 7
 Teacher: Ithi iexplanation pha {The explanation there}, kuthiwa {says} do multiplication before subtraction.
 Learners: Subtraction. (S P6 L3 EE2 lines 167-183)

What follows is a detailed procedure for Rule 2 as presented above:

Rule 2: Processing more than one operation (+, −, ×, ÷):

1. Count the number of operations.
2. Identify the operations.
3. If there are ÷ and × signs, together with + and − signs, you may not proceed from left to right and × and ÷ should be performed first.
4. Insert brackets to indicate where to start the problem.

Consider the procedure for applying the next rule (in the transcript extract that follows and in Figure H.29) which considers processing multiplication and division signs in the same problem:

Rule 3: Processing more than one operation (× and ÷): (See Figure H.29)

1. Division is performed before multiplication BODMAS
2. Use calculator as a resource to check.

Figure H.29: Rule 3 for processing more than one operation (× and ÷) (S P6 L3 EE2)

Teacher: Masikhupheni icalculator. {Who has a calculator with them now?} Let's check kwi calculator {using the calculator} and see which... isinika eyiphi ianswer. {What answer it gives us?} Let's do nge {with} technology. Let's use technology going to help us. Calculate as you see it ibhaliwe pha ebhodini {on the board}. 24 divide by 3 times 4. So which one is correct there?

Learners: Division

Teacher: Yimani kaloku masibuyeni {Wait, let's go back.}. Icalculator {The calculator} is using this thing BODMAS. So ke {now} people the first one is correct. So it's always division before multiplication. Okay ke masibeni sixoxa lento {Let's all work together and discuss this.} (S P6 L3 EE2 lines 250-255)

Consider the computational activity for this evaluative event in Table H.13:

Table H.13: The computational activity for S P6L3EE2

Lesson 3	EE2 [21:03 – 40:36]	Input (domain)	Output (codomain)	Operation
Order of operations		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<u>Multiplication and division of integers:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply or divide signs using the rules of multiplication and division of signs. 3. Multiply or divide whole numbers. 4. Re-attach sign to the final whole number answer

Curriculum and textbooks

Neither the curriculum nor the textbook makes any mention of the order of operations when performing integer arithmetic.

The mathematics encyclopaedia

Integers: see L1 EE1

Regulation of the computational activity

An intricate set of rules served to regulate this evaluative event. BODMAS was one such rule which indicated the order in which to proceed when having to deal with multiple operations in a problem. When using BODMAS, brackets needed to be calculated first followed by division and multiplication which had the same priority as did addition and subtraction. If the problem had several operations of the same priority, one proceeded from left to right. BODMAS could therefore be characterised as a *computational resource* in this evaluative event. I have provided an account of the constitution of BODMAS in Table H.14:

Table H.14: The constitution of BODMAS

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
BODMAS	Proceed in this order: Brackets, Of (\times), Division, Multiplication, Addition, Subtraction	Computational resource	Whole number arithmetic	Operatory properties of integer arithmetic

School P6 Lesson 3 EE3: Description of the commutative property

Describing the computational activity

This evaluative event described the commutative property and the teacher emphasised that students were meant to recognise and apply this property to simplify problems:

Teacher: [...] you can simplify calculations by using the commutative, associative and distributive properties of operation. In this session you will learn how to recognise these properties and apply them to simplify calculations. (S P6 L3 EE2 lines 442-443)

In Figure H.30, he stressed that the order of operations was not important when adding and multiplying integers, as presented in the following transcript extract for the problems, $4 + 5$ and 3×6 :

Teacher: Masenzi kengoku, kuthiwa pha from page 2, kuthiwa properties of operation, masimameleni {We must do now, they say on Page 2, they say the properties of operation, let's listen.}, you can simplify calculations by using the commutative, association and distributive properties of operation. In this session you will learn how to recognise these properties and apply them to simplify calculations. Let's look at this one right, 4 plus 5, 5 plus 4. 4 plus 5 ngubani {what is it equal to?}? U 4 plus 5? {4 plus 5}

Learners: 9

Teacher: What about 5 plus 4?

Learners: 9 (S P6 L3 EE2 lines 441-447)

Teacher: No, because siqale ngabani? {What should we start with?} Ngo 3 saza ngo 6 kanti phasiqale ngabani? {At 3 then followed by 6} Ngo 6 saza ngobani ngo 3. {and then 6, followed 3} Kengoku xa siyibiza le yi {And now, what would you call it} commutative property Ithini? {What is it?} Ithi {It says}. When we add or multiply two numbers you can change the order of the number. Multiply and addition is sometimes easy to do when you change the order of numbers you are working with. So that means le property yokwazi (to be able to apply this property) ukuwaguqulela amanani {change/flip the numbers} we call it a? associ... or commutative property. The order of numbers is not important when we are adding or multiplication [...] (S P6 L3 EE2 lines 458-462)

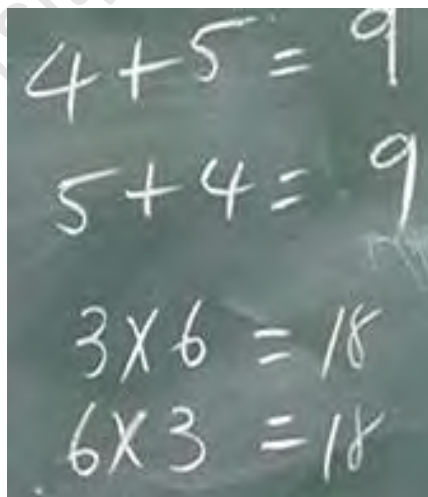

$$\begin{array}{l} 4 + 5 = 9 \\ 5 + 4 = 9 \\ 3 \times 6 = 18 \\ 6 \times 3 = 18 \end{array}$$

Figure H.30: Both addition and multiplication are commutative

In this lesson, the teacher described multiplication in terms of the multiplier and the multiplicand in Figure H.30 when discussing the commutative property, explaining that the multiplier and the multiplicand were interchangeable.

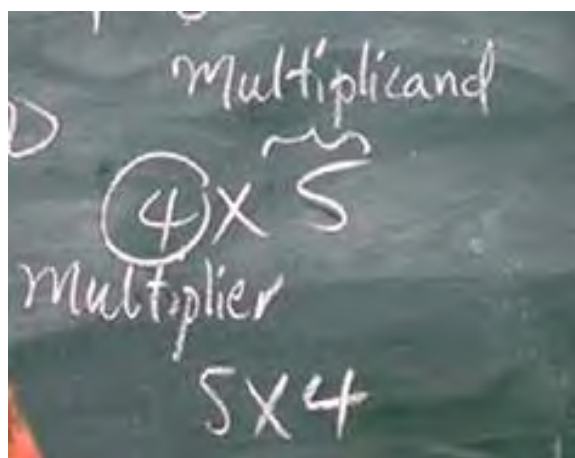


Figure H.31: The multiplier and multiplicand

Table H.15 presents an account of the computational activity for this evaluative event:

Table H.15: The computational activity for S P6L3EE3

Lesson 3	EE3 [45:38 – 47:20]	Input (domain)	Output (codomain)	Operation
Description of the commutative property		\mathbb{N}	\mathbb{N}	<p><u>Multiplication of integers:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply signs using the rules of multiplication of signs. 3. Multiply whole numbers. 4. Re-attach sign to the final whole number answer <p><u>Adding integers with the same sign:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Add the whole numbers. 3. Re-attach the common sign to the final whole number answer.

Curriculum and textbooks

The RNCS does provide commentary on what students need to know with regard to the commutative law:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

We know this when the learner:

Uses a range of techniques to perform calculations including:

- using the **commutative**, associative and distributive properties with rational numbers [...] (DoE, 2002: 73)

The teacher's description of what students needed to know when he announced this topic initially echoes the RNCS' recommendations as well. No textbook was used for this topic and the teacher referred to a worksheet for the entire the lesson.

The mathematics encyclopaedia

The mathematics encyclopaedia provides a detailed account of the commutative property for both multiplication and addition:

Commutative property:

Addition is commutative: $a + b = b + a$

Multiplication is commutative: $a \cdot b = b \cdot a$ (Usiskin *et al.*, 2003: 232)

The set \mathbb{R} of real numbers with the operation $+$ of addition is a commutative group $(\mathbb{R}, +)$.

The set \mathbb{R}^+ of positive real numbers with the operation \cdot of multiplication is a commutative group (\mathbb{R}^+, \cdot) . (Usiskin *et al.*, 2003: 11)

Multiplicand: a number to quantity to be multiplied by another: In 5 times 497, the multiplicand is 497. (World Book Dictionary, 1982: 1366)

Multiplier: a number by which another number is to be multiplied: In 5 times 83 the multiplier is 5. (World Book Dictionary, 1982: 1366)

Students were, however, only presented with specific examples when applying the commutative property rather than a being presented with a general form of the property e.g.: $a + b = b + a$ or $a \cdot b = b \cdot a$.

Regulation of the computational activity

The commutative property was presented as a *computational procedure* in this evaluative event especially since the teacher stated that: '[...] you can simplify calculations by using the commutative, associative and distributive properties of operation' (S P6 L3 EE2 lines 442). Table H.16 provides an account of the constitution of the commutative property:

Table H.16: The constitution of the commutative property

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Commutative property	Addition Multiplication	Computational resource	Whole number arithmetic	$a + b = b + a$ $a \cdot b = b \cdot a$

School P6 Lesson 3 EE4: Description of the distributive property

Describing the computational activity

The distributive property appeared to be the intended object of learning in this evaluative event. Problems were encountered when negative integers were multiplied or divided with positive integers. The emphasis appeared to be on performing the correct operations on the integers, rather than recognising the value of the distributive property at the operative level. Consider Figure H.32 which presents an attempt at showing how the distributive property worked:

Figure H.32 shows two methods for calculating $5 \times (23 + 17)$. Method (a) on the left illustrates the distributive property: $5 \times (23 + 17) = 5 \times 23 + 5 \times 17 = 115 + 85 = 200$. The word 'Multiplicand' is written below the final result. Method (b) on the right illustrates the BODMAS rule: $5 \times (23 + 17) = 5 \times 40 = 200$.

Figure H.32: A choice between the distributive rule or BODMAS

After the teacher offered two presentations of the problem $5 \times (23 + 17)$ in Figure H.32, he posed the question: ‘Which is easier?’ In the first instance (see Figure H.32 (a)), he provided a description of the distributive property and presented it as a rule, where 5 was multiplied by both 23 and 17 in the set of brackets. The distributive property was presented as a *computational procedure* where specific numbers were used rather than presenting a general case for this property.

He then juxtaposed the first attempt in (a) with an alternative method of doing the same problem in (b) (see Figure H.32). He seemed to display a more simplified version of this problem in the second attempt by omitting the multiplication sign. This version of the problem looked less complicated and may well suggest that he was attempting to regulate the solution procedure by simplifying the problem. Or he may simply have been providing a justification for BODMAS, especially since he asked: ‘Which is easier?’

Consider the procedure for (a) and (b) in Figure H.32:

Procedure (a): $5 \times (23 + 17)$

1. Multiply the 5 by both 23 and 17.
2. Add the two answers

Procedure (b): $5(23 + 17)$

1. Perform the operation in brackets first.
2. Multiply 5 by the result in the bracket.

Table H.17 presents an account of the computational activity for this evaluative event:

Table H.17: The computational activity for S P6L3EE4

Lesson 3	EE4 [47:20 – 51:10]	Input (domain)	Output (codomain)	Operation
Description of the distributive property		\mathbb{N}	\mathbb{N}	<u>Multiplication of integers:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply signs using the rules of multiplication of signs. 3. Multiply whole numbers. 4. Re-attach sign to the final whole number answer
				<u>Adding integers with the same sign:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Add the whole numbers. 3. Re-attach the common sign to the final whole number answer.

Curriculum and textbooks

The distributive property is named in the RNCS and students are expected to recognise, describe and use this property:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

We know this when the learner:

Uses a range of techniques to perform calculations including:

- using the commutative, associative and **distributive** properties with rational numbers [...] (DoE, 2002: 73)

In this evaluative event, as with the commutative property in EE3, students were once again only presented with specific examples when applying the distributive property rather than a being presented with a general form of the property e.g. : $a.(b + c) = a.b + a.c$.

For this evaluative event, the teacher also used a worksheet and made no reference to the textbook.

The mathematics encyclopaedia

The distributive property as presented in the mathematics encyclopaedia is not found in the RNCS:

Distributive property:

$a.(b + c) = a.b + a.c$ (Usiskin *et al.*, 2003: 232)

Multiplication:

An algebraic system $(A, +, \cdot)$ consisting of a set A together with two binary operation, + and \cdot , on A is called an integral domain if it has the following properties. For all a, b , and c in A,

Multiplication:

- Multiplication is associative: $a.(b.c) = (a.b).c$
- Multiplication is commutative: $a.b = b.a$
- Existence of multiplicative identity: There is an element $1 \neq 0$ in A such that $a.1 = a$.
- Cancellation property: If $a.b = a.c$ and $a \neq 0$, then $b = c$.
- Distributive property:

$a.(b + c) = a.b + a.c$ (Usiskin *et al.*, 2003: 232)

Integers: see L1 EE1

Again, as with the commutative property, the distributive property was described by the teacher using specific examples, rather than providing a general definition of the property. Although the RNCS names this property, it provides no formal definition.

Regulation of the computational activity

The distributive property served as a *computational procedure* in this evaluative event and this was emphasised by the teacher when he said: '[...] you can simplify calculations by using the commutative, associative and distributive properties of operation' (S P6 L3 EE2 lines 442). BODMAS could also be considered to be a computational procedure which regulated this evaluative event. Table H.18 provides an account of the constitution of the distributive property:

Table H.18: The constitution of the distributive property

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Distributive property	Multiplication Addition Division Subtraction	Computational resource	Whole number arithmetic	$a.(b + c) = a.b + a.c$

School P6 Lesson 3 EE5: Distributive rule over division

Describing the computational activity

The presentation of this evaluative event is similar to the previous one where the teacher presented two methods of doing the same problem i.e. $72 - 48 \div 8$. In the first instance he used the distributive rule to produce a solution followed by using BODMAS to simplify the same problem. I have provided an account of the two procedures and have noted repeated errors in integer arithmetic. In this problem it seemed as though the teacher contradicted his earlier rules regarding BODMAS where he emphasised that division should be done first. Consider the first method where the distributive property is presented in Figure H.33:

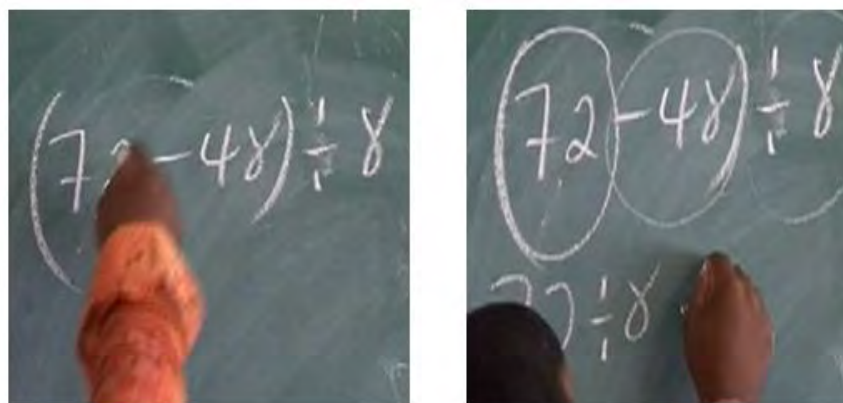


Figure H.33: Applying the distributive rule

Procedure 1: Using the distributive rule

Both 72 and 48 are divided by 8. Uncertainty, however, arises when -48 is divided by 8. The negative sign in front of the 48 together with the presence of a bracket in Figure H.34 causes confusion and it is worth mentioning that the signs and numbers have up until this point been treated as separate entities.

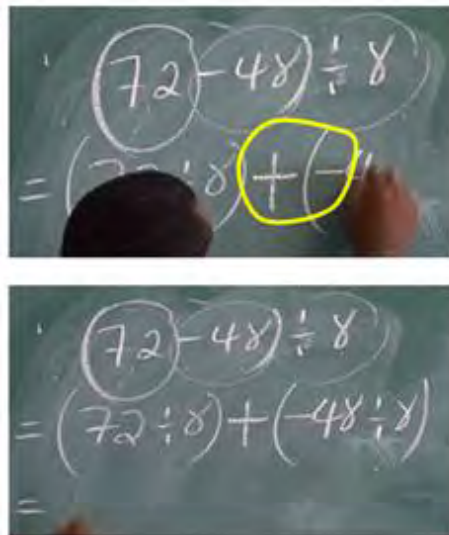

$$\begin{aligned} & (72 - 48) \div 8 \\ &= (72 \div 8) + (-48 \div 8) \\ &= 9 + (-6) \end{aligned}$$

Figure H.34: Brackets and a sign pose a problem

The teacher asks the question: ‘Brackets and a sign mean ...?’ when performing the calculation $9 + (-6)$ in Figure H.35. Students don’t recognise that they have to multiply $9 + (-6)$ to get $9 - 6 = 3$ even though they were taught this concept in L1 EE3. The teacher also mentions that the distributive property would not have worked in this instance if 48 was replaced with 50. This may imply that the distributive property only works for specific numbers and students are therefore not granted access to the generalised property at all.

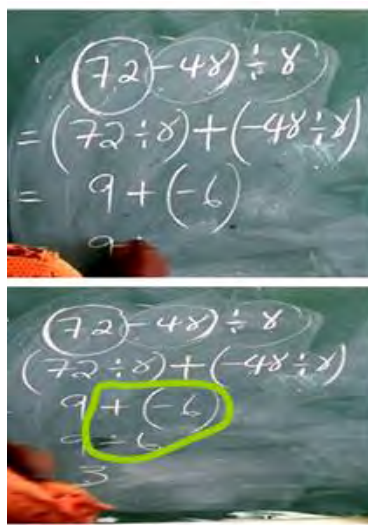

$$\begin{aligned} & (72 - 48) \div 8 \\ &= (72 \div 8) + (-48 \div 8) \\ &= 9 + (-6) \\ &= 3 \end{aligned}$$

Figure H.35: Students have forgotten what ‘brackets and a sign mean’

I have provided a detailed account of the procedure just presented:

Procedure 1:

1. Both 72 and 48 are divided by 8.
2. Each calculation is performed separately in a bracket i.e. $(72 \div 8) + (-48 \div 8)$.
3. Rules for division of integers are applied i.e. $9 + (-6)$
4. Rules for addition of integers with different signs are applied i.e. subtract smaller digit from bigger digit and attach sign of the bigger digit i.e. $9 - 6 = 3$

Procedure 2: Using BODMAS

Now consider an alternative method for calculating $72 - 48 \div 8$. which uses BODMAS. The teacher instructs students to do brackets first in Figure H.36:

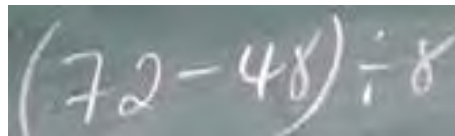

$$(72 - 48) \div 8$$

Figure H.36: Using BODMAS for $72 - 48 \div 8$.

When an answer is requested from students for $72 - 48$, they chant an answer of 9. They possibly assume that they have to divide 72 by 8 and do not listen to the instruction. The teacher proceeds with the problem as follows in Figure H.37:

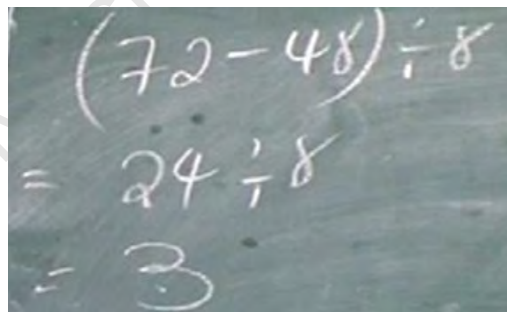

$$\begin{aligned} &(72 - 48) \div 8 \\ &= 24 \div 8 \\ &= 3 \end{aligned}$$

Figure H.37: Using BODMAS

Procedure 2:

1. Brackets are calculated first, using the rules for addition of integers.
2. The answer is divided by 8.

However, if one were to use the teacher's earlier BODMAS rule, $72 - 48 \div 8 = 72 - 6 = 66$.

Table H.19 presents an account of the computational activity for this evaluative event:

Table H.19: The computational activity for S P6L3EE5

Lesson 3	EE5 [51:10-59:15]	Input (domain)	Output (codomain)	Operation
Distributive rule over division				
Description of the two procedures: $(72 - 48) \div 8$		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<ol style="list-style-type: none"> 1. <u>Adding integers with different signs:</u> 2. Separate the sign from the whole number. 3. Subtract smaller whole number from the bigger whole number. 4. Re-attach the sign of the bigger whole number to the final whole number answer <u>Division of integers:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply or divide signs using the rules of multiplication and division of signs. 3. Multiply or divide whole numbers. 4. Re-attach sign to the final whole number answer

The two procedures presented are in fact different versions of the distributive rule.

Curriculum and textbooks

See L3 EE4

The mathematics encyclopaedia

As mentioned in L3 EE4, the distributive property as presented in the mathematics encyclopaedia is not found in the RNCS:

Distributive property:

$a \cdot (b + c) = a \cdot b + a \cdot c$ (Usiskin *et al.*, 2003: 232)

Division:

Division algorithm: If a and b are integers and if $b > 0$, then there are unique integers q and r such that $a = bq + r$ and $0 \leq r < b$. The division problem in the Division Algorithm is $\div b$. That is, a is the dividend and b is the divisor. Here we prove the existence of the integer quotient q and the integer remainder r for the given integers a and b with $b > 0$. (Usiskin *et al.*, 2003: 206)

Integers: see L1 EE1

Natural numbers: see L1 EE1

The teacher, in his presentation of this evaluative event, provided a specific instance of the distributive property over division, and mentioned that there were instances where it would not work. Perhaps he should have stated that the set of integers is not closed under the operation of division.

Regulation of the computational activity

As in the previous evaluative event, the distributive property served as a *computational resource* for obtaining the required solution based on the teacher's expression: '[...] you can simplify calculations by using the commutative, associative and distributive properties of operation' (S P6 L3 EE2 lines 442).

It seems that the procedure of BODMAS was a template for how to distribute whole number arithmetic - merely regulating students in a specific way to reach the required solution. It appears that the actual concept of the distributive property was not really engaged with other than being named and so BODMAS could also be classified *computational resource*.

School P6 Lesson 3 EE6: A description of the utility of the associative, commutative and distributive properties

Describing the computational activity

In this evaluative event the teacher read from the worksheet that he had been referring to for the last three evaluative events. He described the utility of the associative, commutative and distributive properties as allowing one to work faster, to work more accurately, to rearrange numbers, to simplify calculations and to look for combinations of numbers that worked well. He also mentioned that names of the associative, commutative and distributive properties were not important and that one should be able to 'see them'. By this he most probably meant that being able to apply the properties was more important than the actual name of the property.

Table H.20: The computational activity for S P6L3EE6

Lesson 3	EE6 [59:15 – 66:45]	Input (domain)	Output (codomain)	Operation
A description of the utility of the associative, commutative and distributive properties		\mathbb{Z}	\mathbb{N} \mathbb{Z}	<u>Adding integers with the same sign:</u> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Add the whole numbers. 3. Re-attach the common sign to the final whole number answer.

			<p><u>Adding integers with different signs:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Subtract smaller whole number from the bigger whole number. 3. Re-attach the sign of the bigger whole number to the final whole number answer <p><u>Multiplication and division of integers:</u></p> <ol style="list-style-type: none"> 1. Separate the sign from the whole number. 2. Multiply or divide signs using the rules of multiplication and division of signs. 3. Multiply or divide whole numbers. 4. Re-attach sign to the final whole number answer
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Curriculum and textbooks

As described in the previous two evaluative events, the RNCS requires that students perform calculations using the commutative, associative and distributive properties:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

We know this when the learner:

Uses a range of techniques to perform calculations including:

- using the **commutative, associative and distributive** properties with rational numbers [...] (DoE, 2002: 73)

The teacher read to the class that an application of the above-mentioned properties was much more important than actually naming the properties. The worksheet used in the lesson was not obtained by the researcher.

The mathematics encyclopaedia

The three properties, i.e., the associative, commutative and distributive properties are clarified in the mathematics encyclopaedia and they are certainly not presented in this manner in both the RNCS or during the presentation of this lesson:

Associative Property:

An algebraic system $(A, +, \cdot)$ consisting of a set A together with two binary operations, $+$ and \cdot , on A is called an integral domain if it has the following properties. For all a, b , and c in A ,

Addition:

Addition is associative:

$$a + (b + c) = (a + b) + c$$

Multiplication:

Multiplication is associative:

$$a.(b.c) = (a.b).c$$

Commutative Property: see L3 EE3

Distributive Property: see L3 EE4

Regulation of the computational activity

The names of the properties and the associated operational properties were not deemed important by the teacher and he explicitly stated this. He stressed the utility of these properties and it appeared that their use as *computational resources* regulated most of the solution procedures.

University of Cape Town

Appendix I: Analysis for School P7

Generating evaluative events

Table I.1: Evaluative events spanning Lesson 1 at School P7 (S P7 L1 EE1-EE6)

Evaluative Event		Type	Length
1.1	Factors [00:00 – 02:10]	Exercise	01:26
1.2	Generating highest common factors [02:19 – 05:44]	Exercise	01:24
02	Calculating cube roots and cubes [05:46 – 13:09]	Expository	02:00
03	Recap: Types of numbers [13:10 – 14:38]	Expository	02:45
04	Exponents [14:41 – 28:44]	Expository	06:33
05	Classification of numbers - primes, odd, composite, squares, cubes [35:50 – 39:15]	Exercise	05:38
			39:15

School P7 Lesson 1 EE1.1: Listing factors of 20, 24, 30, 36, 40, 48, 60

Describing the computational activity

Lesson 1 commenced with the teacher checking the previous days' homework on the board where he generated what amounts to a table showing the *factors* of a list of even values: {20, 24, 30, 36, 40, 48, 60, 72} in Figure I.1. The list of factors presented in this evaluative event was to serve as a resource for introducing the *highest common factor* as the next topic.

Number	Factors
20	(1, 2, 4, 5, 10, 20)
24	(1, 2, 3, 4, 6, 8, 12, 24)
30	(1, 2, 3, 5, 6, 10, 15, 30)
36	(1, 2, 3, 4, 6, 9, 12, 18, 36)
40	(1, 2, 4, 5, 8, 10, 20, 40)

Figure I.1: Factors of 20, 24, 30, 36, 40, ... (S P7 L1 EE1)

In an attempt to provide a description of the teacher's procedure for generating factors, we note the following interaction between the teacher and students:

Teacher: [...] Twenty goes into forty twice [...]

Teacher: Now if you look at those numbers, you see those numbers are all even numbers. So, therefore, what will also be affected?

Learners: Two.

Teacher: [Writes 1 as a factor of all the numbers] There we put in the two .. for each one.. [...] (S P7 L1 EE1 lines 1-3)

There appeared to be no clear evidence in this lesson of what had been established as the definition of a factor. For the teacher, the definition of a factor was a number that 'goes into' another number (S P7 L1 EE1). The colloquial expression - 'goes into' conjured up a description of a factor in terms of it being an outcome of a process. It also suggested that 'goes into' it may signal divisibility. Because the numbers were all even, one and two were listed as factors of all of them. Consider the following board snapshot in Figure I.2:

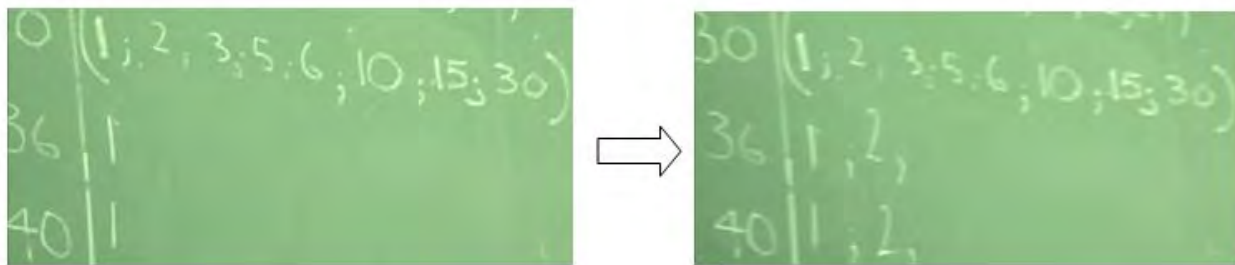


Figure I.2: 'One' and 'two' will always be factors of even numbers (S P7 L1 EE1)

The content meant to emerge from this evaluative event in Figure I.1 were *factors* of a natural number i.e. given $n \in \mathbb{N}$, list the factors of n , from smallest to largest. When attempting to provide a more formal description of the procedure used by the teacher for generating factors of a number, the following propositions emerged:

1. One (i.e. $1 \in \mathbb{N}$) is a factor of every natural number. Hence the list of 1s arranged vertically in correspondence with a vertical list of natural numbers.
2. If n is even, then 2 is also a factor of n . Since all the natural numbers listed are even, 2 is a factor of each of them.

Note that propositions (1) and (2) refer to the existential features of the list of factors of n , but that they do not say anything explicit about co-factors.

The closest we get to a clear statement of what a factor of an integer or a natural number is, is the expression 'goes into': 'twenty goes into forty twice' (S P7 L1 EE1 line 1). The expression 'goes into' was used to indicate that one (natural) number was divisible by another, or that one algebraic expression was divisible by another. The idea seemed to be that one considered the list of natural numbers, from 1 to n , and then selected those numbers from the list that 'goes into' n ; i.e., $\forall m \in \{1, 2, \dots, m\}$, if $m|n$, then m is a factor of n .

Table I.2 presents a description of the computational activity for this evaluative event:

Table I.2: Computational activity for S P7 L1 EE1

Lesson 1	EE1.1 [00:00 – 02:10]		Input (domain)	Output (codomain)	Operation
Checking homework: Listing factors of 20, 24, 30, 36, 40, 48, 60			\mathbb{N}	\mathbb{N}	Division

Curriculum and textbooks

The RNCS states that factors need to be recognised, classified and represented in the statement that follows:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them: [...]

- multiples and factors; [...] (DoE, 2002: 69)

The curriculum does not, however, provide any general statements relating to factors and divisibility. The observer in this particular lesson (S P7 L1 EE1) indicated that textbooks were not apparent and that students were presented with worksheets for the series of lessons. The teacher, however, had access to two textbooks, *Classroom Mathematics* and *Preparing for High School Maths* where the following descriptions of factors were presented (respectively):

Factors of a number always divide into the number without any remainder. For example, the factors of 18 are 1; 2; 3; 6; 9 and 18 since all these natural numbers divide into 18 without remainder. (Laridon *et al.*, 2007: 18)

A factor is a number which divides into another number exactly with no remainder. (Bull & Hepworth, 2008: 22)

It is interesting that the latter description, (also presented in Figure I.3) also makes no reference to co-factors.

SKILL 1 FACTORS

Factors

A factor is a number which divides into another number exactly with no remainder.

Example:

Find the factors of 8, 9 and 12.

Step 1

Write the number pairs which multiply together to give the number. These are the factors.

Step 2

List these factors in numerical order.

8	9	12
1×8	1×9	1×12
2×4	3×3	2×6
		3×4
{1, 2, 4, 8}	{1, 3, 9}	{1, 2, 3, 4, 6, 12}

Figure I.3: A definition of factors in *Preparing for High School Maths* (Bull & Hepworth, 2008: 22)

Now consider what is found in the mathematics encyclopaedia in the passage that follows.

The mathematics encyclopaedia

The proposition that follows, commonly found in texts on number theory, is key to the notion of factorisation and divisibility:

When a relation $c = ab$ holds between integers a, b , and $c \neq 0$, one says that a is a *divisor* or a *factor* of c and that c is *divisible* by a . We also call (this) a decomposition or factorization of c . Clearly b is also a divisor of c and uniquely determined by a . This leads to an observation that is useful in certain problems, namely, that divisors of a number occur in pairs (a, b) . (Ore, 1948: 29; italics in original.)

Consider the formal definitions of even numbers and factors also found in the mathematics encyclopaedia in relation to the method presented by the teacher for identifying even numbers and factors:

Even numbers:

An integer n is even if and only if there exists an integer k with $n = 2k$. (Usiskin *et al.*, 2003: 207)

Factor:

We say that $k \in \mathbb{N}$ is a *factor* or *divisor* of $m \in \mathbb{N}$ if there exists $s \in \mathbb{N}$ such that $m = ks$. We write $k \mid m$. Trivially 1 and m are factors of m ; any other factor is called a *proper factor*. (Stewart & Tall, 1977: 165, italics in original.)

The teacher says the following of even numbers: ‘Now if you look at those numbers, you see those numbers are all even numbers.’ (S P7 L1 EE1 line 1). The teacher’s description stands in place of the encyclopaedic definitions. Perhaps the teacher assumed that students implicitly knew that they needed to check if the values were divisible by two in order to classify them as being even.

Regulation of the computational activity

The notion of a factor is an outcome of a calculation where the teacher’s procedure for generating factors of the given values in Figure I.1 relies on listing factors in order from smallest to largest *and* the use of the everyday informal expression ‘goes into’ which is meant to signal divisibility. This process suggests a sequence of natural numbers: 1, 2, 3, ..., n , ... with individual numbers tested from left to right, until n is reached. The concept of a factor and its related properties do not regulate the procedure as described from the onset; rather the *computational procedure* appears to be the primary object. Table I.3 presents an account of the constitution of a factor for this evaluative event:

Table I.3: The constitution of a factor

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Factors	Colloquial expression: ‘goes into’ to signal divisibility	List of even factors on the board - Computational resource	Division of natural numbers	$\forall m \in \{1, 2, \dots, m\}$, if $m \mid n$, then m is a factor of n .

School P7 Lesson 1 EE1.2: Generating the highest common factor

The teacher's procedure in Figure I.1 entailed listing all the factors of the values that appeared in the problem statements given as homework and then empirically checking which the largest value common to all, as factors, was. The first problem presented required students to use the table in Figure I.1 as a resource for calculating the highest common factor of 20, 24 and 40 (See Figure I.4).

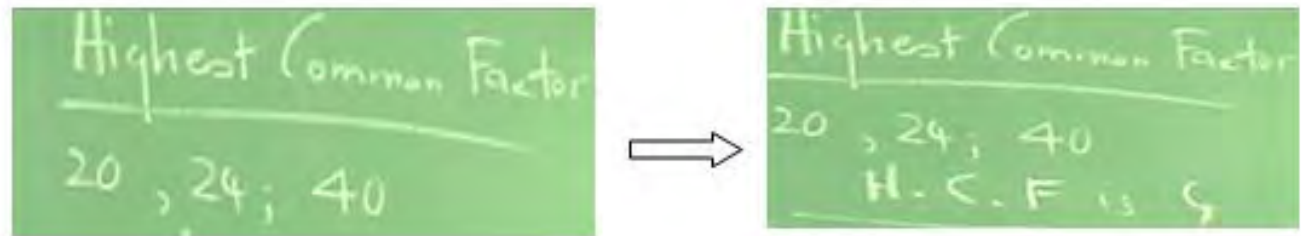


Figure I.4: Calculating the highest common factor of 20, 24 and 40 (S P7 L1 EE2)

An explanation provided by the teacher for finding the highest common factor followed:

- Teacher: [...] That is a fact that the factor of twenty and the factor of twenty-four and the factor of forty. ...
Hmm? ...
Teacher: No. So that's the biggest one. So the highest common factor of those there ... The highest common factor.
Learner: Ten.
Teacher: The highest common factor is four.
Learner: Four.
Teacher: The highest number that can divide into all three. [...] (S P7 L1 EE1-EE2 lines 5-10)

What finding the highest common factor entailed was calculating the greatest common divisor of a list of natural numbers. The problem, especially for large values, was knowing that all the factors were found, especially if one was making use of the teacher's method. Note that the factors in Figure I.1 were always listed in ascending order by the teacher. Another way of doing the problem would have been for the teacher to list the factors as co-factors: $F(20) = \{1, 20, 2, 10, 4, 5\}$. In this way the factor list would more easily have been seen to be complete. Note that $\sqrt{20} \approx 4.47$ and so $4 < \sqrt{20} < 5$, so we would have needed to search for co-factors by searching for divisors up to 4. The integral value of $\sqrt{20}$ would become the upper bound for the values we tested as a potential divisor of twenty. For example, $\text{INT}(\sqrt{72}) = 8$, so we need test $m \in \mathbb{N}$, for divisors and generate the co-factors, and so the factors of 72 from that: $\{1, 72, 2, 36, 3, 24, 4, 18, 6, 12, 8, 9\}$.

Alternatively, the teacher could have used prime factorisations:

$20 = 2^2 \times 5$, $24 = 2^3 \times 3$, $40 = 2^3 \times 5$ from which it is immediately apparent that $2^2 = 4$ is the highest common factor, i.e.

$$\left. \begin{array}{l} 20 = 2^2 \times 5 \\ 24 = 2^2 \times 2 \times 3 \\ 40 = 2^2 \times 2 \times 5 \end{array} \right\} 2^2 = 4$$

The procedure described in this evaluative event can be described in more formal terms:

$$\text{Max} [F(20) \cap F(24) \cap F(40)] = \text{HCF}(20, 24, 40)$$

$$F(n) = \{m \in \mathbb{N} \text{ s.t. } m|n \text{ where } n \in \mathbb{N}\}$$

$$\text{So, } F(20) = \{m \in \mathbb{N} \text{ s.t. } m|20 \text{ where } n \in \mathbb{N}\}$$

$$F(20) = \{1, 2, 4, 5, 10, 20\}$$

$$F(24) = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

$$F(40) = \{1, 2, 4, 5, 8, 16, 20, 40\}$$

$$F(20) \cap F(24) \cap F(40) = \{1, 2, 4\}$$

$$\text{Max} [F(20) \cap F(24) \cap F(40)] = \text{Max} [1, 2, 4] = 4$$

$$\therefore \text{HCF} = 4$$

What follows is an account of the computational activity for this evaluative event in Table I.4:

Table I.4: Computational activity for S P7 L1 EE2

Lesson 1	EE 1.2 [02:19 – 05:44]	Input (domain)	Output (codomain)	Operation
Generating Highest Common Factors		\mathbb{N}	\mathbb{N}	<ol style="list-style-type: none"> 1. Listing factors. 2. Selecting the common factors. 3. Selecting the highest common factor

Curriculum and textbooks

As described in L1EE1.1, the RNCS simply prescribes that factors need to be recognised, classified and represented and makes no mention of the highest common factor at all (DoE, 2002: 69). The textbook, *Preparing for High School Maths*, that may have been referenced in preparation for this lesson lists the factors of a number in ascending order in exactly the same manner that the teacher does at the commencement of this lesson in EE1.1. in Figure I.5:

Highest common factor (HCF)

The highest common factor is the largest of the common factors.

Example

Find the highest common factor of 8 and 12.

Step 1

List the factors of both numbers in numerical order.

Step 2

Locate the common factors.

Step 3

Find the highest common factor.

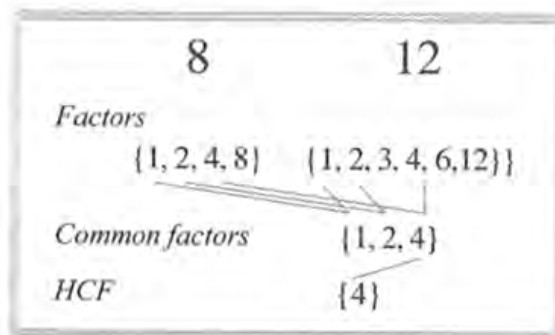


Figure I.5: A definition of highest common factor in *Preparing for High School Maths* (Bull & Hepworth, 2008: 23)

The entire procedure for identifying the highest common factor in Figure I.5 was very similar to how the teacher proceeded in class using his list in Figure I.1. The list of factors of even numbers could therefore be considered as a regulative resource in this evaluative event.

The mathematics encyclopaedia

The description of the highest common factor found in the mathematics encyclopaedia is similar to what is found in the textbook and to what the teacher prescribes. Consider the following formal definition of highest common factor:

Highest common factor:

[...] if a and b are integers, not both zero, then the greatest common factor of a and b , $\text{gcf}(a,b)$ is the unique natural number such that

1. $\text{gcf}(a,b)$ is a factor of both a and b ;
2. If d is any integer that is factor of both a and b , then d is a factor of $\text{gcf}(a,b)$.

For example, $\text{gcf}(-24;30) = 6$ and $\text{gcf}(15;-8) = 1$ (Usiskin *et al.*, 2003: 208-209)

Once again, it appears that the procedure for calculating the highest common factor, emphasised by both the teacher and the textbook, takes precedence over any formal definition that may be offered by the mathematics encyclopaedia.

Regulation of the computational activity

As described in EE1.1, the list of factors generated in ascending order in Figure I.1 was referenced by the teacher as part of the procedure for calculating the highest common factor. The teacher also used terminology that was

unclear at times, especially when indicating to students how to identify the common factor. Consider the following transcript extract where he enquires about the highest common factor of 30, 48 and 72:

- Teacher: [...] Now you look at all the factors of thirty, forty eight and seventy two. Who can tell me, what's the highest common factor of those three numbers? What do you see? Here are thirty factors, here are forty eight factors and there we have seventy two factors. Which is the highest one? Now you need to examine thirty's factors. Let's look at ten. Is ten a factor of forty eight?
- Learners: No.
- Teacher: No, ok. Which one do you think fits in by all three? [Chair noise makes word unclear]. Yes my girl?

He ends off by saying that the common factor of 30, 48 and 72 should 'fit in by all three' which certainly does not provide explicit instructions to students for how to proceed. It appears that the list of factors of even numbers in Figure I.1 serves as a *computational resource* for calculating the highest common factor. Consider the constitution of the highest common factor in Table I.5 for this evaluative event:

Table I.5: The constitution of the highest common factor

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Highest common factor	Textbook procedure List factors in numerical order, find common factor followed by highest common factor	List of even factors on the board - Computational resource	Division of natural numbers	Definition of highest common factor

School P7 Lesson 1 EE2: Calculating cube roots and cubes

Describing the computational activity

Consider the following transcript extracts for how cube roots of small and big numbers are generated as well as an account how the list of cubes is recited by students:

i. How to generate cube roots of small numbers

The teacher provides a procedure for calculating cube roots of numbers in the extract that follows:

- Teacher: [...] Right, class. We've done cubes and cube roots already. The first answer, the cube root of eight. What's the cube root of eight? Hmm?
- Learner: Two.
- Teacher: Two, why two?
- Learner: [Inaudible].
- Teacher: No, it doesn't go into eight, three times. Because two times two times two gives you the eight. What's the cube root of one hundred and twenty-five?
- Learner: Five.
- Teacher: Five, because? Five times five times five, bring you back to one twenty five. [...] (S P7 L1 EE2 lines 22-28)

ii. How to generate cubes

Cubes were generated by the entire class; they were recited in order from one to nine and this became a resource for generating the cube roots of bigger numbers by inspection:

- Teacher: [...] Now we know our cubes already. We know our cubes from one to nine. What's one cubed, hands up? Class, whole class? One cubed?
- Learners: One.
- Teacher: Two cubed?
- Learner: Four.
- Teacher: Two cubed?
- Learners: Eight.
- Teacher: Three cubed?
- Learners: Twenty-seven. (S P7 L1 EE2 lines 38-46)

iii. How to generate cube roots of larger numbers

A method used by teacher for calculating cube roots of larger numbers relied on the list of cubes already listed on the board. More, specifically, the last digit of the given value as indicated by the teacher in Figure I.6 served as a resource for generating its cube root. Consider the transcript extract that follows:

- Teacher: Now we've got these big numbers here. ... [Teacher rubs some writing off the chalkboard.] Okay. ... [Teacher points at $\sqrt[3]{5832}$ / as he speaks. – See Figure I.6] Those people ... with the .. scientific calculators ... I want you .. not to use that special facility on there. How are we going to .. [Teacher points at the expression $\sqrt[3]{5832}$ / again, tapping the chalkboard as he speaks.] We must look for a number which will multiply by itself. Now ... Here you've got to try ... Try twenty times twenty and see what you get.
- Learner: Eight thousand.
- Teacher: What do you get?
- Learner: Eight thousand.

- Teacher: Eight thousand. Now .. that number [Teacher refers to $\sqrt[3]{5832}$ /] is smaller than eight thousand ... so the cube root is going to be less .. than .. twenty. ... Now find a number ... What number do you think it's going to be? [...]
- Learners: [Silent.] (S P7 L1 EE3 lines 34-38)

When students did not respond to the teacher's question, 'What number do think it's going to be?', he asked them to list the cubes from one to nine and relied on this as resource for generating the cube root of 5832:

- Teacher: Right. Now notice eight .. cubed gives you five hundred and twelve. .. Ends in a two. So this number here [Teacher points to /5832/.] ... This number here ... is going to end in a eight. .. Because of that two [Teacher points at /2/ in the expression $\sqrt[3]{5832}$ /.]
- Learner: Sir?
- Teacher: So .. now .. what number .. Now we must try and find it. Yes?
- Learner: Eighteen times eighteen times eighteen.
- Teacher: That's right. .. So the cube root of that is 18. [Teacher writes /=18/ alongside $\sqrt[3]{5832}$ / as he speaks, producing $\sqrt[3]{5832} = 18$ /.] (S P7 L1 EE3 lines 59 - 64)

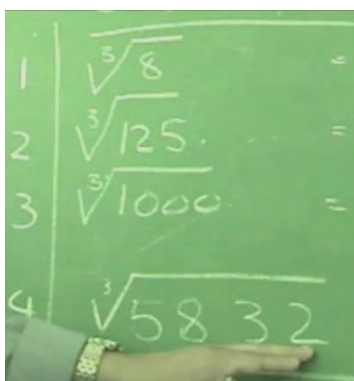


Figure I.6: Calculating the cube root by observing the last digit of the value (S P7 L1 EE2)

Students were advised not to use the cube root function on their scientific calculators, but to rather observe that $8^3 = 512$ and as a result $\sqrt[3]{512} = 8$. Furthermore 512 'ends in a two', so it followed that $\sqrt[3]{5832} = 18$. Because 5832 also 'ends in two', the answer (cube root) would 'end in an eight' (S P7 L1 EE3). The teacher's criteria for selecting what the upper and lower bounds for the desired solution should be were not clear at the onset. He may have been using his knowledge of the answer to select appropriate pairs of multiples of tens to construct bounded intervals. For example, having multiplied $18 \times 18 \times 18$ to get 5832 when he constructed the problem, the teacher knew that 18 would be the solution to the problem of calculating the cube root of 5832, and so he chose to compare 5832 with 10^3 and 20^3 , finding that $10^3 < 5832 < 20^3$ and that the solution lay between 10 and 20. For the student, who had not constructed the problem, the criteria for selecting the upper and lower bounds for the solution remained framed in terms of the teacher's knowledge. The criteria remained context dependent since there was no explicit explanation for how to calculate cube roots.

In essence then, consider the procedure for finding cube roots of 'big numbers':

1. Using the list of perfect cubes from one to nine as a point of reference, look at the last digit of the perfect cube. (The teacher points to the 2 as the last digit in $\sqrt[3]{5832}$ in Figure I.6)
2. The last digit of the perfect cube as well as the last digit of its cube root are used to calculate the cube roots of larger numbers
3. Notice that $8^3 = 512$, so $\sqrt[3]{512} = 8$ and 512 'ends in a 2'. So, because 5832 'ends in a 2', the answer (cube root) will 'end in an eight'. Therefore, $\sqrt[3]{5832} = 18$.

Consider the computational activity for this evaluative event in Table I.6:

Table I.6: The computational activity for S P7L1EE2

Lesson 1	EE2 [05:46 – 13:09]	Input (domain)	Output (codomain)	Operation
Calculating cube roots and cubes		\mathbb{N}	\mathbb{N}	<ol style="list-style-type: none"> 1. Observe the last digit value of the large number to be $\sqrt[3]{\quad}$ 2. Using the list of cubes, compare the last digit value of the large number to be $\sqrt[3]{\quad}$ with this list. 3. If the last digit value of the large number corresponds with the last digit value of one of the cubes, then the cube root of that large value will end in the same value as the cube root selected from the list of cubes.

Curriculum and textbooks

In Learning Outcome 1, the curriculum specifies that students should be able to recognise, classify and represent cubes and cube roots in exponential form:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them:

[...] • numbers written in exponential form including squares and cubes of natural numbers and their square and cube roots (DoE, 2002: 69)

This is precisely what both the teacher and the textbook adheres to. Consider the textbook's account of cube numbers in Figure I.7 which is similar to what both the RNCS and the teacher prescribe for this topic:

Cube numbers are made by multiplying a number by itself twice (3 lots of the number).

Example: 4^3 is read 'four cubed' which is $4 \times 4 \times 4 = 64$

Figure I.7: A definition of cube numbers in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

The textbook does not provide any description of cube roots though. Now consider what the mathematical encyclopaedia presents on this topic.

The mathematics encyclopaedia

The mathematics encyclopaedia's account of a cube root closely resembles what both the teacher and the textbook presents:

Cube roots: the number that produces a given number when used as a factor three times: The cube root of 125 is 5. (World Book Dictionary, 1982: 503)

Cubes: a cubic number is a figurate number of the form n^3 with n a positive integer. The first few are 1, 8, 27, 64, 125, 216, 343, ... (Wolfram.mathworld.com)

The encyclopaedia, however, describes cube roots in terms of its related factors compared to the textbook which references cubes in a more informal manner.

Regulation of the computational activity

The list of perfect cubes which the class recited from one to nine was used as a point of reference to find the cube root of a large number in this evaluative event. When finding the cube roots of larger numbers the starting point for students was to look at the last digit of the number to be cube rooted and use estimation based on the list of perfect cubes to find its cube root. Consider the transcript extract for finding the cube root of a value that ended with a 7 as well as the method for finding what the answer would end with:

Teacher: [...] ... Ok. The next one, you have to try and get a starting point. What starting point can we use here?
Learner: Seven.
Teacher: Hmm? Wait, it's going to end in a, ... your answer is going to end in a?
Learners: Three. (S P7 L1 EE2 lines 81-84)

Note that $\sqrt[3]{27} = 3$ is used as a point of reference for the above calculation and as a result, finding the cube or cube root of a number entails adhering to a *computational activity* rather than referencing the formal definition found in the mathematics encyclopaedia. Table I.7 presents an account of the constitution of a cube root:

Table I.7: The constitution of a cube root

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Cube roots	Smaller numbers: List of cubes from 1 to 9	List of cubes from 1 to 9 - Computational resource	Counting Estimation	Procedure for finding cube root
	Bigger numbers: List of cubes and observe the last digit of value to be $\sqrt[3]{\quad}$	Using the last digit of the perfect cube and the last digit of its cube root – computational resource		

School P7 Lesson 1 EE3: Types of numbers

Describing the computational activity

In this evaluative event the teacher references whole, natural odd, even, composite, prime numbers less than ten, multiples, factors, prime factors, squares, square roots, cubes and cube roots in order to present the class with an overview of what they had already covered in previous lessons and what would be tested on in future.

Teacher: Seventy four. ... Right. Now let's stop for a minute. Let's look at all the work you've done because all of these have been done so far; it's all going to be tested. Let's start off with, we started off with whole and natural numbers, even numbers, odd numbers, multiples, prime numbers, composite numbers carried on and then we've got all the prime numbers less than ten; then we went on to prime factors. We went on to factors, squares, square roots, right. [...] (S P7 L1 EE3 lines 95-97)

These topics were not taught during this evaluative event. I have, however, provided an account of the objects that were involved in the computational activity for the above-mentioned topics in Table I.8:

Table I.8: The objects for S P7L1EE3

Lesson 1	EE3 [13:10-14:38]	Types of numbers	Input (domain)	Output (codomain)	Operation
Whole , Natural, Odd, Even, Composite, Prime numbers < 10, Multiples, Factors, Prime factors, Squares, Square roots, Cubes, Cube Roots			Not applicable	Not applicable	Not applicable

Curriculum and textbooks

Consider how the RNCS' prescriptions regarding types of numbers only relate to being able to recognise, describe and represent numbers rather than providing any formal definition:

RNCS:

LEARNING OUTCOME 1 NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

Recognises, classifies and represents the following numbers in order to describe and compare them: [...]

- numbers written in exponential form including squares and cubes of natural numbers and their square and cube roots; [...]
- multiples and factors; [...] (DoE, 2002: 69)

The textbook which may or may not have been referenced in prior lessons provides definitions and descriptions of the afore-mentioned topics in Figure I.8, Figure I.9, Figure I.10, Figure I.11., Figure I.12, Figure I.13 and Figure I.14:

DEFINITIONS :

A **prime number** has only two factors – itself and one.

A **composite number** has more than two factors.

Factors are the whole numbers which divide exactly into the number.

Common factors are those factors which are shared between two or more numbers.

Multiples of a number can be found in the 'multiplication table' of that number.

Example: The first 6 multiples of 10 are 10, 20, 30, 40, 50, 60.

Figure I.8: 'Definitions' in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

SKILL 3 COMPOSITE NUMBERS			
Composite numbers are the whole numbers which have more than two factors.			
The set of composite numbers is: {4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26,... etc.}			
Except for the number 1 (which has only one factor), all the whole numbers are either prime or composite.			
No.	Products	Factors	
2	2×1	[1, 2]	Prime
3	3×1	[1, 3]	Prime
4	$4 \times 1, 2 \times 2$	[1, 2, 4]	Composite
9	$9 \times 1, 3 \times 3$	[1, 3, 9]	Composite
12	$12 \times 1, 2 \times 3$ 3×4	[1, 2, 3, 4]	Composite

Figure I.9: Definition of composite numbers in *Preparing for High School Maths* (Bull & Hepworth, 2008: 24)

SKILL 5 MULTIPLES

The multiples of a number are obtained by multiplying it by the natural numbers.

Example:

Find the first five multiples of the number 8.

Solution:

The first five multiples of 8 are:
8, 16, 24, 32, 40

1st multiple is $1 \times 8 = 8$

2nd multiple is $2 \times 8 = 16$

3rd multiple is $3 \times 8 = 24$

4th multiple is $4 \times 8 = 32$

5th multiple is $5 \times 8 = 40$

Figure I.10: Definition of Multiples in *Preparing for High School Maths* (Bull & Hepworth, 2008: 24)

SKILL 4 PRIME FACTORS

Every composite number can be shown as the product of a string of prime numbers. These are known as its prime factors.

A factor tree can be used to break the number down.

Examples:

Find the prime factors of:

(a) 18

(b) 100

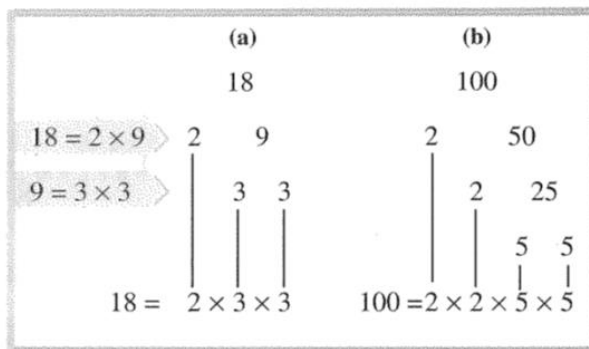


Figure I.11: Definition of Prime factors in *Preparing for High School Maths* (Bull & Hepworth, 2008: 24)

Square numbers are made by multiplying a number by itself once.

Example: 9^2 is read 'nine squared' which is $9 \times 9 = 81$

Cube numbers are made by multiplying a number by itself twice (3 lots of the number).

Example: 4^3 is read 'four cubed' which is $4 \times 4 \times 4 = 64$

Figure I.12: Definition of squares and cubes in *Preparing for High School Maths* (Bull & Hepworth, 2008: 20)

SKILL 7 SQUARE ROOTS

Square roots are related to the squares of number. In fact they are the complete opposite. Finding the square root of a number means undoing the square.

The symbol for square root is $\sqrt{\quad}$.

Example:

Find (a) $\sqrt{36}$ (b) $\sqrt{16}$
(c) $\sqrt{100}$ (d) $\sqrt{49}$

(a) $\sqrt{36} = 6$ because $6 \times 6 = 36$
(b) $\sqrt{16} = 4$ because $4 \times 4 = 16$
(c) $\sqrt{100} = 10$ because $10 \times 10 = 100$
(d) $\sqrt{49} = 7$ because $7 \times 7 = 49$

The **square root** of a number is a number which, when multiplied by itself, is equal to that number. The sign used is $\sqrt{\quad}$

Example: $\sqrt{64}$ is read 'the square root of 64' which is 8, because $8 \times 8 = 64$

Figure I.13: Definition of square roots in *Preparing for High School Maths* (Bull & Hepworth, 2008: 25)

SKILL 6 SQUARE NUMBERS

Square numbers are obtained by multiplying the number by itself. The symbol used for squaring a number is a small 2 positioned on the top right-hand side.

The first square numbers are {1, 4, 9, 16, 25, 36...}

Examples:

Find the value of:

- (a) three squared
(b) six squared

3^2 three squared
 $= 3 \times 3$
 $= 9$

6^2 six squared
 $= 6 \times 6$
 $= 36$

5^2 squared
five squared

Figure I.14: Definition of squares in *Preparing for High School Maths* (Bull & Hepworth, 2008: 25)

I am not able to relate this to lesson content that may have transpired in the classroom in prior lessons and re-iterate that it merely stands as a point of reference for what may have been covered or used during lesson preparation relating to these topics.

The mathematics encyclopaedia

Consider the detailed formal definitions presented in the mathematics encyclopaedia relating to types of numbers and how this stands in contrast to what the textbook presents:

Even numbers: see L1 EE2

Factors: see L1 EE2

Composite numbers: An integer > 1 is prime if and only if its only integer divisors are itself and 1; otherwise it is composite. (Usiskin *et al.*, 2003: 209)

Prime numbers < 100 : An integer > 1 is prime if and only if its only integer divisors are itself and 1; otherwise it is composite. (Usiskin *et al.*, 2003: 209)

Multiples:

In the case in which the quotient resulting from the division of a by b is an integer, denoting it by q , we have $a = bq$, i.e. a is equal to the product of b by an integer. We will then say that a is divisible by b or that b divides a . Here a is said to be a multiple of b and b is said to be a divisor of the number. The fact that b divides a is written as: $\frac{a}{b}$. (Vinogradov, 1954: 1-2; italics in original.)

Prime factors:

Suppose that n is a natural number greater than 2 and that p_1, p_2, \dots, p_k are the distinct prime factors of n written in increasing order; that is $p_1 < p_2 < \dots < p_k$. For each prime factor p_i of n , let m_i be the number of times that p_i occurs as a factor of n in its prime factorization. Then a prime factorization of n can be expressed in the following form:

$$n = p_1^{m_1} \cdot p_2^{m_2} \cdot \dots \cdot p_k^{m_k}$$

This is called the standard or canonical prime factorization of n . (Usiskin *et al.*, 2003: 220)

Natural numbers: The natural numbers are the familiar counting numbers 1, 2, 3, 4, 5, ... [...] We write \mathbf{N} for the set of all natural numbers. (Stewart & Tall, 1977: 12-13)

Odd numbers: An integer n is odd if and only if there exists an integer k with $n = 2k + 1$. (Usiskin *et al.*, 2003: 207)

Squares: the product obtained when a number is multiplied by itself (Barnhart & Barnhart, 1982: 2029)

Cubes: the product obtained when a number is used three times as a factor: 125 is the cube of 5, because $5 \times 5 \times 5 = 125$ (Barnhart & Barnhart, 1982: 503)

Whole numbers: One of the numbers 1, 2 or 3 ... also called the counting numbers or natural numbers. 0 is sometimes included in the list of whole numbers.

Square roots:

What power of a , when multiplied by itself, gives a ? We recognize the answer as the square root of a . [...] think of a square root of a as $a^{\frac{1}{2}}$. (Usiskin *et al.*, 2003: 12)

Cube Roots : see L1 EE3

Unlike the mathematics encyclopaedia, most of the textbook definitions provide a procedure for finding different types of numbers and all of them provide an example of the type of number presented without providing definitions of the different types of numbers.

Regulation of the computational activity

The *general description* of the type of number provided in the textbook, together with an example is the central means of regulation in this evaluative event.

School P7 Lesson 1 EE4: Exponents

Describing the computational activity

This evaluative event introduced the topic, powers:

Teacher: [...] Let's first take this heading down, powers. We've got powers of two. The first one. Now anything ... now I'm not going to explain this to you, I'm just gonna tell you. Anything to the power of zero always comes to one. That you're gonna learn more about in Grade nine and ten. Now two to the power of one is just plain two. Now what is two squared? Take out two fingers, and it's two times? Two. And what do you get?

Learners: Four. (S P7 L1 EE4 lines 105 - 108)

In this transcript extract, students were told to simply accept the fact that any value raised to the power of zero would have an answer of one. The definitions of 2^0 , 2^1 and 2^2 that were presented in the given transcript extract (S P7 L1 EE4) were an indication of the types of descriptions that circulated in this particular pedagogic context. The teacher stated that $2^0 = 1$, 2^1 is 'just plain two' and 2^2 was calculated as follows: 'Take out two fingers, and it's two times? Two' (S P7 L1 EE4). What *was* made explicit to students is what needed to be done at the computational level when presented with a whole number raised to a power. When the teacher asked a student to explain what the superscript in the exponential expression represented in Figure I.15, he referred to it in very general terms. Consider the following transcript extract:

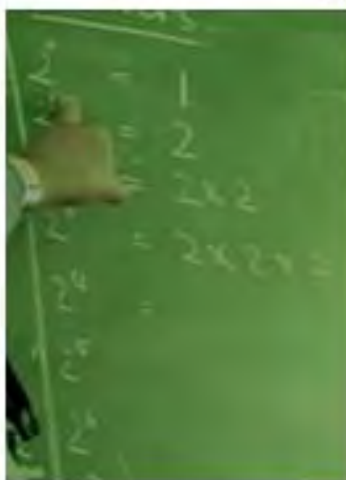


Figure I.15: 'What do we call that here?' (S P7 L1 EE5)

Teacher: [...] Right, complete all of these powers here. The two, what do we call the one, two, three, four and so on? What do we call that here? .. That small number? .. Hello?

Learners: Exponents.

Teacher: Exponents. Or another word? Another word for exponents? Is? Index. [...] (S P7 L1 EE5 lines 109-112)

In this transcript extract, exponents are referenced as ‘that small number’ or the ‘index’. The mathematical object, namely, the exponential expression is announced but not defined in any precise manner. It is described in spatial terms by observing the position of the superscript of the exponent in relation to the base rather than providing a precise mathematical definition. Table I.9 presents an account of the computational activity for this evaluative event:

Table I.9: The computational activity for S P7L1EE4

Lesson 1	EE4[14:41-28:44]	Input (domain)	Output (codomain)	Operation
Exponents		N	N	Multiplication

Curriculum and textbooks

The curriculum does make reference to numbers written in exponential form, referencing both squares and cubes of natural numbers that also fall into the category of numbers that can be written in exponential form:

Recognises, classifies and represents the following numbers in order to describe and compare them:

[...] • numbers written in exponential form including squares and cubes of natural numbers and their square and cube roots; [...] (DoE, 2002: 69)

No mention is, however, made of the general form that an exponential expression takes. The textbook, *Preparing for High School Maths*, does provide a definition of square numbers and cube numbers in Figure I.11 but it remains framed in very general terms i.e. ‘square numbers are made by multiplying a number by itself once’ and ‘cube numbers are made by multiplying a number by itself twice (3 lots of the number)’ (Bull & Hepworth, 2008: 20). Both definitions are accompanied by an example and the exponential expression, as such, is never referenced in terms of its constituent components i.e. the base and the exponent.

The mathematics encyclopaedia

Consider the following definitions relating to the exponential expression found in the mathematics encyclopaedia:

Exponent: An exponent is the power p in an expression of the form a^p . The process of performing the operation of raising a base to a given power is known as exponentiation. (Wolframalpha.com)

Exponential expression: a mathematical expression consisting of a constant raised to a power.

These definitions are not found in the textbook or in the curriculum.

Regulation of the computational activity

Counting off on fingers and using the calculator to confirm answers are the necessary regulative resources for computing problems in this evaluative event. Both counting as well using the calculator could therefore be classified as *computational resources*. It is also apparent that the general structure of an exponential expression, as one that comprised of a base and an exponent, was not made explicit at any point during both lessons. The teacher preferred to use a *general description* of this concept together with specific examples. Table I.10 presents an account of the constitution of an exponential expression:

Table I.10: The constitution of an exponential expression

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Exponents	Counting on fingers to denote the exponent Calculator Specific examples	General description of a mathematical concept	Counting Multiplication	Definition of an exponential expression: any number raised to a power n could be calculated by multiplying the number by itself n times.

School P7 Lesson 1 EE5: Classification of numbers

Describing the computational activity

This evaluative event is very brief and entailed that students choose from the list of numbers given: 1, 2, 4, 5, 6, 8, 24, 33, 37, 40 and classify them based on the description provided of prime, odd, composite, square and cube numbers in previous lessons. No teaching actually transpired, except that students were instructed as to what to do. The textbook provides a definition of prime numbers, odd numbers, composite numbers, squares and cubes which can be categorised as a general description. So the domain, codomain and operation are not applicable in this instance. (See Table I.11)

Table I.11: The computational activity for S P7L1EE5

Lesson 1	EE5[35:50- 39:15]	Input (domain)	Output (codomain)	Operation
Classification of numbers – primes, odd, composite, squares, cubes		Not applicable	Not applicable	Not applicable

Curriculum and textbooks

See L1 EE3

The mathematics encyclopaedia

Prime numbers: see L1 EE3

Odd numbers: see L1 EE3

Composite: see L1 EE3

Squares: see L1 EE3

Cubes: see L1 EE4

Regulation of the computational activity

See L1 EE3

School P7 Lesson 2

Generating evaluative events

Table I.12: Evaluative events spanning Lesson 2 at School P7 (S P7 L2 EE1-EE5)

Evaluative Event	Type	Length
01 Powers [00:00 – 02:32]	Expository	02:32
02 Square roots and cube roots [02:32 – 02:55]	Expository	00:23
03 Finding the first two-digit number, the digits of which has a sum of 12 and finding the first three digit number, the digits of which has a sum of 14 [02:55 – 09:20]	Expository	06:25
04 Recap of the meaning of 5^2 [09:30 – 10:50]	Expository	01:20
05 Listing odd numbers, even numbers greater than 30, prime numbers and composite numbers greater than 30 [11:30 – 14:15]	Exercise	02:25
		38:51

School P7 Lesson 2 EE1: Powers

Describing the computational activity

As already described in L1 EE4, the notion of an exponential expression was not defined in any precise way in this evaluative event. Rather, there seemed to be a reliance, firstly, on using ‘fingers’ to denote the power that the base had been raised to and, secondly, on the use of a calculator as a means of confirming the answer for both Lesson 1 and Lesson 2. The following transcript extract confirmed this:

Teacher: [...] Which is the better way? Four cubed or four to the power of three.
 Some learners: Four cubed.
 Some learners: Four to the power of three.
 Teacher: Four cubed is better. Okay? Next one. And what does four cubed mean? Take three fingers.
 Learner: Four ...
 Teacher: And now?
 Learners: Four times four times four.
 Teacher: If you want to use your calculators or if you have a calculator like me. Right. (S P7 L2 EE1 lines 8-14)

Consider the following instance where students were unable to calculate the value of 2^4 ; their answer was eight:

Teacher: [...] Two to the power of four. Take your four fingers and take.
 Teacher and Learners: Two times two times two times two.
 Teacher: And that gives you eight, right? ... Right?
 Learners: Yes, sir. (S P7 L2 EE1 lines 14 -18)

In the transcript extract just presented, students were instructed to use four fingers to count off multiplying two times two times two times two. The criteria for reproducing the required solution were obviously inadequate since students agreed with the incorrect answer provided by the teacher. In the transcript extract that follows, the teacher once again provided an incorrect answer, either as a means of generating a correct answer or to test if students understood the concept of the exponent in the extract for calculating 10^3 :

Teacher: [...] Ten to the power of three. Thirty. Okay?
 Learners: Yes. Yes.
 Teacher: Now where do we come with? What is it supposed to be? Three fingers.
 Teacher and Learners: Ten times ten times ten.
 Teacher: Is that thirty?
 Learners: No sir. (S P7 L2 EE1lines 25-30)

Counting off on fingers and using the calculator to confirm answers were the central means of regulation for computing such problems. The general form of an exponential expression was not made explicit at any point during both lessons, i.e. structure of the exponential expression as one which comprised of a base and an exponent, and the general idea that any number raised to a power n could be calculated by multiplying the number by itself n times. The teacher, however, preferred to use specific examples during the exposition of this topic. Consider the computational activity for this evaluative event in Table I.13:

Table I.13: The computational activity for S P7L2EE1

Lesson 2	EE1[00:00- 02:32]	Input (domain)	Output (codomain)	Operation
Powers		\mathbb{N}	\mathbb{N}	Multiplication

Curriculum and textbooks

See L1 EE4

The mathematics encyclopaedia

Exponential expressions: see L1 EE4

Exponents: see L1 EE4

Cubes: see L1 EE2 & EE3

Squares: see L1 EE3

Regulation of the computational activity

In this instance, the idea of an exponential expression emerges as the outcome of some activity. Counting on one's fingers as well as using the calculator are the means of regulation in this evaluative event and these could be classified as *computational resources*.

School P7 Lesson 2 EE2: Square roots and cube roots

Describing the computational activity

This evaluative event is rather brief and simply entailed the teacher quizzing students on square roots and cube roots, suggesting that they should be memorised when he said: ‘But you should know it straight away like that ...’ S P7 L2 EE2 line 37. Consider the following interaction for finding square roots:

Teacher: The square root of one. This you calculate again, do you? ... But you should know it straight away like that. The square root of one?
Learner: One. (Teacher: The square root of one?)
Learner: Is one sir.
Teacher: I told you earlier on, check. ... Come and get the right one here. Next one. Square root of three, of nine.
Learner: Three. . (S P7 L2 EE1lines 36 - 40)

Table I.14 presents an account of the computational activity for this evaluative event:

Table I.14: The computational activity for S P7L2EE2

Lesson 2	EE2[02:32- 02:55]	Input (domain)	Output (codomain)	Operation
Square roots and cube roots		\mathbb{N}	\mathbb{N}	Square rooting (Reciting squareroots)

Curriculum and textbooks

See L1 EE3

The mathematics encyclopaedia

Square root: see L1 EE3

Cube root: see L1 EE2

Regulation of the computational activity

Memorising the list of square roots and cube roots is the primary means of regulation in this evaluative event.

School P7 Lesson 2 EE3: Finding the first two-digit number that has a digit sum of 12 and finding the first three digit number that has a digit sum of 14

Describing the computational activity

This evaluative event entailed students being able to identify the first two digit number, the biggest two digit number, the first three digit number and the largest three digit number. Once this was accomplished by all, they were required to find the first two digit number that had a digit sum of twelve as well as the first three digit number that had a digit sum of 14.

Consider the first two digit number, 10, as well as the biggest two digit number, 99, as well as the method employed for working out their digit sums: $10: 1+0=1$ $11: 1+1=2$ $12: 1+2=3$. In the transcript extract that follows, the teacher requested that students calculate the first two digit number that had a digit sum of twelve:

Teacher: [...] Now where is your first two-digit number where the sum of the digits is twelve?
Learner: Six.
Teacher: That's not that even part of the.. one of the answers. It's a two-digit number. Where's the first one?
Learner: Six.
Teacher: Six is not a two-digit number.... What's the first two-digit number where the sum of the digits is twelve. ...?
Learner: Six.
Teacher: Hmm?
Learner: Six. [05:00]
Teacher: But it's a one-digit number. It must be a two-digit number. What's the first two-digit number where the digits will add to twelve.
Learner: [Inaudible].
Teacher: Let me see who can get that. Instead of me giving you those kind of chocolates I'll give you a real chocolate. Hmm? Twelve? No, my boy. The digits in twelve are one and two which add up to three.
Learners: [Inaudible].
Teacher: What's the first two-digit number where the digits add up to twelve. It will be thirty-nine. It can't be in the twenties. The biggest number that you can get is twenty-nine, which will give you eleven. The first one will be thirty-nine. Three and nine adds up to twelve. Then in the forties you find one. Which number will be in the forties.... Two numbers that add up to twelve in the forties. [06:00] (S P7 L2 EE1 lines 52 - 72)

The student that answered the teacher in the above transcript extract repeatedly answered the question incorrectly. It appeared as though he did not understand what a two-digit number was. Perhaps he was dividing two (the two digit number) by twelve (the sum of the digits) to get an answer of six. If this was the case, then this particular student had no idea of what was being asked and was therefore unable to relate a two digit number to its associated digit sum. The correct answer for this problem was 39, since $3 + 9 = 12$.

Students later identified the first three digit number as being 100 and the largest three digit number as 999. They were then requested to calculate the largest three digit number that had a digit sum of 14. They added up the digit sum of 100 and all agreed that that $1+0+0 = 1$:

Teacher: [...] The first four three-digit numbers where the sum of the digits is fourteen. Three-digit numbers. The digits add up to fourteen . What's the first three-digit number? One hundred. What do the digits add up to in a hundred..... What are the digits in a hundred. One.
Teacher & Learners: Zero and zero.
Teacher: What do those digits add up to?
Learner: One. (S P7 L2 EE1 lines 78 - 83)

Finding the largest three digit number with a digit sum of 14 proceeded in the following manner: $100:1+0+0=1$, $107:1+0+7=8$ followed by calculating the digit sum of 119, 129, 139 so as to estimate whether the answer of 14 had been reached yet. This time a student correctly answered 149 because $1+4+9=14$:

Learner: One hundred and forty-nine.
Teacher: Hundred and forty-nine she said. Is she right? That's the first one. One and nine at the end makes ten and four in the middle makes fourteen. [...] (S P7 L2 EE1 lines 96 - 97)

Consider Table I.15 which presents an account of the computational activity for this evaluative event:

Table I.15: The computational activity for S P7L2EE3

Lesson 2	EE3[02:55- 09:20]	Input (domain)	Output (codomain)	Operation
Finding the first two-digit number that has a digit sum of 12 and finding the first three digit number that has a digit sum of 14		\mathbb{N}	\mathbb{N}	Addition

Curriculum and textbooks

The curriculum prescribes the following for Learning Outcome 1(Numbers, Operations and Relationships): ‘The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems’ (DoE, 2002:61).

The mathematics encyclopaedia

Most students were familiar with calculating the digit sum of values and have no problem understanding its universal meaning i.e. Sum: An amount obtained as a result of adding numbers. As mentioned previously, no textbook or worksheet material was gathered that references this particular topic.

Regulation of the computational activity

Students were generally engaged in adding whole numbers and were regulated by using trial and error and estimation as a means for calculating the correct answer.

School P7 Lesson 2 EE4: Recap of the meaning of 5^2

Describing the computational activity

Consider the following transcript extract where the teacher responded to a learner's incorrect interpretation of 5^2 as he walked around the classroom checking learners' notebooks:

- Teacher: Five squared. What does five squared mean? You see, this is something you mustn't do. [Teacher writes on board. (See Figure I.16) Five squared equal ten. He is saying five times two. What is he supposed to say?
- Learner: Five times five.
- Teacher: Five times five. Five squared is two fives being multiplied together. [...] (S P7 L2 EE4 lines 104 - 107)

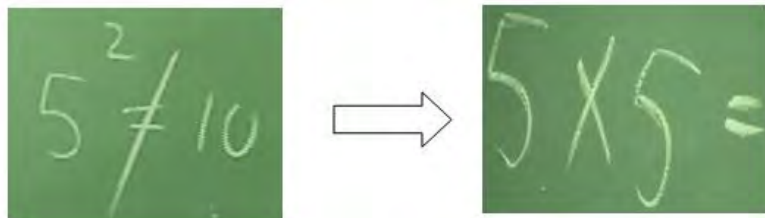


Figure I.16: A corrected interpretation of five squared (S P7 L2 EE4)

It is interesting that this episode occurred in Lesson 2 (EE4), even though a list of squares and cubes were generated the previous day during Lesson 1. This particular lesson on the previous day entailed using expansion e.g. $2^3 = 2 \times 2 \times 2$, counting off using 'fingers' and confirming answers using a calculator. The definition of 5^2 that the teacher prioritised in this particular lesson was framed in terms of how to compute its value i.e. 'Five squared is two fives being multiplied together. [...]' (line 108). There was no formal description of the object 5^2 in terms of its existential features and properties, even though the teacher asked: ' [...] What does five squared mean? [...]' As discussed in both L1EE4 and L2EE1, the form of an exponential expression comprising of a base and an exponent was never defined. The teacher had up until then only used specific examples to describe powers and exponents. Consider Table I.16 which provides an account of the computational activity for this evaluative event:

Table I.16: The computational activity for S P7L2EE4

Lesson 2	EE4[09:30- 10:50]	Input (domain)	Output (codomain)	Operation
Meaning of 5^2		\mathbb{N}	\mathbb{N}	Multiplication

Curriculum and textbooks

See L1 EE3

The mathematics encyclopaedia

Squares: see L1 EE3

Regulation of the computational activity

See L1 EE3

School P7 Lesson 2 EE5: Types of numbers

Describing the computational activity

This evaluative event comprises of a series of exercises that were discussed and marked. The mathematical objects to be acquired in this evaluative event included listing odd numbers, even numbers greater than 30, prime numbers as well as listing composite numbers less than 30. Since this particular evaluative event entailed checking a homework exercise, very little reference was made to the existential features relating to these topics. Perhaps these features relating to the various topics were mentioned in lessons prior to the three lessons in the archive for School P7.

Consider the following extract where the teacher provided an incorrect answer as a means to test whether students were able to identify the list of prime numbers smaller than five and as well as a list of composite numbers smaller than thirty:

Teacher: [...] Prime numbers. One. Right?
Learners: Yes, sir. No. No, sir.
Teacher: Huh?
Learners: No sir. No.
Teacher: Why?
Learner: Yes sir. One only had one factor, sir.
Teacher: Is one a prime number? So where's your first prime number?
Learners: Two.
Teacher: Two, then?
Learners: Then nothing. That's all.
Teacher: Hmm?
Learners: That's all.
Teacher: At least you're starting to understand prime numbers. Then composite less than thirty. [...]
Learner: [Inaudible.]
Teacher: Hmm?
Learner: [Inaudible.]
Teacher: Less than thirty.
Learner: [Inaudible.]
Teacher: Hmm?
Teacher: [...] Okay. Composite numbers less than thirty are four, six, okay okay. I'm gonna stop there now. Look here. Go home. Go over everything. [...] (S P7 L2 EE8 lines 140 - 162)

The teacher provides a definition of composite numbers, factors and prime numbers which can be categorised as general descriptions. So the domain, codomain and operation are not applicable in this instance. (See Table I.17)

Table I.17: The computational activity for S P7L2EE5

Lesson 2	EE5[11:30- 14:15]	Input (domain)	Output (codomain)	Operation
Types of numbers		Not applicable	Not applicable	Not applicable

Curriculum and textbooks

See L1 EE4

The mathematics encyclopaedia

When considering this evaluative event and the topics covered, there is a cluster of inter-related contents that are related to the topic: Prime numbers and composite numbers. These include factors, co-factors, divisors, divisibility, products and usually, integers. Outside of dictionaries, appropriate descriptions are found in texts on number theory, an example of which follows.

The number 1 has only one positive divisor, namely 1. In this respect the number 1 stands alone in the sequence of natural numbers. Every integer, greater than 1, has no fewer than two divisors, namely 1 and itself; if these divisors exhaust all the positive divisors of an integer, then it is said to be prime. An integer >1 which has positive divisors other than 1 and itself, is said to be composite. (Vinogradov, 1954: 14; italics in original.)

The definition presented draws out the set of interconnected existential features of a number when it can be described as a prime number or composite number. In terms of this definition, the recognition of one number as prime or as a composite entails an activation of a network of ideas, as indicated earlier, that are much richer than the idea of simply reciting a list of primes or composites.

Regulation of the computational activity

This evaluative event is regulated by *general descriptions* (dealt with in lessons prior to this one) of primes, odds, composites, squares and cubes.

School P7 Lesson 3

Generating the evaluative events

Table I.18: Evaluative events spanning Lesson 3 at School P7 (S P7 L3 EE1)

Evaluative Event	Type	Length
01 Drawing bar graphs [00:00 – 31:02]	Expository	31:02
		31:02

School P7 Lesson 3 EE1: Drawing bar graphs

Describing the computational activity

The topic, bar graphs, is announced explicitly by the teacher and he requested that students should not enquire about the previous days' work. It also appeared that the he expected students to work independently during this particular lesson after providing them with an explanation of the announced topic. The bar graph presented to students was a representation of Grade 8 – 12 student enrolment at a high school for a particular year. The transcript extract suggests that they are required to listen to his instructions and then complete the activity:

Teacher: What's this talking for? My girl, you're asking me questions. Now what did I say? I'll give you an excellent chance to go over this work. Right? Now don't ask me any questions today. I'm giving you a chance today. [...] [Writes on board] / Bar Graphs/. (S P7 L3 EE1 lines 2-3)

In the transcript extract that follows, the teacher pointed to a representation of a bar graph on the board, and said:

Teacher: [...] That's the work we're doing today ... bar graphs. Now any graph [points to bar graph on board] is a picture of a ... of a set of numbers. [...] (S P7 L3 EE1 lines 7-8)

He described a graph as 'a picture of a ... set of numbers' (S P7 L3 EE1 line 7-8) which suggested that he was focusing primarily on what the graph should look like. Later, in this lesson the teacher stressed that mathematics was produced by following rules and listening to the instructions provided by teachers:

Teacher: [...] Now I want you to follow instructions. It's not a joke. Sit up. I don't know. You people must learn to follow instructions. In maths you've got to learn to go by the rules. If you don't go by the rules then you're gonna make mistakes. If you don't pay proper attention then you're gonna get the wrong answers. You must learn to listen to teachers. That's what this ... now, one of the things about this lesson is that one of these days you're getting a project. That's why I want to teach you about bar graphs because you're going to have to do it all on your own. Is that clear? (S P7 L3 EE1 lines 110-114).

He also emphasised that the layout of the bar graph needed to be exactly as he has presented it. Students painstakingly drew the table in Figure I.17 and the graph in Figure I.18, wasting valuable lesson time. The procedure for producing the content, in this instance the bar graph, relied on what the final 'picture' looked like and can be described as follows:

1. A table of values (data) measuring a certain variable is plotted on the bar graph. (See Figure I.17)
2. Identify the variable on the vertical axis (with the appropriate calibration) and label it. (See Figure I.18)
3. Identify the variable on the horizontal axis (with the appropriate calibration) and label it. (See Figure I.18)
4. Provide the graph with a title.
5. Each bar must have a width of 2mm and should be shaded in.
6. Bars should have spaces in between them.

Grade	8	9	10	11	12
Enrollment	250	310	205	190	135

Figure I.17: The table of values (S P7 L3 EE1)



Figure I.18: Identifying and calibrating the vertical and horizontal axes and reproducing the bar graph (S P7 L3 EE1)

In Figure I.18 the teacher provided learners with a scale, indicating that 2 spaces on the vertical axis represented 50 units and that the horizontal axis should be calibrated in 1cm units. Although the teacher indicated which values were to appear on the vertical axis, he was not explicit about how to construct the vertical scale from the table of values in Figure I.17:

- Teacher: [...] You use two spaces here. [Referring to the vertical axis.] Okay? Just let's .. let's just put in something else so long. This point is going to be nought. After two spaces we're going to have fifty... after another two spaces .. one hundred. And then that will be a hundred and fifty and so we go along. Alright? So how many spaces are you going to need altogether here? Come, come, come, come! We need two spaces from nought to fifty. Another two .. another two .. another two. Okay? Then we come to two hundred .. then to two Fifty .. then three hundred .. then three hundred and fifty. How many spaces? That would be four hundred
- Learner: Four hundred. (S P7 L3 EE1 lines 18-24).

The criteria made available to the students to reproduce a bar graph remained framed in terms the strict procedure that needed to be adhered to. Furthermore, students were also told to refrain from asking questions at the commencement of the lesson (line 3). After learners had completed their attempts of these graphs, the teacher noticed that a few of the students were experiencing difficulty in drawing these graphs and the following interaction ensued:

- Teacher: [Holds up a learner's book] We said two spaces for every fifty and therefore it should be sixteen spaces. What did this boy do? Did he especially close his ears? Then we got other people that are using two centimetres here like that girl there at the back. Now I want you to follow instructions. (S P7 L3 EE1 lines 108-110)

Any deviation from the procedure for drawing the bar graph produced an outcome which was not regarded as being legitimate by the teacher, even when students accidentally used a different scale on the horizontal or vertical axis and produced a valid solution. As already mentioned the emphasis in this evaluative event appeared to be focused on following the teacher's prescribed procedure precisely and replicating his exact representation of the bar graph. Consider the computational activity for this evaluative vent in Table I.19:

Table I.19: The computational activity for S P7L3EE1

Lesson 3	EE1[00:00- 31:02]	Input (domain)	Output (codomain)	Operation
Drawing bar graphs		\mathbb{N}	\mathbb{N}	Counting and drawing

Curriculum and textbooks

Bar graphs are referenced in the RNCS in Learning Outcome 5 (Data Handling) and students are expected to both display and interpret data from bar graphs:

LEARNING OUTCOME 5 DATA HANDLING

The learner will be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation.

Draws a variety of graphs by hand/technology to display and interpret data including:

- bar graphs and double bar graphs;
- histograms with given and own intervals [...] (DoE, 2002: 91)

The teacher adhered to this curriculum prescription rather stringently and placed great emphasis on the procedure for producing this graph. Consider how the textbook defines a bar graph in Figure I.19:

SKILL 4 CONSTRUCTING BAR GRAPHS

This method of displaying information is based on drawing rectangles for each category. The taller the rectangle the larger the number in that category.

Figure I.19: The definition of a bar graph in *Preparing for High School Maths* (Bull & Hepworth, 2008: 138)

The textbook's presentation of this topic was quite similar to the teacher's method and almost provided a checklist for what a bar graph should look like in Figure I.20:

Bar charts must have:

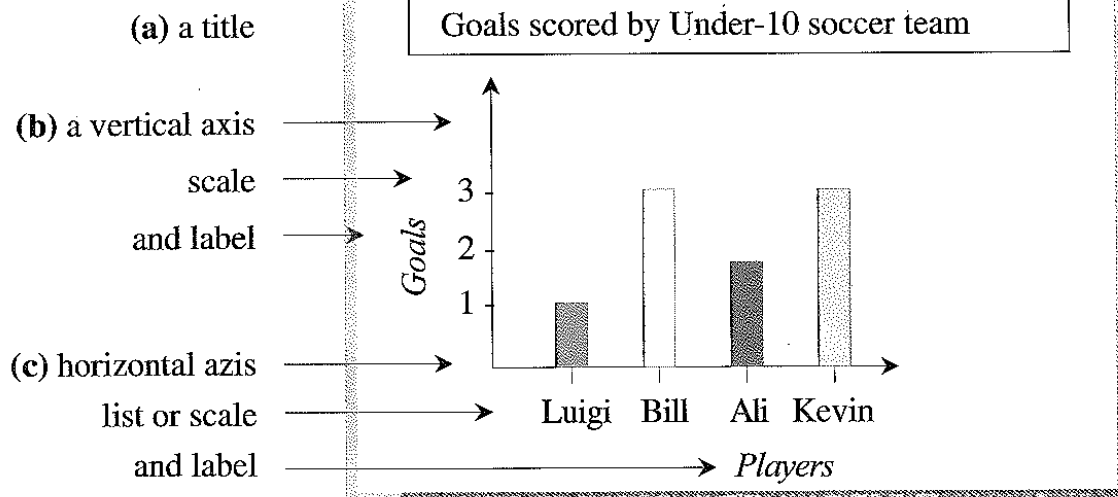


Figure I.20: 'Bar charts must have' in *Preparing for High School Maths* (Bull & Hepworth, 2008: 138)

Figure I.20 describes three key features of bar graphs that are also emphasised by the teacher.

The mathematics encyclopaedia

The following definition is not provided by curriculum, textbook or the teacher in the presentation of this lesson:

Bar graph: A bar graph is a pictorial rendition of statistical data in which the independent variable can attain only certain discrete values. The dependent variable may be discrete or continuous. The most common form of a bar graph is a vertical bar graph, also called a column graph. (Whatis.com)

This mathematical definitions are contrasted against the textbook's description of a bar graph in Figure I.19 which describes it as a 'method of displaying information (which) is based on drawing rectangles for each category [...]' (Bull & Hepworth, 2008: 138) as well as the teacher's definition of it as being 'a graph is a picture of a set of numbers' (line 7-8).

Regulation of the computational activity

The means of regulation in this evaluative event relies on the *computational procedure* provided by the teacher and on the *symbolic representation* of the bar graph, especially since he says that 'a graph is a picture of a set of numbers' (line 7-8). These certainly stand in place of any sort of formal definition of what exactly a bar graph might be. Table I.20 presents an account for how a bar graph is constituted in this lesson:

Table I.20: The constitution of a bar graph

Announced topic	Procedures used	Regulation of the computational activity	Realised content	Substitutes for
Bar graph	Measuring Drawing – vertical and horizontal axis with a scale and title	Procedure - Computational resource A reliance on what solution may look like	Replica of teacher's bar graph Counting	Definition of a bar graph Definition of independent and dependent variable

University of Cape Town

Appendix J: Realisation of the content across the five schools

School	Announced topic		Realised content	
P1	1	Prime factorisation (Factor tree method)	Multiplication of whole numbers	Basic arithmetic
		Prime factorisation (Ladder method)	Division of whole numbers	
	2	Multiples	Multiplication tables	Basic arithmetic
	3	Lowest common multiple (LCM)	Multiplication tables	Basic arithmetic
	4	Highest common factor (HCF)	Multiplication of whole numbers	Basic arithmetic
P2	5	Plotting co-ordinates	Order of plotting points Counting Pictorial representation	Counting
	6	Translations	Sliding a figure to a new position without turning it Counting	Counting
P3	7	Prime numbers	Prime numbers	Classification of whole numbers
	8	Composite numbers	Composite numbers	Classification of whole numbers
	9	Factors	Factors	Classification of whole numbers
	10	Highest common factor	Multiplication of whole numbers	Basic arithmetic
	11	Equivalent fractions, mixed numbers and improper fractions	Multiplication and addition of whole numbers Fractions are treated as pairs of whole numbers	Basic arithmetic
	12	Addition of fractions with the same denominator	Multiplication, division and addition of whole numbers	Basic arithmetic
	13	Addition of fractions with different denominators	Multiplication, division and addition of whole numbers	Basic arithmetic
P6	14	Integer addition when signs are the same	Addition of whole numbers	Basic arithmetic

School	Announced topic		Realised content	
	15	Integer addition when signs are different	Whole number subtraction Counting using the Poker Chip model	Basic arithmetic
	16	Integer multiplication	Whole number multiplication	Basic arithmetic
	17	Integer division	Whole number division	Basic arithmetic
	18	BODMAS	Whole number arithmetic	Basic arithmetic
	19	Commutative property	Whole number arithmetic	Basic arithmetic
	20	Distributive property	Whole number arithmetic	Basic arithmetic
P7	21	Factors	Division of whole numbers	Basic arithmetic
	22	Highest common factor	Division of whole numbers	Basic arithmetic
	23	Cube roots	Counting Estimation	Counting
	24	Exponential expressions	Counting Multiplication of whole numbers	Counting
	25	Bar graph	Replica of teacher's bar graph Counting	Counting

Appendix K: Regulation of the content across the five schools

K.1 Forms of regulation at School P1

For all four topic areas, the procedures presented by the teacher and the textbook served as computational resources and for two of these topics there was also a reliance on what the solution should look like, especially since the ‘ladder template’ for prime factorisation was emphasised by the teacher. The computational procedures were the dominant means of regulation for all of the evaluative events at School P1, followed by a reliance on what the solution should look like for 3 of the 14 evaluative events as displayed in Table K.1.

Table K.1: Forms of regulation at School P1

Evaluative Event	Lesson 1						Lesson 2					Lesson 3		
	1	2.1	2.2	2.3	3.1	3.2	1.1	1.2	2	3	4	1.1	1.2	2
Interconnected propositions														
Computational resources	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Reliance on what solution looks like	✓				✓	✓								
General Descriptions														

K.2 Forms of regulation at School P2

Table K.2 displays that at School P2, 4 of the 5 evaluative events were regulated by computational resources, 2 of the 5 evaluative events were regulated by a reliance on what the solution should look like while 1 of the 5 presented a general description of a mathematical concept.

Table K.2: Forms of regulation at School P2

Evaluative Event	Lesson 1		Lesson 2	Lesson 3	
	1	2	1	1	2
Interconnected propositions					
Computational resources	✓	✓	✓		✓
Reliance on what solution looks like	✓		✓		
General Descriptions				✓	

K.3 Forms of regulation at School P3

The results displayed in Table K.3 for School P3 show that mathematics is constituted in a manner which does not grant students access to the main principles and interconnected propositions of mathematics since 5 of the 7 evaluative events were regulated by the use of computational resources, 3 of the 7 by a reliance on what the solution should look like and another 3 out of 7 by providing students with general descriptions of mathematical concepts.

Table K.3: Forms of regulation at School P3

Evaluative event	Lesson 1			Lesson 2			Lesson 3
	1	2	3	1	2	3	1
Interconnected propositions							
Computational resources			✓	✓	✓	✓	✓
Reliance on what solution looks like					✓	✓	✓
General Descriptions	✓	✓	✓				

K.4 Forms of regulation at School P6

For 9 of the 11 evaluative events the procedures prescribed by both the textbook as well as the teacher served as computational resources and this was by and large the dominant means of regulation for constituting the the topic of integer arithmetic at school P6. For 2 of the 11 evaluative events, the use of a mathematical proposition regulated the elaboration of the content. Table K.4 displays this result.

Table K.4: Forms of regulation at School P6

Evaluative Event	Lesson 1			Lesson 2		Lesson 3					
	1	2	3	1	2	1	2	3	4	5	6
Interconnected propositions								✓	✓		
Computational resources	✓	✓	✓	✓	✓	✓	✓			✓	✓
Reliance on what solution looks like											
General Descriptions											

K.5 Forms of regulation at School P7

At School P7, 9 of the 12 evaluative events were regulated by means of computational procedures while 4 of the 12 evaluative events were regulated by general descriptions of mathematical concepts. Only 1 of the 12

evaluative events was regulated by a reliance on what the solution looked like. If one considers how mathematics is constituted for this particular teacher at School P7 on the whole, the data in Table K.5 shows that the computational resource is the primary means of regulation, followed general descriptions and a reliance on what the solution looked like.

Table K.5: Forms of regulation at School P7

Evaluative Event	Lesson 1						Lesson 2					Lesson 3
	1.1	1.2	2	3	4	5	1	2	3	4	5	1
Interconnected propositions												
Computational resources	✓	✓	✓		✓		✓	✓	✓	✓		✓
Reliance on what solution looks like												✓
General Descriptions				✓	✓	✓					✓	

Appendix L: Transcript records

SCHOOL P1

LESSON 1 EE1-EE3.2

S P1 L1 EE1: Prime factorisation of natural numbers - fragment of a lesson

Time	#	Speech
00:00	001	Teacher: Prime factors, remember? So we said there were two methods. There was the factor tree, which this method.
	002	Learner: The ladder.
	003	Teacher: Or it was that ladder one.
	004	Learner: The ladder.
	005	Teacher: And then, but we gave you, there were four different options on the board. And each option gave the same answer. So for the learners that
	006	Learner: Miss [inaudible] in the exam miss, where you can either write that or that.
	007	Teacher: No, you yes! You will be told factorize, write the number as a product of it's prime factors and then you must choose whichever method you,
	008	Learners: Yes, miss.
	009	Teacher: All the examples look like this.
	010	Learners: Yes, miss.
	011	Teacher: So, but in the exam we won't specify, we won't say you must use this method or that. It will simply say, write it as a product of its prime

S P1 L1 EE2.1: Multiples of natural numbers

Time	#	Speech
	012	Teacher: Okay, then in the lab now, we started with multiples and we said that the multiples are the answers that that you get when you do what?
	013	Learner: Times.
	014	Teacher: When you?
	015	Learner: Times.
	016	Teacher: Times what?
	017	Learners: Times by the [inaudible].
	018	Learner: A prime number
	019	Teacher: By the prime number?
	020	Learner: No [Several learners all talk together].
01:59	021	Teacher: No. Okay, let's say I ask you for multiples of five. Remember we did factors, hey? So now we're doing multiples.
	022	Learner: Natural numbers.
	023	Teacher: So I'm asking you now for the multiples of five.

024 Learner: Five, ten.
 025 Teacher: It's five.
 026 Learner and Teacher: Ten, fifteen, twenty.
 027 Teacher: So we said, how do we get to those answers?
 028 Learner: We times by.
 029 Teacher: We multiply it by?
 030 Learner: Five.
 031 Learner: By one.
 032 Teacher: By one.
 033 Learners and Teacher: by two, by three.
 034 Teacher: The answers are the multiples so the multiples of five will be five.
 035 Learners: Ten, fifteen, twenty, twenty-five, thirty, thirty-five, forty, forty-five.
 036 Teacher: Shh, shh thank you. Then we did the speed test. Now the speed test was all about multiples. You had the multiplication table was?
 037 Learner: Twelve.

S P1 L1 EE2.2: Addition of fractions with same denominators

Time	#	Speech
03:00	038	Teacher: Twelve, but of course there are many others. Now why do we need to know multiples? Because we might eventually be asked to [writes on board], what is a quarter plus a quarter?
	039	Learner: A half.
	040	Another Learner Two quarters.
	041	Teacher: A half?
	042	Learner: Yes.
	043	Learner: No.
	044	Teacher: Is it?
	045	Learner: No.
	046	Learner: No, miss.
	047	Teacher: Okay, somebody's saying it's a half.
	048	Learner: A whole, ma'am.
	049	Teacher: Somebody's saying two quarters.
	050	Learner: Shh.

S P1 L1 EE2.2: Addition of fractions with same denominators

Time	#	Speech
	051	Learner: A whole.
	052	Teacher: Shh, right let's see.
	053	Learner: A half.
	054	Learner: Three quarter.
	055	Learner: The denominator stays the same.
	056	Teacher: The denominator stays the same. Darren, Darren what's a denominator?
	057	Learner: Miss?
	058	Teacher: Yes?
	059	Learner: The number at the bottom.
	060	Teacher: The number at the bottom of the fraction. He's saying it's two quarters because the number at the bottom stays the same and then we add the top numbers,
	061	Learners: No, miss.
	062	Other Learners: No.
	063	Other Learners: Yes, miss.
	064	Teacher: Shh, okay, who's saying no?
04:00	065	Learner: Lift up your hand.
	066	Teacher: Right, what is that?
	067	Learner: A whole.
	068	Teacher: Right, if I cut that in half.
	069	Learner: Half.
	070	Teacher: Then this is one out of the two parts. How many parts do I have? I've got two.
	071	Learners: Two, miss.
	072	Teacher: But this one out of the two parts is called?
	073	Learners: A half.
	074	Teacher: And this is?
	075	Learners: A half.
	076	Teacher: Right, if I take that same thing and I [illustrates on board] and I cut it into how many parts?
	077	Learners: Four.
	078	Teacher: Then this is one over four, that's another one over four but didn't we say that's a half?
	079	Learners: Yes, miss.
	080	Teacher: So therefore, a half is the same as?
	081	Learner: Two quarters.

S P1 L1 EE2.3: Addition of fractions with different denominators

Time	#	Speech
	082	Teacher: Right, now why do I need multiples? Because when I add a quarter to a quarter, when these are the same, then it's easy to add, hey? But what if I ask you
	083	Learners: [Inaudible.]
	084	Teacher: Then, is it, can I give an answer immediately?
	085	Learner: No, miss.
	086	Teacher: Is it? Why not?
	087	Learner: They haven't the same denominator, miss.
	088	Teacher: Right, and how do we do that?
	089	Learners: [All talk at once – inaudible].
	090	Teacher: By finding the lowest.
	091	Learners and Teacher: Common multiple.
	092	Teacher: That's why we need to know the multiples of numbers. Now, how do I find a lowest common multiple? What does common mean?
	093	Learners: [All talk at once – inaudible].
	094	Learner: Use the most.
	095	Teacher: Use the most, used frequently.
	096	Learner: Appears in both.
	097	Teacher: Appears in both. Right. So Aswan?
	098	Learner: It's six, miss.
	099	Teacher: It's six. Now how do we get six? Let's look at the multiples of two.
	100	Learner: Two.
	101	Teacher: What are the multiples of two?
06:00	102	Learners: Two, four, six, eight, ten.
	103	Learner: etcetera, etcetera
	104	Learners: Twelve, fourteen, sixteen, eighteen.
	105	Teacher: Okay. Right. That's enough. Let's take the multiples of three.
	106	Learners: Three, six, nine, twelve, fourteen, fifteen, eighteen, twenty-one.
	107	Teacher: Right, now common, we said is in both so we've got more than one that's common. What's all common?
	108	Learner: Six, twelve.
	109	Teacher: Six is common
	110	Learner: Twelve.
	111	Teacher: And if we were to go on, there would be more but we are looking for the?
	112	Learner: Lowest.

S P1 L1 EE2.3: Addition of fractions with different denominators

Time	#	Speech
07:00	113	Teacher: So what would that be?
	114	Learner: Six.
	115	Another Learner: Three.
	116	Learners: Six, miss.
	117	Teacher: Now, once we find the lowest common multiple, we will convert the halves to sixths and we'll convert the thirds to sixths and when they're both sixths then we can add them together, okay? Now how many sixths give you a half?
	118	Learners: Three.
	119	Teacher: And how many give you a third?
	120	Learners: Two.
	121	Teacher: And now we get an answer of?
	122	Learners: Five.
	123	Teacher: Five over?
	124	Learners: Six.

S P1 L1 EE3.1: How to calculate lowest common multiples

Time	#	Speech
08:00	125	Teacher: Okay, so that is the reason why it's important for us to know our multiples because if we don't know multiples then we can't find lowest common multiples and then we find it difficult to add fractions where denominators are different. Okay, now do we all have these books? [Holds up a copy of "Preparing for High School Maths".]
	126	Learners: Yes, miss. No miss.
	127	Teacher: Who's sitting next to somebody? You can have one at every.
	128	Learner: Yes, miss.
	129	Teacher: Right, page, page twenty.
	130	Learner: Miss, we don't have one, miss.
	131	Teacher: Right, who's got two books?
	132	Learner: (names of students unclear)... have two books.
	133	Teacher: I think we'll share...(rest of sentence unclear). Right, can you all open on page twenty-eight.
	134	Learner: Yes, miss.
	135	Teacher: It says, shh. Guys.

S P1 L1 EE3.1: How to calculate lowest common multiples

Time	#	Speech
09:00	136	Learner: Miss.
	137	Teacher: Number Theory practise. Now we've done factors right, we've done um, prime numbers. Shh. Mister. [Pauses for silence.] Right, now we're going to be busy with multiples. Look at Skill Five. Skill Five, multiples. Page twenty-nine. The heading is on page twenty-eight, the exercise on page twenty-nine. Look at Skill Five, multiples. [Writes on board.] Right, this is on page twenty-nine. Okay, it says here, now you must always follow the instructions, hey? If they ask you to list the first four multiples, you may not list eight of them. You must follow the instructions correctly. Yusri? Where's? Are you paying attention?
	138	Learner: Response unclear.
	139	Teacher: Okay. Skill Five. List the first four multiples of eight, five, six, ten then, the next question says list the second four multiples of, now the first four of eight, five, six and ten then, the next four of six, four, seven, twelve and eleven and then, the exercise that I've just shown you, the list, find the lowest common multiple of; now there are pairs of numbers: of twelve and eight, twelve and eighteen and I want all of you to use this method. You're going to use this method where you're going to list your multiples and then you're going to circle the lowest one and then you're going to write. [Illustrates on board.]
	140	Learner: Student mumbling unclear
	141	Teacher: The lowest common multiple of two and three is [writes on board]. Okay, this is how you're going to do all the sums. Yes?
	142	Learner: Now miss, if the number that side is bigger than the other one, if you can find like, let's say, three and five miss, can you like divide twelve by three then
	143	Teacher: But why, why are you dividing twelve by four?
	144	Learner: Miss, but isn't it the same like the lowest common multiple?
	145	Teacher: So what are you saying? What will the answer be? If I give you two numbers like twelve and three.
	146	Learner: Miss, then you leave three like that miss, and then you take twelve and divide it by four, miss.
	147	Teacher: And then you get three. So you're saying the lowest common multiple will be three. Now is three a multiple of twelve?
	148	Learner: No.
	149	Teacher: List the multiples of twelve for me. What's twelve times one?
	150	Learner: Twelve, twenty-four.
	151	Teacher: Twenty-four.
12:00	152	Learner: Thirty-six.
	153	Teacher: So will your answer be three?
	154	Learner: No.
	155	Teacher: That's why I said rather use this method first. We were going to look for short-cuts later. Let's not use the short-cuts now. Let's do it properly then we won't make unnecessary mistakes.
	156	Learner: (mumbles about fractions)
	157	Teacher: The what?
	158	Learner: The fractions.
	159	Teacher: What about the fractions?
	160	Learner: Must first get it.

S P1 L1 EE3.1: How to calculate lowest common multiples

Time	#	Speech
13:00	161	Teacher: What fraction?
	162	Learner: Like, like miss said first find the lowest common multiple, like that method.
	163	Teacher: No, no, no I just used this example to show you why we need to find lowest common multiples. Forget about that. [Wipes board clean.] Don't get confused now with fractions. We're not doing fractions, hey? I just used that as an example to show you that when we add fractions, we need to find the lowest common multiple to be able to add them or subtract them or whatever, okay? But you must simply, let's do the first one together. I'm gonna skip to number nine.
		Teacher: Right, right, what is the lowest common multiple of twelve and eight? So let's start with twelve.
	164	Learner: Twelve, twenty-four.
	165	Teacher: It will be twelve.
	166	Learner: Twenty-four, thirty-six, forty-eight.
	167	Teacher: Forty?
	168	Learner: Forty-eight.
	169	Learner: Sixty.
	170	Teacher: What's after that?
	171	Learner: Sixty, seventy-two.
	172	Teacher: Seventy-two. Now most times, we don't have to go further than that, right? Let's do the multiples of eight.
	173	Learner: Eight.
	174	Teacher: Eight.
	175	Learner: Sixteen, twenty-four, thirty-two, forty, forty-eight.
	176	Teacher: I'm not gonna go any further because I've already noticed there's something that is common.
	177	Learners: ...Twenty-four, twenty-four [Inaudible.]
	178	Teacher: Twenty-four. Therefore the lowest common multiple, lowest common multiple of twelve and eight is?

S P1 L1 EE3.2: Calculating lowest common multiples

Time	#	Speech
14:00	179	Learner: Twenty-four.
	180	Teacher: And then you go with number ten. Start.
	181	Learner: Miss, must we do the whole of Skill Five, miss.
	182	Teacher: Yes, the whole of Skill Five. It is from number one to number twenty.
	183	Learner: Yes, miss.
	184	Teacher: Right, people, before you start with Skill Five, make sure you've written down those examples of this morning.
	185	Learner: Yes, miss.
	186	Teacher: And underneath that you write Skill Five, the page, starting with number one. What?
	187	Learner: [Inaudible.]

S P1 L1 EE3.1: How to calculate lowest common multiples

Time	#	Speech
	188	Teacher: Underneath this morning's work.
15:00	189	Learner and Teacher: converse. [Inaudible.]
	190	Teacher: Start where you've stopped working, hey?
16:00	191	
	192	Learner: Miss, er, [inaudible.]
	193	Teacher: You start with number one, hey.
	194	Learner: Must we first write the multiples and then write the er, answers, the multiples of twelve until maybe seventy-two.
	195	Teacher: Ya.
	196	Learner: Or must you just write the last.
	197	Learner: They say you must write the first four.
	198	Teacher: They say, in that exercise they don't tell you how many you must write down. They just say find the lowest common multiple. Sometimes after three, you
	199	Learner: [Inaudible.]
	200	Teacher: No, no, you start at number one.
	201	Learner: [Inaudible.]
	202	Teacher: Write the first four of eight. The first four of five, the first four of six, the first four of ten.
	203	Learner and Teacher: [Inaudible.]
	204	Teacher: No, I don't want answers, use that method. Write down the multiples of both numbers then you circle the common one.
	205	Learner: Miss, his nose is bleeding
18:00	206	Teacher: [Inaudible] the secretary. Do you often get nose bleeds? Go to the secretary.
	207	Learner: He's bleeding
	208	Learner: [to Teacher – inaudible].
	209	Teacher: Don't be lazy.
	210	Learner bangs head on desk
	211	Learner: Ow!
	212	Teacher: Right, if you've done numbers one to five, if you've got the answers of one to five, you put up your hand.
19:00	213	Intercom Buzzer: Good Morning, school. I'm sorry for this interruption. Can the following people please come down to the office? Leilah Abrahams, Aleah
		Hussain, Zakie Colea and Kayli Swartz. Leilah Abrahams, Aleah Hussain, Zakie Colea and Kayli Swartz. Can you please come down to the office now? Thank you.
19:46		End of Lesson.

SCHOOL P1
LESSON 2 EE1.1 – EE4

S P1 L2 EE1-EE4

S P1 L2 EE1.1: Filling in missing multiples and number patterns

Time	#	Speech
00:00	1.	Teacher: Tule, number one.... Okay, let's go with number four.
	2.	Learner: Number four.
	3.	Teacher: Who's? Who's group number four?
	4.	Learner: This one.
	5.	Teacher: You people?
	6.	Learner: Yes Miss, we are.
	7.	Teacher: Right Alex. ... One a. The number pattern, it's ...[writes on board] and remember ... you must fill in missing multiples so what, ... which multiples do
	8.	Learner: It's er...
	9.	Teacher: Multiples of four [writes answers on board]. ...
	10.	Learner: Four, eight, twelve ...
	11.	Teacher: Right, all got that?
	12.	Learner: Yes, Miss.
	13.	Learner: Must we mark it?
00:41	14.	Teacher: You mark it with your pencil ... and people listen; only, only if your answer is different, you put up your hand because we want to go through this so
	15.	Learner: Twenty, twenty-five.
	16.	Teacher: Now there's no twenty-five, the... the pattern ended at twenty. See here
	17.	Learner: Eight.
	18.	Another Learner: Eighteen.
	19.	Teacher: Right, shh
	20.	Learner: Eighteen.
	21.	Teacher: No, no, no. ... let's go.
	22.	Learner: Eighteen.
	23.	Teacher: Eighteen. [Writes answers on board.]
	24.	Learner: Thirty-six, forty-five.
	25.	Teacher: Now remember ... they gave ... they only gave these two hey ...and you had to fill in those three.

S P1 L2 EE1.1: Filling in missing multiples and number patterns

Time	#	Speech
	26.	Learner: Yes, Miss
	27.	Teacher: So which ... which multiplication table was this?
	28.	Learner: Nine.
	29.	Teacher: These are the multiples of?
	30.	Learner and Teacher: Nine.
	31.	Teacher: Right, then we've got e, d.
	32.	Learner: Four, Miss.
	33.	Teacher: Four.
	34.	Learner: Six.
	35.	Learner: Eight, ten, twelve. [Teacher writes answers on board.]
	36.	Teacher: Okay, that was straight-forward. Next group, Yusri,. [02:00]
02:04	37.	Learner: Five hundred.
	38.	Teacher: Five hundred.
	39.	Learner: Four hundred.
	40.	Another Learner: Six hundred.
	41.	Teacher: Six hundred. Right, six hundred ... they gave ... eight hundred [writes answers on board].
	42.	Learner: Nine hundred.
	43.	Learner: One thousand.
	44.	Teacher: And they gave a thousand.
	45.	Learner: One thousand one hundred.
	46.	Teacher: So we see that we are ... going up in hundreds. ... And ,, then f.
	47.	Learner: But Miss.
	48.	Teacher: They give eight ... two spaces ... and then thirty-two.
	49.	Learner: [Chair noise makes learner's words unclear] Eight.
	50.	Teacher: How much? ... Sixteen, twenty-four and?
	51.	Learner: Eighty-two.
02:52	52.	Teacher: Also the ... eight times table. Now, number two. Find out for each of these pairs of numbers whether the first number... [03:00] [learner approaches
	53.	Learner: Early for him, yes, Miss.
	54.	Learners: [Inaudible commenting].

S P1 L2 EE1.2: How to check if a number is a multiple of another

Time	#	Speech
	55.	Teacher: Okay, right question two. Find out for each of these pairs of numbers whether the first number is a multiple of the second and what did I say you
	56.	Learner: Yes or no.
	57.	Teacher: What must you say?
	58.	Learners: Yes or no.
	59.	Teacher: Right, so they give you ... [writes on board]
	60.	Learner: Yes.
	61.	Teacher: Is the first number a multiple of the second number?
	62.	Learners: Yes, Miss.
	63.	Teacher: Right, b.
	64.	Learner: No.
	65.	Teacher: Is three, is twenty-seven a multiple of three?
	66.	Learners: Yes. Yes.
	67.	Learner: No.
	68.	Teacher: C. Right, who didn't have that? ... Who didn't have that answer? ...
	69.	Learners: [Inaudible].
	70.	Teacher: One, have you got that answer?
	71.	Learner: <u>Ja</u> {Yes}, I've got that answer. [04:00]
	72.	Another Learner: But I didn't [inaudible].
04:02	73.	Teacher: But I hear people say no. ... You all got it? ... Right, is five a multiple of...
	74.	Learner: Yes, Miss.
	75.	Teacher: Sorry, is thirty a multiple of five?
	76.	Learner: Yes, Miss.
	77.	Another Learner: Say yes, Miss. [Teacher writes on board.]
	78.	Third Learner: Yes, Miss.
	79.	Teacher: Is thirty-six a multiple of nine?
	80.	Learner: Yes, Miss.
	81.	Teacher: And is nine thousand one hundred and eighty a multiple of nine?
	82.	Learner: Yes, Miss.
	83.	Learner: Yes, ma'am
	84.	Teacher: How do we know that ... that is so?

S P1 L2 EE1.2: How to check if a number is a multiple of another

Time	#	Speech
	85.	Learners: [All call out various answers at the same time – inaudible].
	86.	Teacher: How do we calculate it?
	87.	Learners: Divide by. [learners give different answers – inaudible].
	88.	Another Learner: Times [other learners talking ...inaudible].
	89.	Teacher: Shh.
	90.	Learner: Divide by nine.
	91.	Teacher: And what answer did you get?
	92.	Learner: One thousand and twenty.
	93.	Teacher: One thousand and?
	94.	Learner: Twenty.
04:43	95.	Teacher: One oh two ... zero okay. So it's yes. ... [Writes answer on board.] So nine thousand one hundred and eighty is a multiple of nine ... and he said to
	96.	Learner: Find the ...

S P1 L2 EE2: Lowest common multiple

Time	#	Speech
	97.	Teacher: Find the lowest common multiple of each of the following sets of numbers. Now the method that we used yesterday was where we list the multiples,
	98.	Learner: Six.
	99.	Learner: Six, Miss.
	100.	Learner: Miss and us?
	101.	Learner: Six, Miss.
	102.	Teacher: Who didn't answer?
	103.	Learner: Group six.
	104.	Teacher: Group six.
	105.	Learner: And group two, Miss.
	106.	Teacher: Group six. Right, shh. Okay we're gonna get to you. Right, shh, three a. ... Let's start, three a. The multiple, the lowest common multiple of two and
	107.	Learner: Fourteen.
	108.	Another Learner: Fourteen.
05:54	109.	Teacher: No. Who's group six? I hear other people shouting out. Right. Now, group six. [06:00] Each one of you must get a turn. Let's start with two and seven.
	110.	Learner: Fourteen.

S P1 L2 EE2: Lowest common multiple

Time	#	Speech
	111.	Teacher: Fourteen... [Writes on board.] How did you get fourteen?
	112.	Learner: [Inaudible.]
	113.	Teacher: What did we do? We listed the multiples of two. [Writing on board.] So what did you get?
	114.	Learners: Two, four, six, eight, ten, twelve, fourteen. [Teacher writes all the answers on the board.]
	115.	Teacher: Then of seven it was?
	116.	Learners: [Inaudible] twenty-one, twenty-eight, thirty-five.
	117.	Teacher: Right. So we did, we used this method hey, we circled it [circles it on board] and we said therefore the LCM is?
	118.	Learner: Fourteen.
	119.	Teacher: Right. Now this is a long and tedious method but you did it yesterday. Today we're gonna use a different method. ... So we're just gonna write down answers. Only if you don't have the answer, am I gonna show you the multiples, circle the common one and show how we got our answer. Right, so what have
	120.	Learner: Miss.
	121.	Teacher: The LCM ...
	122.	Learners: [Inaudible].
06:57	123.	Teacher: No, no, no. Only that group. ... Nine and twelve. [07:00]
	124.	Learner: Thirty-six.
	125.	Teacher: Thirty-six. L.C.M of nine and twelve. ... [Writes on board] ... The lowest common multiple of five and eight. ... Five and eight?
	126.	Learner: Forty.
	127.	Another Learner: Forty.
	128.	Teacher: Forty. And remember, we listed the multiples of five ... we listed the multiples of eight ... and the lowest one which appeared in both, which was
	129.	Learner: Thirty-six. [Teacher writes on board.]
	130.	Teacher: Now e. The lowest common multiple of three, five and six. Akeeqah?
	131.	Learner: Thirty.
07:57	132.	Teacher: Thirty. ... [08:00] ... Does everybody have that answer ... e? Let's see ... put up your hand
	133.	Learner: Thirty.
	134.	Teacher: Lowest common multiple?
	135.	Learner: Thirty.
	136.	Teacher: Also thirty.
	137.	Learner: Thirty. [Teacher writes on board.]
	138.	
	139.	

S P1 L2 EE2: Lowest common multiple

Time	#	Speech
08:44	140.	Teacher: Right, now the second part of the page.
	141.	Learner: I have that!
	142.	Teacher: Okay, let's see who, who, who managed to get to the second part of the page. Let's see?
	143.	Learner: Can't say, we didn't learn that (inaudible – learners talking in the background.)
	144.	Teacher: Okay, right so ...
	145.	Teacher and Learners: Highest ... common ... factor.
	146.	Teacher: Right, so put down your pens. ... Let's take the factors of sixteen ...[writes on board] and [09:00] twenty-four. Right, who did it? Who did this
	147.	Learner: Miss, I wrote down all the factors of six and forty-four and then I (word unclear) which one was the highest.
	148.	Teacher: Sixteen and twenty-four.
	149.	Learner: Don't be versin!
	150.	Teacher: Oh, you did the sum. Six and forty-four. Right. Let's do the sum that he did ... six and forty-four. You wrote down what? The factors of six.
	151.	Learner: Yes, Miss
	152.	Teacher: Right, so what did you get?
	153.	Learner: Two times three.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
10:00	154.	Teacher: Two times three. [Writes on board] Did you write it as two times three or did you write it like that? [Writes
	155.	Learner: Two rand three.
	156.	Teacher: Right, okay
	157.	Learner: And one and six.
	158.	Teacher: And one and six. So we put it in that order. Right, but you didn't have to put it in that order, hey?
	159.	Learner: I did.
	160.	Teacher: Okay. Then, of forty-four.
	161.	Learner: [10:00] Two times twenty-two.
	162.	Learner: Miss, there's someone's bread
	163.	Learner: It's yours
	164.	Teacher: [Turns from board] What?
	165.	Learner: Gemors! It's yours!

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
	166.	Learner: It is!
	167.	Learner: It's bread, Miss!
	168.	Learner: It's yours
	169.	Learner: I don't have peanut butter on my bread
	170.	Learner: Give it to me, man
	171.	Teacher: Put it in...okay, leave it there, leave it there for this time, for this and at the end of the period, you put it in the bin. Okay, are you listening
	172.	Learner: Yes, Miss
	173.	Teacher: Right, firstly, do you agree that the factors of six are one, two, three and six.
	174.	Learner: Yes
	175.	Teacher: Right, because all of them will divide into six without giving a remainder. Right, now the factors of forty-four. What have you got?
	176.	Learner: Two and twenty-two.
	177.	Teacher: Two and thirty-two.
	178.	Learner: Twenty-two, twenty-two.
	179.	Teacher: Twenty-two.
	180.	Learner: Eleven and four.
	181.	Teacher: Eleven and four. Let's put it there.
	182.	Learner: One and forty-four.
10:55	183.	Teacher: And one and forty-four. Right, so they're all jumbled up, hey, but it doesn't matter. [11:00] So it's one, two, four, eleven, twenty-two and
	184.	Learner: <u>Nee</u> {No}.
	185.	Teacher: Right, now they're asking you for the...
	186.	Learner: Highest common.
	187.	Teacher: Highest common one and what was your answer?
	188.	Learner: Twenty-two.
	189.	Another Learner: Two.
	190.	Third Learner: No, not twenty-two.
	191.	Fourth Learner: Two.
	192.	Teacher: Why are you saying twenty-two? Who says twenty-two?
	193.	Learner: Him!
	194.	Teacher: Why?
	195.	Learner: Miss, um it appears in both.
	196.	Teacher: Where's, where's, over here?

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
	197.	Learner: Here, Miss.
	198.	Teacher: William? That's a one, that's a two, that's a three, that's a six.
	199.	Learners: [Inaudible.]
	200.	Teacher: Okay, remember people, firstly it must be common. Common means it must appear in both sets so...
	201.	Learner: Two.
	202.	Teacher: Two.
	203.	Learner: Miss?
	204.	Teacher: And there we've got a two. [Circles the twos on the board.] Yes?
	205.	Learner: Oh, now I understand.
	206.	Learner: And the one.
	207.	Teacher: The one is also common but it must be the ...
	208.	Learner: Highest.
11:59	209.	Teacher: And it's not the highest. Okay, [12:00] so the highest one is?
	210.	Learner: Two.
	211.	Teacher: So the highest common factor ... is?
	212.	Learner: Two.
	213.	Teacher: Right. ... Now since you didn't do ... the other sums, I'm ... I'm not gonna continue with the other method I wanted to show you till you
	214.	Learner: [Inaudible].
	215.	Teacher: Right, who still doesn't understand how to do the factors of a number? ... Your hand up?
	216.	Learner: No, Miss.
	217.	Teacher: Okay, we all know how to get the factors. We ask ourselves which numbers multiplied with each other give us that number or we say which numbers can divide into that number and not give a remainder. So, what are the factors of forty-two? [Writes on board.]
13:00	218.	Learner: [13:00] Twenty-one.
	219.	Learner: Six.
	220.	Teacher: Right, let's start with the very smallest number. Let's start with which number?
	221.	Learners: One.
	222.	Teacher: Will one go into forty-two?
	223.	Learners: Yes, Miss.
	224.	Teacher: How many times?
	225.	Learners: Forty-two times.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
	226.	Teacher: Then we'll try the next number which is?
	227.	Learner: Two.
	228.	Teacher: Will two go into that?
	229.	Learner: Yes, Miss.
	230.	Teacher: Now remember, one times the forty-two will give you forty-two and the two times what will give you forty-two?
	231.	Learner: Twenty-one.
	232.	Teacher: Twenty-one, right. Now let's see will three go into... [interrupted]
	233.	Learner: Yes, Miss.
	234.	Teacher: Right, three goes there, how many times?
	235.	Learner: Four, no.
	236.	Learner: No Miss, three can't go into forty-two.
	237.	Teacher: Okay let's... it's three, carry one.
	238.	Learners: [Inaudible.]
	239.	Teacher: Fourteen.
	240.	Learner: Fourteen, Miss.
	241.	Teacher: Will four go there?
	242.	Learner: No, Miss.
	243.	Teacher: And six ... and what other number?
	244.	Learner: And eight, Miss.
	245.	Teacher: Eight.
	246.	Learners: Seven.
13:59	247.	Teacher: Seven. [14:00] [Writes on board.] Let's try eight. On your calculator, try eight.
	248.	Learners: No.
	249.	Teacher: Forty-two divide by eight.
	250.	Learner: No, Miss.
	251.	Teacher: Do you get a remainder?
	252.	Learners: Yes.
	253.	Teacher: So is it a factor?
	254.	Learner: No.
	255.	Teacher: Right, so we got, any, anything else? Any other number?
	256.	Learner: No.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
	257.	Teacher: Right, let's go to fifty-six. ... Factors of fifty-six. Again we start with?
	258.	Learners: One.
	259.	Teacher: [Writes on board.] One and fifty-six. Two and?
	260.	Learner: Twenty-eight.
	261.	Learner: Yup.
	262.	Learner: And seven and four.
	263.	Teacher: Will three go in there?
	264.	Learner: No, Miss.
	265.	Learner: Seven.
	266.	Teacher: Will four go in there? Four?
	267.	Learner: No, Miss.
	268.	Teacher: Four?
	269.	Learner: Yes, Miss.
	270.	Teacher: How many times?
	271.	Learner: Fourteen.
	272.	Learner: Point fourteen.
	273.	Learner: And seven, Miss.
	274.	Teacher: Seven.
	275.	Learner: Yes, Miss.
	276.	Learner: Times eight.
	277.	Another Learner: Eight times.
	278.	Teacher: Seven times what?
	279.	Learner: Eight.
	280.	Teacher: Anything else? ... You sure?
	281.	Learners: Yes, Miss.
	282.	Teacher: You tested nine?
15:00	283.	Learner: Yes, Miss. [15:00]
	284.	Teacher: Right, so we got ,, those are the factors. Now the highest one which is common; now let's look at all the common ones. There is a one
	285.	Learner: Sorry, man.
	286.	Teacher: Two is common.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
	287.	Learner: Miss, fourteen.
	288.	Another Learner: Fourteen.
	289.	Teacher: Seven is common. Fourteen is common.
	290.	Learners: Miss, fourteen, Miss.
	291.	Teacher: So the highest one which is common is...
	292.	Learners and Teacher: Fourteen.
	293.	Teacher: So my highest common factor ... is? [Writes on board all the time.]
	294.	Learner: Fourteen.
15:39	295.	Teacher: Now you see that the bigger the number becomes, the more factors you have ... so ... surely there must be a method where ... you're going to take less time ... and it's actually going to be easier. Because some of you are going to skip numbers. Some of you will have one, two, three and then maybe fourteen but you might not see the six and the seven. [16:00] So there must, the other method is the method that we use where we factorize a number ... into the product of it's prime factors. And let's just see what we get if we use that method on these two numbers. ... Right, so you've all marked this.
	296.	Learner: Yes, ma'am
	297.	Teacher: [Cleans board.] Let's take forty-two ... and over here ... let's make fifty-six. Right. Who can use this method, not the factor tree method,
	298.	Learner: Prime factors?
	299.	Teacher: Yes, remember we're only doing prime numbers, hey? ... [To learner at the board.] What did you do? The factor tree?
	300.	[Teacher checks the learner's work.]
16:57	301.	Teacher: Okay, you did that. That's a factor tree. [Learner sits down.] You can use that. There's nothing wrong with that. [17:00] Who, who used,
	302.	Learner: Twenty-one.
	303.	Learner: Twenty-one times.
	304.	Teacher: Twenty-one and then the next number that we try is?
	305.	Learner: Three.
	306.	Teacher: Three. ... And three goes into there?
	307.	Learner: Seven.
	308.	Another Learner: Fourteen times.
	309.	Third Learner: [Inaudible.]
	310.	Teacher: And then we say? Seven ... goes there?
	311.	Learners: Once.
	312.	Teacher: So the prime factors of ... that make up forty-two are?
	313.	Learner: Two, three.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
17:58	314.	Another Learner: Seven.
	315.	Teacher: Two times three times seven. ... Right, let's take the next number, fifty-six. Now, who can use this method to do fifty-six? ... Come, we
	316.	Learners: Twenty-eight. [18:00]
	317.	Another Learner: Lowest is twelve.
	318.	Learner: Fourteen.
	319.	Teacher: And then two goes into twenty-eight?
	320.	Learners: Fourteen.
	321.	Teacher: And then two goes into fourteen?
	322.	Learners: Seven.
	323.	Teacher: And then seven goes there?
	324.	Learners: Once.
	325.	Teacher: Right, so the prime factors of fifty-six .. are two times
	326.	Learners: Two.
	327.	Teacher: Times.
	328.	Learners: Two,
	329.	Teacher: Times seven... Now, what is common?
	330.	Learners: Seven.
	331.	Teacher: Yes, there's a seven here and there's a seven there.
	332.	Learners: A two.
	333.	Another Learner: Two.
	334.	Third Learner: And up.
	335.	Teacher: And what else do you see in both?
	336.	Learner: Two.
	337.	Learner: Four.
	338.	Teacher: A?
	339.	Learner: Four.
	340.	Teacher: A two.
	341.	Learner: And a one.
	342.	Learner: One.
	343.	Learner: There's still one left.

S P1 L2 EE3: Highest common factor of two numbers

Time	#	Speech
18:58	344.	Teacher: Now was there a one there?
	345.	Learner: No, Miss.
	346.	Teacher: Why not one here? Because what ... what kind of factors are we using here?
	347.	Learner: Prime.
	348.	Teacher: Prime factors and is one a prime number?
	349.	Learner: No.
	350.	Another Learner: No, Miss.
	351.	Teacher: Right, so what is common? [19:00] A?
	352.	Learner and Teacher: Two.
	353.	Teacher: And a?
	354.	Learner: and Teacher: Seven.
	355.	Teacher: And so what is two times seven?
	356.	Learner: Fourteen.
19:54	357.	Teacher: And so the highest common factor is fourteen. Instead of listing your multiples like this ... if we factorize it like this ... and we write
	358.	Learner: Seven and three.
	359.	Teacher: And so, what did you still get? Two times seven times three. You see, it's still that? [20:00]
	360.	Learner: Two to the power of.
	361.	Teacher: Two to the?
	362.	Learners: Power of three.
	363.	Teacher: Then, only two to the power of one would have been common, hey? So no matter what method you use, we would get the same thing. So
	364.	Learner: Yoo! [shouting]

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
20:29	365.	Teacher: Who did the H.C.F. of ninety-six and one oh eight? Who did the highest common factor of ninety-six and one oh eight. ... Right, let's do it. Let's
	366.	Learner: Little
	367.	Teacher: Little. ... [Goes to learner.] Mister, what's going on? You sick, hmm? ... I don't know what?
	368.	Learner: It's my head, it's paining.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
	369.	Another Learner: He's tired.
	370.	Teacher: You got a headache. ... Right people, what number do we start with?
	371.	Learner: Two.
	372.	Teacher: Start with? Two.
	373.	Learner: Ninety-six.
	374.	Learners: [Inaudible.] [Teacher walks around the room, checking work.]
21:57	375.	Teacher: And I'm going to do; remember in your group, [22:00] you must look to see if the other person's answer is the same as yours.
	376.	Learner: Yes, Miss.
	377.	Teacher: When you've done that method, you're going to do the multiples at the bottom of it using your calculator. ... And one oh eight ... and then we're going to check to see if you get to the same answer. So you know what we can do? Two of you do the top method and the other two do the bottom method and then you see if we get the same answers.
	378.	Learners: [Inaudible.] [Lots of activity from learners.]
22:48	379.	Teacher: Now the people with calculators, you try the bottom one it's easy to find the multiples on the calculator. ... [23:00] Why are you people not in a group? Why did you rearrange your tables?
	380.	Learner: Miss... [Inaudible.]
	381.	Learner: So Miss put it so.
	382.	Learner: Miss.
	383.	Learner: Wait.
	384.	Learner: Miss, the answer is six.
	385.	Teacher: The what? The page. The what?
	386.	Learner: The answer is six.
	387.	Teacher: The answer is?
	388.	Learner: Six.
	389.	Teacher: The answer is six. Okay. Have you got a method? ...
	390.	Learner: You lie. You got a calculator here now.
	391.	Teacher: It doesn't matter... [Rest of sentence inaudible.]
	392.	Learner: See, now I need my calculator. ...
	393.	Learner: Now... [Inaudible.]
	394.	Teacher: But now what about this, this ... [Inaudible.] So you must make the circle... [Inaudible.]
	395.	Learner: The one with the most factors?
	396.	Teacher: Hey?

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
23:51	397.	Learner: All of it?
	398.	Teacher: You see, this is common, right? This is also common. [24:00] ... Okay, let's see. ...
	399.	Learner: Miss, he's disturbing me.
	400.	Teacher: Okay, so what were you going to multiply now? Right, now the multiples, what are you going to do?
	401.	Learner: [Inaudible.]
25:00	402.	Teacher: Where's your paper. Let's see what... [Inaudible,]
	403.	Learner: [25:00] Miss?
	404.	Teacher: I'm coming now. [Learners noisy.]
	405.	Teacher: The multiples. Just write, okay first you must put it in the proper mode, hey? You must put it
	406.	Learner: Miss!
25:59	407.	Teacher: That's your first one then your next one.
	408.	Learner: Oh, the highest, you're gonna see now.
	409.	Learner: Miss Heneke.
	410.	Teacher: Yes? Right. [26:00] I'll be with you right now. Let's see what, are you winning? ... Is that all that's common? I see other things that's common.
	411.	Learner: [Inaudible.]
26:59	412.	Teacher: <u>Ja</u> {Yes}, you must do one there. Right, what are you doing? Same method?
	413.	Learner: Miss?
	414.	Teacher: So what have you got? Right, now ,, write down the prime factors here and here and show me what's common. Common means they appear in both. Circle. Every time a number appears in both, circle that number and then see what... [Inaudible. Learners noisy.]
	415.	Learner: Miss?
	416.	Teacher: Yes, so what have you got? [27:00] You've got two times two times [inaudible.] ... [Speaks to class.] Right, I see everybody's using, who's doing multiples on the calculator?
27:40	417.	Learner: No Miss, not me,
	418.	Teacher: Not multiples, sorry, factors, factors. ... Factors? Divide two into ninety-six ... then you get? ... Right. Then go on, so two is a factor and forty-eight is a factor.
	419.	Learner: So now divide it by two. You divide it by two.
	420.	Teacher: You get twelve. Okay. What did he get? You also got twelve? Okay, now factorize it, not by using prime factors. Use your calculator or just work out. Forty-eight is a factor. All these numbers here are factors, okay and [28:00] then see which one is common.
	421.	Learner: Miss, must you find the highest, Miss?
	422.	Teacher: Hey?

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
28:54	423.	Learner: The highest.
	424.	Teacher: The highest common one.
	425.	Learner: Miss, it's three.
	426.	Teacher: Hey
	427.	Learner: It's three.
	428.	Teacher: Is three, the highest common one? And aren't there twos that's common.
	429.	Learner: There is, Miss.
	430.	Teacher: So let's look at all that's common. [Goes around the room.]
	431.	Learner: Miss, it's twelve, Miss.
	432.	Teacher: Right, let's do it people. Right, who can come and do c on the board? Come. ... You come and do one oh eight, Yusri.
	433.	Learner: [Does the sum on the board.] ... [29:00] ...
	434.	Teacher: Come, you do the one oh eight.
	435.	Learners: [Inaudible.]
	436.	Learner: Ninety-six, forty-eight. [Inaudible.] [Yusri goes to sit down and another learner goes to the board.]
29:50	437.	Teacher: Right, so now ... okay that's fine. ... I hear people saying that they've got an answer. See if everybody in your group has got the same answer. Darren what did you get?
	438.	Learner: Zeroes.
	439.	Other Learners: [Inaudible. Class noisy.]
	440.	Teacher: It must be a ... [inaudible] number. It must be [inaudible].
	441.	[Learner at the board turns to the class for help.]... [30:00] ...
	442.	Learner: Nine, nine.
	443.	Another Learner: Not there! That's a three there!
	444.	Teacher: [Helps learner at the board.] You've got nine out of twenty-seven.
	445.	Learner: So it's three, Miss.
	446.	Teacher: So are you gonna choose two again? What's your next number?
	447.	Learner: He don't know.
	448.	Teacher: What's your next number?
	449.	Learners: Three. Three. Three.
	450.	Teacher: No.
	451.	Learner: Three.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
30:58	452.	Teacher: The numbers right down are all prime numbers, hey? ... [Teacher waits while learner completes the sum.] ...How do you get this? [With another group of learners.]... .. Show me. Okay. Two is common. What else is common?
	453.	Learner: There at the bottom.
	454.	Teacher: There's a two and there's also a two, so that's also common.
	455.	Learners: [Inaudible.] [31:00] Learner goes to work at the board.] Do the activity. [Class noisy].
	456.	Teacher: Okay, he's got shh, shh. Right people he's got two squared, three to the third. This one has got one, two, three, four, five. How do I write that?
	457.	Learner: Two and a five.
	458.	Teacher: Two to the? ... Five. ,, , Times three.
	459.	Learner: Miss?
32:00	460.	Teacher: That's fine. People, if you have not written it in this form, ... Akeeb, Raees turn around. ... Do all of you have two to the power of five times three?
	461.	Learner: [Inaudible]
	462.	Teacher: Who hasn't got this? [32:00] ...
	463.	Learner: [Inaudible]
	464.	Teacher: ... And then you got, for this one you've got, two squared and three times three to the power three.
	465.	Learner: [Inaudible.]
	466.	Teacher: Now if you didn't write it in this form, you could have written it as one, two, three, four, five. Right, these are the factors of ninety-six ... and these are the factors of ... one oh eight. That's two times two, times three times three, right. Let's look what's common. What does common mean? Common says it appears ?
	467.	Learner: Appears in both.
	468.	Second Learner: More than once.
	469.	Third Learner: Two and a three.
	470.	Fourth Learner: Something that appears the most.
32:56	471.	Fifth Learner: Something that appears more than once.
	472.	Teacher: Something that appears the most.
	473.	Learner: Eraser.
	474.	Teacher: So if we um ... if we're talking about [33:00] um ... something that we've got in common. Songs that we like in common, then it's songs that appears the most, that we both like. But in this case if we say, something is in common, something is common, then it means it appears in both. You will see it here. You will see it here and you will see it here. So there's a two. You agree that two is common? But there's another two that's common. ... Okay? ,, , So it's two times two. This two is common, it's there and there. This two is common as well. And then, there's a three as well. ... So what is common?
	475.	Learner: Three.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
33:48	476.	Another Learner: Two.
	477.	Teacher: Two multiplied with another two or two squared. There's two times and two times two which is four times three which is twelve, so my highest [34:00] common factor is twelve.
	478.	Learner: I told you so.
	479.	Learner: Yo! {Wow!}
	480.	Teacher: Now if we use the other method, what'll we have? Right, what are the factors of ninety-six?
	481.	Learner: It's one.
	482.	Teacher: One.
	483.	Learner: Two, four, six.
	484.	Teacher: Two.
	485.	Learner: Three, four.
	486.	Teacher: Three.
	487.	Learner: Four.
	488.	Teacher: Four.
	489.	Learner: Six, eight, twelve, sixteen, twenty-four, thirty-two, forty-eight.
	490.	Teacher: [Writes on board.] This is if we use the other method, hey? And then what have you got for one oh eight?
	491.	Learner: One, two, three, four.
	492.	Teacher: Four?
	493.	Learner: Yes Miss, four.
	494.	Learner: Six, nine.
	495.	Teacher: Nine.
34:59	496.	Learner: Twelve, eighteen, twenty-seven, thirty-six, fifty-four, one oh eight.
	497.	Teacher: I'm working with one person. [35:00] And the rest? ... Do you all understand how to get the factors?
	498.	Learner: Yes, Miss.
	499.	Teacher: Do you see how long this method is? And you could easily skip one or two numbers and if we look at this then one is common, two is common, three is common. four, six but the highest one which is common is?
	500.	Learner: Twelve.
	501.	Another Learner: Twelve.
	502.	Teacher: So whether you use this method or whether you use the bottom method, you get to the same answer which is? Twelve. Right, let's go to the next sum. Now ... the highest common factor of fifteen and thirty-six.
	503.	Learner: No Miss, we didn't do that.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
35:50	504.	Teacher: We're doing it now, d, fifteen, thirty-six. Right, we're going to use this method, it's much easier. ... Easier, simpler, quicker. [36:00]
		Factors of fifteen.... Prime.
	505.	Teacher: Three.
	506.	Learner: Three.
	507.	Teacher: Three goes into fifteen?
	508.	Learner: Five.
	509.	Another Learner: Five times.
	510.	Teacher: And five goes there?
	511.	Learner: Once.
	512.	T: Once. So the factors of fifteen is?
	513.	Learner: Divided by.
	514.	Another Learner: Three.
	515.	Teacher: Times.
	516.	Learner: Five, Miss.
	517.	Teacher: Are you paying attention? ... Right, tell me what to do with thirty-six. Two into thirty-six?
	518.	Learner: Eighteen.
	519.	Teacher: Okay, I want him to do it.
	520.	Learners: [Laughter.]
	521.	Teacher: Right, shh. Leave him, leave him. Sit with it. He must learn. ... Come.
36:59	522.	Learner: Miss, he did make himself [37:00] ... [Inaudible]
	523.	Teacher: NO, let him. Right, now you got eighteen. Can two go into eighteen. [Writes on board.]
	524.	Learner: Yes, Miss.
	525.	Teacher: How many times?
	526.	Learners: Nine.
	527.	Learner: Nine.
	528.	Teacher: Now we go, are we gonna try two again?
	529.	Learner: No.
	530.	Teacher: So we go to our next prime number which will go there.
	531.	Learner: Three times.
	532.	Teacher: Andikaya?
	533.	Learner: Yes, Miss.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
	534.	Teacher: Do you agree the next number we're going to choose is three?
	535.	Learner: Yes, Miss.
	536.	Teacher: So three goes into nine?
	537.	Learner: Three times.
	538.	Teacher: And then three goes there?
	539.	Learner: Once.
	540.	Teacher: Right, so what are the factors of thirty-six?
	541.	Learner: Two times two times three to the power of two.
	542.	Teacher: Two times two.
	543.	Learner: Two times three.
	544.	Teacher: Two to the power two times three to the power of...
	545.	Learner and Teacher: Two.
	546.	Teacher: Let's see what's common. What's common?
	547.	Learner: Three.
	548.	Teacher: What's common?
	549.	Learners: Three.
37:59	550.	Teacher: So what's my answer? [38:00]
	551.	Learner: Three.
	552.	Teacher: ... People, why am I now saying three times three?
	553.	Learner: Ur...yes, Miss.
	554.	Teacher: Aswan? Three, why? No, you tell me why. Why is the answer three and not three times three? ... These are the factors of fifteen, these are the factors of thirty-six. Why is my answer three?
	555.	Learner: Cause the three is only appears once.
	556.	Teacher: ... Because three only appears once in both of them. There's a three here and there's a three here which means it's common and remember we're looking for a common one. It must be a factor. It must be common and it's highest one. Right, so the highest common factor is?
	557.	Learner: Three.
38:48	558.	Teacher: Three. ... Three e. [Writes on board.] ... Right ... look for the ... find the highest common factor of twenty-two, fifty-five, eighty-eight. [39:00]
	559.	Learners: [Inaudible.]
	560.	Learner: Twenty-two, Miss.
	561.	Learner: Twenty-two.
	562.	Learner: It's eleven, Miss.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
	563.	Teacher: There's two people, three people saying one thing, somebody else said, ... who said, who's saying something else?
	564.	Learner: It's eleven.
	565.	Another Learner: Eleven.
	566.	Teacher: Eleven.
	567.	Learners: Eleven.
	568.	Teacher: And then, look at f. Shh. You're still gonna have to show me how you got to the answer, hey? Look at f. What you think it's gonna be?
	569.	Learner: Six.
	570.	Teacher: Six? You sure?
	571.	Learner: Yes.
	572.	Learner: No.
	573.	Learner: Yes, Miss.
	574.	Learner: Two.
	575.	Teacher: Right, you do e and f quickly. ... Right, who's done with e and f?
	576.	Learner: I'm already done.
	577.	Second Learner: Okay.
	578.	Third Learner: What?
	579.	Fourth Learner: I didn't see your work.
	580.	Learners: [Inaudible.] [40:00]
40:06	581.	Teacher:Right, where's e?
	582.	Learner: [Inaudible], Miss.
	583.	Teacher: No, there. That's your worksheet. So now you must, in your maths book, hey, write down today's date.
	584.	Learners: [Activity – Inaudible.]
	585.	Learner: Eleventh of April.
40:47	586.	Teacher: Right ... remember, only when you're done, put up your hands. Let me mark your work of yesterday. ... [Goes around the class, marking work.]
	587.	Learners: [Activity – Inaudible.]
	588.	Learner: Miss?
	589.	Teacher: Hey?
41:23	590.	Bell rings.
	591.	Teacher: Right, listen to me. Remember the story, what we do in class.
	592.	Learners: Homework.
	593.	Teacher: Finish your homework. Now you've only got e and f, right, of yesterday's work. ... Right, we'll start this ... Monday.

S P1 L2 EE4: Preferencing the ladder method for finding highest common factor between two numbers

Time	#	Speech
41:56	594.	Learner: [Inaudible.]
	595.	Teacher: No, English. Remember we've already used our English period.
	596.	Learner: Miss, tell him. Maths, Miss.
	597.	Learners: ... [42:00] [Inaudible.]
	598.	Teacher: Reading. Okay guys, e and f for homework. If you've done e and f, it means you don't have homework.
	599.	Learners: [Noisy – inaudible.]
42:41	600.	Teacher: Right, anybody else finished with e and f?
	601.	Learners: [Noisy – inaudible.]
	602.	End of lesson.

SCHOOL P1**LESSON 3 EE1.1 – EE2**

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
00:00	1.	Teacher: Right, now today we're going to do some problem solve hey.
	2.	Learners: [Very noisy.]
	3.	Teacher: [Closes the door.]
	4.	Learner: Miss, it's hot.
	5.	Teacher: I know it's hot; I just want the noise out.
	6.	Teacher: [Inaudible. In conversation with someone at door].
	7.	Teacher: Right, we just, I just want the um classes to settle and once they settle then I can open the door again. I know it's very hot. ... Right, I need you to sit in your groups. I see people sitting, ... that's a group ... but your tables are supposed to be rearranged the way, it's supposed to be like this for the maths session. Come. ... And these people here as well.
01:00	8.	Learners: [Inaudible]
	9.	Teacher: [Opens door.] Anyway, it doesn't matter, to the front, as long as you can face ... everybody can face me. ... Right. ... I need to just quickly recap. Where's the duster? ... Right. In the group, when last we met, we said, I want ... the only person who is going to be lying like that is Munier because he hurt himself. But the rest of you, and I know it's difficult, because it's so hot, but you need to listen and concentrate, hey?
	10.	
	11.	
02:00	12.	Learner: Miss, where did Munier hurt himself?
	13.	Another Learner: His arm.
	14.	Teacher: His arm, his hand.
	15.	Learner: It's swollen.

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
	16.	Teacher: The two of you. ... Must... are you part of that group?
	17.	Learner: Miss, he's dying. [Inaudible] ... Knocked down by Lithle. [Inaudible].
	18.	Teacher: Right. ...
	19.	Learners: Inaudible. Move tables around.]
	20.	Teacher: Okay, right, this is the worksheet. ... Right? Now which of these sums must still be marked?
	21.	Learners: Miss. [Inaudible] Eighteen, nineteen and twenty.
	22.	Teacher: Did we mark everything?
	23.	Learner: No, Miss.
	24.	Learner: Yes
02:56	25.	Learner: No, Miss.
	26.	Teacher: Right. Now in your group, I'm going to give you a problem to solve. In fact there are [03:00] two problems on the page but we're going to do them one at a time. ... And I'm going to let you solve them practically, in other words, I'm going to give you the strips of paper that they talk about. ... Now you must be in a group, hey? Groups of four otherwise there's not enough to go around. [Hands out the strips of paper.]
03:27	27.	Learner: Miss, [Inaudible]
	28.	Teacher: Right. ... One person in the group ... will...
	29.	Learner: [Inaudible] [Class very noisy].
	30.	Teacher: If somebody doesn't understand, right, the one person in the group will try to explain it to the others but let me first read it with you. And maybe you'll all understand what's going on. Right, look at number two. ... Right, number two. [04:00] ...
04:02	31.	Learner: I can't see.
	32.	Teacher: Right, now I know you can't see the page, hey, because it's only one amongst all of you.
	33.	Learner: Make your table right.
	34.	Teacher: Laeeq, what's going on?
	35.	Learner: Miss, he says he can't see but tell him make the table right.
	36.	Teacher: Turn your table around so you can see. ... Now remember you're going to have this page to refer to ... if you don't understand what I'm trying to tell you. You can read this over and over till you understand. Number two. David has three strips of colourful leather and they measure nine centimetres, [writes on board] twelve centimetres and eighteen centimetres. Right, so he's got three strips of colourful leather. The strips measure, one of them measure nine centimetres, another measure twelve and another measure eighteen. Now, he wants to cut them [05:00] into shorter, equal lengths to make a tassle to hang on his bicycle handle bars.
05:07	37.	Learner: Miss, what's a tassle?
	38.	Teacher: Right, who of you have seen a tassle?
	39.	Learner: [Puts up his hand.]
	40.	Teacher: What is a tassle?
	41.	Learners:[Inaudible.] The strips hanging out at the sides of the handle bars.
	42.	Teacher: Right, the strips hanging out at the sides of the handle bars and I have decide.. I've done this so that you, just in case you don't have the colour paper, right? So if we put a few together then it will make a tassle. But now you will have to cut these to the proper measurements that you see on your page, hey or on the board. Right, now he's got three strips but they're not of equal length. The ones that I am going to give you are of equal length. You will cut them into; one of nine, one a bit longer which is twelve and then the longest one which is eighteen. So you will have one of nine, one of [06:00] twelve and one of eighteen. ... Now, he wants to cut them, all of them, into shorter, equal lengths to make a tassle which will hang on his bicycle handle bars. What is the longest length that he can cut the strips into so that no pieces are left over?

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
06:25	43.	Learners: Twelve. Three. Nine. Nine.
	44.	Teacher: No, I, now we're starting to guess. Now, you're going to ... you got the problem in front of you. You're going to get the strips of paper. You're going to work out, whether it is by calculation or by folding, you're going to figure out into what the longest length can be for these strips to make the tassle but there must be no pieces left over.
	45.	Learner: [Raises his hand.]
	46.	Teacher: And remember, all of them must be the same length. Now if you've got an answer, don't shout it [07:00] out. You must convince your group that the answer that you got is the right one. Explain it to them. Show it to them, right? And then we will compare the results of each group. So every group will get these and together we must sit and figure it out. And people you must try to be involved in this problem, right? Contribute. See, and I know it's hot [opens door], ... but you must try to figure out something, okay? So somebody must take out a page maybe a calculator. You definitely need a ruler, a pair of scissors. If you don't have a scissors, fold, tear. Right, so let's see. ... Um, I'll give each group six and you can see if you can figure it out. [Hands out strips of paper.] ... [08:00] Remember the question is in front of you. If you're struggling, read it again and again. ... Three, right, somebody must have a ruler, take out a ruler, measure the pieces.
08:24	47.	Learners: [Inaudible.] [Noisy.]
	48.	Teacher: ... Right, while you're concentrating on that, I've given you a problem. [Inaudible] and concentrating on that ...
	49.	Learner: Yusri, where's your ruler?
	50.	Teacher: Come, sit up straight and you can see if you can solve this problem.
	51.	Learners: [Inaudible.]
09:02	52.	Learner: Now this is in centimetres
	53.	Teacher: Come, sit up, sit up [inaudible].
	54.	Learners: [Inaudible.]
	55.	Teacher: Hey? You already got an answer? Okay, now you explain to them. Right, now show them, show them with this, show them how it works.
	56.	Learners: [Inaudible.] Don't be rude.
	57.	Learner: You're supposed to make... [Inaudible]
	58.	Teacher: [Goes around checking learners' progress.]
	59.	Learner: Miss, how many strips must each person have, Miss?
	60.	Teacher: Hey?
	61.	Learner: [Inaudible.]
	62.	Teacher: Well, I've given two lots at a table, right? But if you want to do your own one ... have you got your own paper? Take out, it doesn't have to be green, it can be white also if you want to do your own. [10:00] Or let's see if there's more here. ...
10:04	63.	Learners: Miss ... [Inaudible]
	64.	Teacher: Now here's one for each one.
	65.	Learners: [Inaudible] half ... eighteen minus six [inaudible].
	66.	Teacher: No, only three. Three per person hey, so you must [inaudible] there.
	67.	Learners: Miss, Miss [Inaudible]
	68.	Teacher: You must speak English.
	69.	Learner: [Inaudible].
	70.	Teacher: But you must give him a hearing, so speak English and then he will understand.
	71.	Learners & Teacher: [Inaudible discussion.] [11:00] [Inaudible.]
11:06	72.	Learner: Miss?

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
	73.	Learners & Teacher: [Inaudible discussion.]
	74.	Learner: Miss I [inaudible].
	75.	Learners: [Inaudible.]
	76.	Teacher: You must explain to him and they must be in agreement with whatever you are saying. And show him there on the paper [inaudible].
	77.	Learners: [Inaudible].
	78.	Teacher: Little, you must [inaudible].
	79.	Learners: [Inaudible].
	80.	Teacher: [Inaudible] ... Little, cut them, one nine, one twelve, one eighteen. [Inaudible.]
	81.	Learners: [Inaudible]. ... [12:00] [Inaudible.]
12:01	82.	Learner: Miss. Miss. [Inaudible].
	83.	Teacher: So what is, what is, this is your twelve, right [inaudible]. You can cut another one if you want to. Cut it thinner if you want to. [Inaudible]. ... Right, can I lend one of these rulers? Take either one of these rulers to that group that doesn't have a ruler.
	84.	Learner: [Inaudible.]
	85.	Teacher: No, the length, the length.
	86.	Learner: [Inaudible.]
	87.	Teacher: One nine, one twelve and one eighteen. [13:00] ... Right, there we go.
13:05	88.	Learner: Thank you, Miss.
	89.	Teacher: [Walks around classroom, stops at one group.] ... One is nine, one is twelve and one is eighteen. eighteen there, now.
	90.	Learners: Is that the eighteen?
	91.	Teacher: What, where the[inaudible], measure that [inaudible].
	92.	Learners & Teacher: [Inaudible discussion].
14:00	93.	Learner: Miss.
	94.	Learner: It's nine.
	95.	Learner: Can you also like add a piece to one line?
	96.	Teacher: No, no remainder, what did they say? They say here ... no pieces must be left over. You can't add, there can't be any left. They must be worked out to exactly the same length. ... Right, now you've got your nine, you got your twelve and your eighteen line. You must think into how many pieces must this be divided so that you get the same size and nothing is left over.
	97.	Learners: Yes, shh
	98.	Learner: Guys, listen. ... Wait.
	99.	Learners: [Inaudible.]
14:59	100.	Teacher: Right, every group will [15:00] have one spokesperson who's going to explain to the class how you arrived at your answer. ... Where's, have you got your lines? Are you helping him?
	101.	Learners: [Inaudible.] [Class extremely noisy.]
	102.	Learners & Teacher: [Inaudible discussion].
	103.	Learner: It's nine.
	104.	Learner: Nought, comma five.
	105.	Learner: [Inaudible]. [16:00] [Inaudible.]
16:14	106.	Teacher: Listen to what he's explaining and then see if you agree and then show them on the paper. ... Right, let me see how far we are.

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
	107.	Learner: Miss.
	108.	Learners: [Inaudible.] [Class extremely noisy.]
	109.	Teacher: [Inaudible]. You got an answer? Then you explain it to them how you got your answer.
16:50	110.	Learners & Teacher: [Inaudible discussion].
	111.	Teacher: ..[Inaudible] [17:00] and see and if you perhaps have extra pieces left over if it will [inaudible] exactly. Exactly the same size [inaudible]. ... You also done? Hey? Explain how you get that .
17:14	112.	Learner: Miss, we're also done, Miss.
	113.	Teacher: [Inaudible.]
	114.	Learner: Miss, we're also done.
	115.	Teacher: Right, shh, I'm gonna be with you now.
	116.	Learners & Teacher: [Inaudible discussion].
	117.	Learner: Miss, they're spitting goeters [stuff] here.
	118.	Learners & Teacher: [Inaudible discussion].
	119.	Learners: [Inaudible.]
	120.	Teacher: Now show me your sum. Where's your eight centimetre piece? Show me your eight centimeter piece. [Inaudible]. [18:00] [Inaudible.] Okay. Cancel.
18:05	121.	Learners: [Inaudible].
	122.	Teacher: So what is this now? ... [Inaudible.]
	123.	Learner: Three.
	124.	Teacher: Three. ... [Inaudible] [19:00] nine, now twelve. [Inaudible.] If three is left and you put, where're you gonna put... there, there and there?
19:17	125.	Learners: [Inaudible.]
	126.	Teacher: Now, you see, he's dividing this up equally but what does the question say? Is he allowed to do that? Is he allowed to [inaudible]. Is this piece shorter than that piece? This piece here. Is this not shorter than [20:00] this? They say shorter pieces [inaudible] all the pieces or you must cut into shorter, equal lengths. In other words, you can't ... unless they're all gonna be like this. You can't have that and that and that. What he's done is, he's divided it equally amongst the three hey, so you're, the method of dividing equally is fine but you can't apply this method to this question. This question is this, all your pieces must be the same length. ... So,
20:39	127.	Learner: You see the point is, Miss [inaudible].
	128.	Learner: Miss? Miss.
	129.	Teacher: It must all be the same length. It must all be the same length, [21:00] so. So, you see, so you are saying all of them [inaudible] but now [inaudible] is there, try it now. If you didn't cut this up into one, there [inaudible]. You can't shoot him down, he came up with that idea. Now you explain why your idea might be a better idea because he's got the right idea but he shouldn't have cut that one up. ... Right. Shh. Right guys. It sounds like every group has got an answer.
	130.	Learner: Yes, Miss.
	131.	Another Learner: Yes, Miss.
	132.	Teacher: Right, people ...
22:00	133.	Learner: Miss.
	134.	Teacher: Right, group number nine. I'm going to be with you in a minute but listen the rest of you. Shh. ... Just hold on. About five groups have got the correct answer. Now if you have done, if you've done the sum, you must convince everybody at your table that, that is the correct answer. Then I want you to look at number three. I'm going to come and work with those who're still trying to do number two. Right, listen to number three. Shh. Aswan, does your group agree with your answer?
	135.	Learner: Yes, Miss.

S P1 L3 EE1.1: Practical problem solving: Making a bicycle tassle

Time	#	Speech
	136.	Teacher: Right and then, um ...
	137.	Learner: Finally.
	138.	Another Learner: Miss, he [inaudible].
	139.	Teacher: Kyle, your group has got an answer?
	140.	Learner: Yes.
	141.	Teacher: Right and then Rudy's, um ... Yusri's group has got an answer. This group has got, in fact most of the groups have an answer and most of the answers are correct. Shh. [23:00] Right. Listen to the second, the third problem on the page. ... A train from Soweto arrives at Park Station, shh. Guys ...
23:11	142.	Learner: I have an answer.
	143.	Teacher: Okay, now if you have an answer then you must discuss it with them. Don't shout out, hey?
	144.	Learner: Listen to [inaudible].
	145.	Teacher: Right. This is for all of you. I'm gonna go back to, who's still. Let me just quickly see, who's still busy trying to figure out number two's problem? ... Just that two groups?
23:50	146.	Learner: Put your hand up.

S P1 L3 EE1.2: 'Using mathematical knowledge' – LCM and HCF to solve problem

Time	#	Speech
	147.	Teacher: Now I'm gonna work with you now. ... Right, listen to the third ... [closes the door]. Right. ... Number three. A train from Soweto. ... Guys, I'm gonna keep you in after school, hey.
	148.	Learners: Shh, shh.
	149.	Teacher: If you're not gonna concentrate now then we stop, and then when the bell rings, we continue. ... A train from Soweto arrives at [24:00] Park Station every six minutes. A train from Springs arrives at Park Station every eight minutes. So the two trains have arrived at the same time, just now. How long will it be before this is going to happen again?
24:18	150.	Learner: Huh?
	151.	Teacher: Now. Read that question over and over and somebody in the group will explain or read it to you and together you will try to figure it out while I'm trying to see what those two groups are struggling with and then we will discuss this further. Okay? ... Right. Let's start here. Put it [inaudible]. This is your [inaudible] units. What did you figure out so far?
	152.	Learner: [Inaudible]. [25:00] [Inaudible].
25:03	153.	Teacher: [Inaudible] one, nine, twelve. They say into how many pieces can you cut equal pieces? Can it be cut [inaudible].
	154.	Learner: Oh, now I see.
	155.	Teacher: But I gave you a lot so that you can try it out and you can figure it out and you mustn't [inaudible] and it's bound to come out. So, this piece [inaudible]. What do you think into how many pieces can each of them be cut ... so that they're all the same length. ... What length will you choose that that [inaudible].
	156.	Learner: Nine, I'll choose nine, Miss.
	157.	Teacher: Hey? You will choose?
	158.	Learner: Nine.
	159.	Teacher: But if you choose nine. Look at this one, if you cut it into equal pieces, each piece [inaudible] if you choose nine and a half with nothing left [26:00] over?
26:01	160.	Learner: [Inaudible].
	161.	Teacher: You must put this on here. You mustn't have a remainder. It says here, you must cut them shorter into equal lengths to make a tassel. What is the longest length so that no pieces are left over?
	162.	Learner: Do they say you can add on to this?
	163.	Teacher: You must, you must divide this [inaudible].
	164.	Learner: [Inaudible.]
	165.	Teacher: Right, show them how or explain to them how. [Moves to next group.] Right, you did your three strips. Okay, nine divide by [inaudible]. I must divide each of this into the same amount, into equal lengths. ... It must be a number that will be able to go into all three [27:00] because the pieces must be of equal lengths, there must be no divide, no remainder.
27:06	166.	Learner: Miss, will it be three?
	167.	Teacher: Will it be three? See if it will work to make a tassel. If, if you got a piece of nine, can you divide this [inaudible].
	168.	Learner: Yes, Miss.
	169.	Teacher: Can you? There will be no divider left? Can you do the same with these strips?
	170.	Learner: Yes, Miss

S P1 L3 EE1.2: 'Using mathematical knowledge' – LCM and HCF to solve problem

Time	#	Speech
	171.	Teacher: And can you do the same with this [inaudible]?
	172.	Learner: Yes
	173.	Teacher: [Inaudible]. ... What do you think? Okay, right. [28:00] ... Right guys. ... Okay. Shh. Thank you, Alex. Right. ... We're going to ask Lyle, not Lyle, Kyle. ... Kyle, come to the board ... and now I want you to listen, shh ... and then I want you to hear if you agree with him. Right, explain to us what you do.
28:35	174.	Learner: This one here?
	175.	Teacher: Yes
	176.	Learners: Um, our group, um, we looked at which numbers can divide into each, um, each strip without leaving a remainder and then we got three because three can go into nine centimetres, twelve, um, can go into nine, twelve and eighteen and it left um no pieces and it left no pieces over.
	177.	Teacher: Okay. So his tassel, he's saying his tassel he, [29:00] they divided into pieces of three centimetres in theirs. Each. And each group got that answer.
29:12	178.	Learners: Yes, Miss.
	179.	Teacher: Right, now if you didn't have the paper, what, what mathematical um knowledge could you use to solve that problem?
	180.	Learner: Division
	181.	Teacher: Division of what?
	182.	Learners: Three
	183.	Learners: Of, of, of eighteen. Eighteen.
	184.	Teacher: Eighteen.
	185.	Learner: Plus twelve.
	186.	Learners: Nine divided by three, Miss
	187.	Teacher: Nine divided by three ... would give you how many pieces?
	188.	Learner: Three.
	189.	Teacher: Three pieces, right.
	190.	Learners: [Inaudible].
	191.	Learner: Or you could have added all that up and divided it by three.
	192.	Learners: Three.
	193.	Teacher: And that would be three pieces.
	194.	Learner: Divided by thirteen
	195.	Teacher: And there would be? Four and there would be? Six. But what, what else could you use to find that answer of three?
	196.	Learners: Miss, you add it all up.
	197.	Teacher: You add it all up.
	198.	Learner: Add it all up and divide it by thirteen.
30:00	199.	Teacher: By thirteen?
	200.	Learner: Huh?
	201.	Learner: By three.
	202.	Learner: Divide by thirteen, you get three, Miss
	203.	Teacher: Okay, let's see. That's what? Twenty-one and thirty-nine divide by ...
	204.	Learner: Three.

S P1 L3 EE1.2: 'Using mathematical knowledge' – LCM and HCF to solve problem

Time	#	Speech
	205.	Teacher: Thirteen. Why did we divide by thirteen?
	206.	Learner: Then you get ... then you get [inaudible].
	207.	Teacher: But why did you choose thirteen to divide it by?
	208.	Learner: Because all together you get thirteen pieces, Miss
	209.	Teacher: But how, how did you know you must get thirteen pieces?
	210.	Learner: Miss, because we [inaudible].
	211.	Learners: Miss. Miss. Miss.
	212.	Teacher: Because you divide it by three. That's why you got to thirteen. Okay, so you divide. What can we use, what have we been doing lately people, we've been?
	213.	Learner: The L.C.M., Miss
	214.	Teacher: Finding the L.C.M. and the H.C.M.
	215.	Learner: Highest common factor.
	216.	Teacher: Highest common.
	217.	Learners and Teacher: Factor.
	218.	Teacher: Right. [Writes on board.]... [31:00] Right, people, remember there are two methods. The one where we're going to test the factors ... and then we're gonna find the common one and then there's the other one where we use the prime factors and we see what's common. So let's first do the one where we list. All the different factors. ... All the different factors of nine. It would be?
	219.	Learners: One, three, five nine. Seven, nine. Seven.
	220.	Teacher: Right, the factors of twelve.
	221.	Learners: One, two, three, four, six, twelve. One three.
	222.	Teacher: And the factors of eighteen.
	223.	Learners: One, two, three, nine, six, nine, eighteen.
	224.	Learner: Twenty-two.
32:00	225.	Teacher: Right, besides the number one. The highest common factor is?
	226.	Learners: Three.
	227.	Learner: Six. Six.
	228.	Teacher: Okay, now there's a six here and a six here but there's not a six there so six is not common.
	229.	Learner: Waa
	230.	Teacher: And then, there's a nine here and a nine but not there, so therefore nine is not common. So the only one is?
	231.	Learners & Teacher: Three.
	232.	Teacher: Right, let's take the other method. ... The, let's take nine ... and we're using prime factors now so will we choose number two?
	233.	Learners: No.
	234.	Teacher: No. We go to the next one which is?
	235.	Learners: Three.
	236.	Teacher: And we say three goes into nine?
	237.	Learners: Three.
	238.	Learners: Three times.
	239.	Teacher: And three goes there?
	240.	Learners: Once.

S P1 L3 EE1.2: 'Using mathematical knowledge' – LCM and HCF to solve problem

Time	#	Speech
	241.	Teacher: Right, let's go, twelve. We will start with number?
	242.	Learners: Two.
	243.	Teacher: Two. Darren, are you watching? ... Right. Two, Darren, two goes into twelve?
	244.	Learner: [Inaudible].
	245.	Teacher: Shh.
	246.	Learner: Six.
	247.	Teacher: Six. Then, which prime number will we take?
	248.	Learner: Three.
	249.	Another Learner: Two.
	250.	Teacher: Two. Two goes into six?
	251.	Learners: Three.
33:00	252.	Teacher: Three. And then we will take?
	253.	Learners: Three.
	254.	Teacher: Three. Goes there?
	255.	Learner: One.
	256.	Learner: Once.
	257.	Teacher: And then for eighteen?
	258.	Learners: Two.
	259.	Teacher: Right, let me first see. ... Right, Abdul.
	260.	Learner: Nine.
	261.	Learner: Two.
	262.	Learners: Nine.
	263.	Teacher: Shh
	264.	Learner: Eighteen goes.
	265.	Teacher: I hear people say nine. Why are they not choosing nine?
	266.	Learners: Because it's the highest. The highest. Lowest one [Inaudible].
	267.	Teacher: Shh. Right. He's saying we're looking for, are we looking for the lowest, common multiple?
	268.	Learner: Yes, Miss.
	269.	Teacher: We're looking for the?
	270.	Learners: Highest common.
	271.	Learners and Teacher: Highest common factor.
	272.	Teacher: But what kind of factors are we using?
	273.	Learners: Prime.
	274.	Teacher: Prime and is nine a prime factor?
	275.	Learner: Yes.
	276.	Teacher: Who says yes? Rudy? Rudy explain why. Why it's prime. ... Or, or tell us if it's not, say why. Who agrees that nine is a prime number? ... Is nine not a prime number?
	277.	Learner: It is, Miss.
34:00	278.	Teacher: Why is it?

S P1 L3 EE1.2: 'Using mathematical knowledge' – LCM and HCF to solve problem

Time	#	Speech
	279.	Learner: Miss, from our notes, Miss.
	280.	Learners: [Laughter.]
	281.	Teacher: Okay, I don't know who you got your notes from. ... Why is nine not a prime number?
	282.	Learner: There's more than one factor, more than two factors.
	283.	Another Learner: Three and nine.
	284.	Teacher: Right, Alex don't shout out because one can divide into it and ...
	285.	Learner: [Inaudible].
	286.	Teacher: Because the factors of nine are one, three and nine. A prime number has only, how many factors?
	287.	Learner: Two.
	288.	Another Learner: Two.
	289.	Teacher: Okay. So let's go, into eighteen?
	290.	Learners: Two.
	291.	Learner: Three.
	292.	Another learner: No, Miss.
	293.	Teacher: Two goes in?
	294.	Learners: Nine
	295.	Teacher: Three goes in?
	296.	Learners: Three times.
	297.	Teacher: Now, shh. ... Now we're gonna write down our factors of nine. ... three times three. Of twelve ...
	298.	Learner: Two times two times three.
	299.	Teacher: Two times two times three. And of eighteen?
	300.	Learners: Two times three times three.
35:00	301.	Teacher: And then we can see that there's a three there, here's a three here, there's a three here so the highest common factor is?
	302.	Learners: Three.
	303.	Teacher: Okay? So we could have done that sum mathematically without any strips. Right, quickly. Number three. Shh. ... Have you got an answer? Right, don't shout it out and you group agrees with your answer?
	304.	Learner: No.
	305.	Another Learner: No, we don't see it ma'am.
	306.	Teacher: Right, shh. Okay.
	307.	Learner: I see it.
	308.	Teacher: Right, people.

S P1 L3 EE2: Practical problem solving: Train arrival times

309. Learner: Don't know.
310. Teacher: Number three. I want you all to look at your page ... or listen. A train from Soweto arrives at Park Station every six minutes. ... The train from Soweto arrives every six minutes. ... The train from Springs arrived every eight minutes. Right, so there's two trains [36:00] arriving at Park Station; one from Soweto, one from Springs. The one from Soweto comes every six minutes, the one from Springs every eight minutes. Now the trains from Soweto and Springs have just arrived at the same time, right now, they've both come to the same station. How long will it be before this is going to happen again when they both come?
311. Learner: Twenty-four minutes, Miss.
312. Teacher: Twenty-four minutes. How'd you get your answer?
313. Learner: Yes, Miss.
314. Another Learner: Right.
315. Teacher: You say after twenty-four minutes and they'll be there again. Why?
316. Learner: It's the lowest common multiple.
317. Teacher: Right, because now we have found the lowest common multiple. Okay. So in your maths notebooks, let's write, let's write down these sums. [Cleans board.] ... Your heading, highest common factor, lowest common multiple.
318. Learner: Miss, must you keep the strips in your book?
319. Teacher: What?
- 37:00 320. Learner: Must you keep the strips in your book?
321. Teacher: The strips?
322. Learners: Yes.
323. Teacher: No, not necessary. Right, shh. Right guys, listen to me. You now need to share the page because I want you to write down the problem. Remember this page I use in other classes as well, hey? So you write down the problem. Number two. ... It is the ninth. Right, number two. [Writes on board.] ...
324. Learner: [Approaches teacher to ask a question.]
325. Teacher: This front group. Where's the ruler that Kyle loaned you?
326. Learner: Here's it, Miss but that boy, Miss.
327. Learner & Teacher: [Inaudible discussion].
328. Learner: Here's it, Miss.
329. Learner & Teacher: [Inaudible discussion].
- 38:00 330. Teacher: Right, shh. ... I want to see all of you working now, hey? ... Right. Now if you are sitting so that you can't see the page, I'm writing this question up on the board. [Writes on board.] ...
331. Learner: [Inaudible.]
332. Teacher: [Writes.] ... [39:00] ... Either write it down from the page or you write it from the board.
- 39:30 333. Learner: Must we write it down, ma'am?
334. Learners: [Inaudible.]
335. Teacher: [Writes again.] ... [40:00] ... Right, that's your question. ... [41:00] ... [Inaudible.] Right, but now you must write down the question, hey because [inaudible]. Right people and then, you must fill in your answer, hey? All the answers that we've worked out. ...
- 41:38 336. Learner: Miss, [inaudible].
337. Teacher: The what?
338. Learner: [Inaudible].
339. Teacher: Yes. Number three. [Writes on board.] ...
- 42:00 340. New Learner: [Enters room.] Miss, ... [inaudible]...asks if you have... [inaudible].

S P1 L3 EE2: Practical problem solving: Train arrival times

341. Learners: Go away. We don't have.
 342. New Learner: [Leaves.]
 343. Teacher: [Writes on board.] ... [A racket outside. Teacher leaves the room.]
 344. Learners: [Inaudible]. ... Shh, Shh. [Inaudible]. ... [43:00] Shhh. Shhh. [Inaudible]. ... Shh. [Inaudible]. ... Shh. [Inaudible]. ...
 44:07 345. Teacher: [Returns and continues to write on board.] ... [44:00] ... [Closes the door.]
 346. Learner: Miss is that... [Inaudible], two and ... [Inaudible]
 347. Learner: Miss.
 348. Learners: It's hot.
 349. Teacher: I know but we, there's too much noise outside. Um a ... [inaudible] of. And now what is the longest length he can cut the strips into so that no pieces [inaudible]
 350. Learner: [Interrupts by approaching teacher.] [Inaudible.]
 351. Teacher: Yes.
 352. Learner: It's hot in here, Miss.
 353. Teacher: Right, I'm gonna open up in soon, as soon as they've settled. [Writes on board.] ... [45:00] ... Right, you, if you are struggling, if you can't see the board please, there's a page on your table. ... [Writes on board.] ...
 45:44 354. Learner: Miss? [Inaudible.]
 355. Teacher: What?
 356. Learner: [Points out a mistake on the board.]
 357. Teacher: [Corrects error.] ... [46:00] Is that what you mean? ... And I would have thought you weren't hungry. ... Why did they eat?
 46:22 358. Learner: I didn't eat [inaudible].
 359. Another Learner: Miss?
 360. Learner: [Inaudible.]
 361. A Learner: [Approaches the teacher.] [Inaudible.]
 362. Teacher: Yes. Right, your two questions are there. Please put down your answers, hey? [Walks around the room.] ... Yes. Right, now do you know, where are your answers? ... Remember, I don't [47:00] just want to see three and twenty-four. I want to see how you got to that answer. So the question two, there's a method on, there's two methods on the board. Only choose one. ... And then explain how you got to your answer of question number three. ... Come, finish up.
 47:33 363. Learner: He was right, Raees.
 364. Second Learner: [Inaudible]
 365. Third Learner: [Raises his hand.] [Inaudible.]
 366. Teacher: Okay, but now you have to write the question there. Put the question here. ... [48:00] ... Right people, let me just remind you that for those of you who did not finish your summaries ...
 48:11 367. Learner: Summaries?
 368. Teacher: English. Summaries. ... I'm going to be checking it tomorrow.
 369. Learner: Miss.
 370. Teacher: Hey?
 371. Learner: [Inaudible.]
 372. Teacher: Your mommy. ... No, if you finish I want to sign your book. ... Ryan. I'm watching you. ... Let's see. ... [49:00] And then when you are done. ...
 ...
 Now. So where's your method? How'd you get to this? ... Right people, shh, if you did not use, Darren, are you listening? If you did not use this

S P1 L3 EE2: Practical problem solving: Train arrival times

- method but in words you said, cut the strip into pieces to measure three centimetres each etcetera, then you write that as your answer, right? If you didn't do it mathematically like this but you solved it using English ... then that's fine but then I want to see that answer. Don't just tell me three and twenty-four. I want to see how you arrived at your answer.
- 49:44 373. Learner: Word sum.
374. Teacher: It can be a word sum, it can be an answer which is in words. [Checks learners' work.] That's fine. ...Right?
375. Learner: Yes Miss.
376. Teacher: Now you must do the next one. ... [50:00] ... Right, but now just explain what's happening to the sums. ...
- 50:09 377. Learner: [Inaudible.]
378. Teacher: From what is this?
379. Learner: [Inaudible.]
380. Teacher: Let's see the website. [Inaudible.] But what is the problem. Do you know [inaudible] virus.
381. Learner: Miss? Miss?
382. Learners: [Inaudible. Very noisy.]
383. Teacher & Learners: [Inaudible discussion.]
- 51:00 384. Teacher: Right. ... You can start finishing up now.
385. Learner: Miss?
386. Teacher: Put up your hand if you're done.
387. Learners: [Inaudible].
388. Teacher: ... Remember that we're all suffering hey, you're not the only one that suffers.
389. Learner: Miss [inaudible].
390. Teacher: I can see you're right. ... Right people, if you're not using those strips then you must see that you dump them in the bin. All the papers on the floor, pick up.
391. Learner: The cleaners, Miss.
392. Teacher: Who are the cleaners today?
393. Learner: Like this, Miss.
394. Learner: Aaah.
395. Teacher: Right. ... Right, everybody put their own, you put your own papers in the bin.
396. Learner: Miss.
397. Teacher: Naeem will do the sweeping. ... The people who put up their hands. [52:00] ... Come, Raees where's your work?
398. Learner: [Inaudible.]
399. Learners: You must clean alone. You must clean alone.
400. Teacher: I, you can only pack away your book if I sign it.
401. Learner: Miss, here Miss
- 52:30 402. End of lesson.

SCHOOL P2

LESSON 1: EE1-EE2

S P2 G8 L1 EE1: Plotting co-ordinates

Time	#	Speech
1.		Learners arriving for a mathematics class.
2.		Teacher: Ok, take out your books. Before we start, I must introduce to you... Sorry! Sorry! Please listen! I must introduce to you Mr Roger, he is coming from UCT
3.		Learner: Yes
4.		Teacher: He will be with us just for this period, <u>nhe?</u> {right?}
5.		Learners: Yes
6.		Teacher: So behave yourself. Take out your books. <u>Khawulezisa, khawulezisa.</u> {Hurry up, hurry up}. Did you take out your books?
7.		Learners: Yes
8.		Teacher: Ok! Without wasting more time.... Take out your textbook, the green one. We going to use the green one. Don't fight, you must share the book.
9.		<u>Zamani ukubolekisana ngeencwadi.</u> {Try and lend each other the textbook.} <u>Asinazo ezinye</u> {we do not have more}. Turn to page two 0 three (203)
10.		Ok, We already did the co-ordinate <u>nhe?</u> {right?} You know how to read the ordered pairs in the Cartesian plane, <u>nhe?</u> {right?}
11.		Learner: Yes, what page?
12.		Teacher: Page two 0 three (203), Page two 0 three <u>nhe?</u> {right?}
13.		Learners: Yes.
14.		Teacher: Ok, there is no need of noise <u>ke bethuna</u> {people}. I want you to plot the points of ordinate, <u>siyavana?</u> {Do we understand each other?}
15.		Learners: Yes
16.		Teacher: Because we already know how to read, <u>nhe?</u> {right?} in the Cartesian plane. Now I want you to plot in the Cartesian plane and I've got the worksheet
17.		for you will give you in groups. One in a group. In a group, you must choose one leader, so do not fight <u>bethuna</u> {people} work together, <u>nhe?</u>
18.		We are not fighting. I said in each group, choose a scribe. Who is the scribe here? Be sure of that. Is this one? So what is the problem now?
19.		Please do not make a noise. And here. (pointing to the group) Who is the scribe? Who is the scribe here? One person. Who is the scribe here?
20.		Learners:
21.		Teacher: Is a scribe Now let us go there and see. (Reading from the worksheets) So you are going to plot the points and join the points <u>nhe?</u>
22.		{right?}
23.		Learners: Yes
24.		Teacher: By order, <u>nhe?</u> {right?} Use the ruler by joining the points according to the order <u>nhe?</u> {right?} and then in brackets the grid should be ten units
25.		across and five units up. But I have already done for you, <u>nhe?</u> {right?}. You will just only plot the points, <u>siyavana?</u> {do we understand each
26.		other?}. Ok! Let us do with out making the noise, <u>nhe?</u> {right?} Work together, it must not be the scribe <u>bethunana</u> {people}. Work together.

S P2 G8 L1 EE1: Plotting co-ordinates

Time	#	Speech
	27.	Learners: Yes ma'am
	28.	Teacher: Ok! Please be quiet <u>bethunana</u> {people}.
	29.	Teacher: Here is the point, point A has two and one, that means the values are two and one, and that two is for what? Which value? x and y <u>nhe?</u> {right?}. So this is x and y.
	30.	You must plot there, the values of x, the value of x is two and one, and then one, where is one? Where they meet in the point. Where?
	31.	Teacher: Ok! Are you finish? Did you join the points? Join, you must join the points <u>nhe?</u> {right?} Join the points in order.
	32.	Teacher: (explaining to another group) <u>Bhala</u> {write} point A , point B, and join. Please hurry up. I do not have time. Sorry class! Sorry! Please listen
	33.	<u>bethunana</u> {people}, we see that we've done a mistake, <u>nhe?</u> {right?}, of mixing x and y, don't mix x and y. You always start by x. <u>Ababantu bathethayo kengoku?</u> {Who is busy talking?}.
	34.	Don't mix up <u>bethunana</u> {people} x and y, you always start by x because it is an order pair, <u>nhe?</u> { right?}, so you must start by x always and
	35.	follow y <u>nhe?</u> {right?}
	36.	Learners: Yes
	37.	Teacher: y must follow as always, so in these points you have got... you have got A, A is two and y, <u>nhe?</u> {right?} So that two stands for, is the value of x,
	38.	<u>nhe?</u> {right?} and y is the value of y. <u>Siyavana bethunana?</u> {do we understand each other?}
	39.	Learners: Learners continue working on their worksheets.
	40.	Teacher: Did you finish that one? Ok! Three (b) is saying that, what is the picture? What is the picture?
	41.	Learners: Boat <div style="text-align: center;"> $\swarrow \leftrightarrow \leftrightarrow \leftrightarrow \searrow$ $\downarrow \rightarrow / \quad \quad \quad \leftrightarrow \leftrightarrow$ $\backslash \quad \quad \quad /$ $\leftrightarrow \leftarrow \leftrightarrow \leftarrow /$ </div>
	42.	Teacher: Boat or ship, <u>nhe?</u> {right?}
	43.	Learners: Yes
	44.	Teacher: Ok. I want you to remind you, in the co-ordinates you have x and y, it is in this order(x;y), you always start by x then y. So the first number is x which is two and the second number is y which is one, and what else? Most of you, you mix x and y. Any question so far?
	45.	Learners: Ask the teacher.

S P2 L1 EE2: Plotting co-ordinates (Homework)

Time	#	Speech
	46.	Teacher: Ok let us do this for the homework, <u>nhe?</u> (right?). Don't make a noise. I will write on the board the homework, <u>siyavana?</u> {do we understand each other?} <u>Homework</u> Plot and join the following co-ordinates from point A-H. A (1,2) B (7,2) C (7,4) D (6,4) E (6,5,5) F (6,5.5) G (3,4) H (1,4)

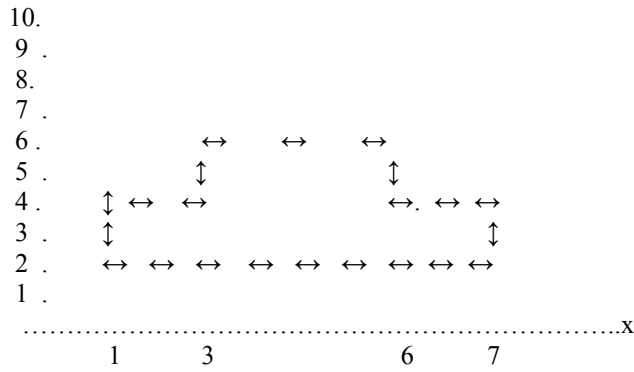
SCHOOL P2**LESSON 2 EE1**

S P2 L2 EE1: Plotting and joining co-ordinates

Time	#	Speech
	47.	Teacher: Can I see your books, all of you. Give the homework. <u>Khawuleza, khawuleza!</u> {Hurry up, hurry up!}. Where is your homework? { <u>Khawulesiza!</u> {Hurry up!}}.
	48.	Teacher: Do not change you homework, <u>nhe</u> . {right!} We are going to do the corrections, <u>nhe?</u> {right?} Don't worry, if you are wrong, <u>nhe?</u> {right?}
	49.	Learners: Yes Miss {teacher}
	50.	Teacher: We will do the corrections, don't rub anything, <u>siyavana?</u> {do we understand each other?} That boy, it seems as if he didn't do his homework now.
	51.	Are we together <u>bethunana?</u> {people?}
	52.	Learners: Yes
	53.	Teacher: Oh! I lost my..... <u>Akukho mntu one tshoko apha?</u> {Does anyone have a chalk here?}
	54.	Learner: No Miss
	55.	Teacher: I lost my chalk. {looking for it on her table}. <u>Iphi?</u> {where is it?}.
	56.	Learners: <u>Nantsi.</u> {Here it is.}

57. Teacher: Ok! Homework, i-corrections zayo nhe. {Doing corrections of the homework}.

Y



58. Teacher: Ok! Kuthwa apha {It says here} join the following co-ordinates from point A up to H, nhe? {right?}.

59. Learner: Yes Miss

60. Teacher: Ok. Point A is one and two. X is? x is what? What is the value of X?

61. Learner: One

62. Teacher: One nhe? {right?}, and for the value of Y?

63. Learners: Two

64. Teacher: Who can come to the board and plot these points? Point A, anyone must come and plot.

65. Learners: Going to the board to plot the points

66. Teacher: Ok! Is she right?

67. Learners: Yes

68. Teacher: Ok! It must be straight {going to the board to check if the learner plotted the points accurately}. Point B is seven and two. Seven for?....

69. Seven for what? Seven is the value of what?

70. Learners: X

71. Teacher: And two is the value for?

72. Learners: Y

73. Teacher: Who can come and plot on the board?

74. Teacher: Seven and two. Is she correct?

75. Learner: Yes

76. Teacher: Huh?

77. Learners: Yes

78. Teacher: Are you sure?
79. Learner: Yes
80. Teacher: Seven and two. {Pointing to the board} Ok! Sit down. The next point is point C, seven and four, seven and four... Ok! Where is the point?
81. Yhu! {Oh!}. I can't see. OK! Point C is seven....Here is seven nhe? {right?} {Pointing to the board}
82. Learner: Yes
83. Teacher: Mayicace nhe?! {let it be clear, ok!}. Must be clear. Ok! Point D...six and four....Yiza! {come!}.
84. Learner: {Learner going to the board to plot the points of six and four.}
85. Teacher: Six and four let us check. Any question? Everyone...Do you understand?
86. Learner: Yes
87. Teacher: All of you?
88. Learner: Yes
89. Teacher: OK! Let us come to point E, six and five and a half. Six and five and a half. Six and five and half.
90. Learner going to the board to plot the points of E.
91. Teacher: Ok! Masiyichecke {Let us check it}. Not accurate. We can't be accurate cause we don't have a ruler, nhe? {right?}
92. Learner: Yes
93. Teacher: But it will be accurate in your worksheets because it is a grid, it already has a grid that is accurate, they have squares nhe? {right} that are measured.
94. Let us come to point F, three and five and a half, three for X, five and a half for Y.
95. Teacher: Ok! Let us check this one. G is three and four, point G is three and four.
96. Learners: Learner going to the board to plot the points of G.
97. Teacher: H is one and four. Ngubani ongekazi? {Who has not come up to plot on the board?}... one and four.
98. Learners: Learner going to the board to plot the points.
99. Teacher: Ok! Let us check if it is correct. So it says in the worksheet, we need to join the points, nhe? {right?}
100. Learners: Yes Miss {teacher}
101. Teacher: So we need to do what now?
102. Learner: Join
103. Teacher: In order nhe? {right?} Ok! Can you join for me, for us.
104. Learner: Learner going to the board to join the points.
105. Teacher: Try to reach point B. {on the board}
106. Learners: Yes Miss {teacher}
107. Teacher: Is she correct?
108. Learners: Yes
109. Teacher: Can I get your books signed and checked. {Teacher going around checking the homework}. Who doesn't understand this?
110. Does anyone don't understand how to plot the points?

111. Learner: No Miss. Ukhona Miss. {There is one teacher}
112. Teacher: Ngophi? {Who's that}. Who don't know how to plot? (Teacher explaining to the learner who doesn't understand). Ok class! We finish doing the co-ordinates, nhe? {right?}. For tomorrow, for the lesson of tomorrow will do the transformation, nhe? {right?}.
113. Learner: Yes
114. Teacher: Now what I need you to come with, mirror siyavana? {do we understand each other?}, mirror...what else... ok! And also the ...something like, like a hardcover, or a box or cardboard, something that is hard, nhe? {right?}
115. Learners: Yes
116. Teacher: So, come with a mirror or a box siyavana? {do we understand each other?}
117. Learner: Yes
118. Teacher: Tomorrow we will do transformation. We finish with co-ordinates. That is all.

SCHOOL P2

LESSON 3 EE1-EE2

S P2 L3 EE1: Transformation geometry - translations

Time	#	Speech
1.	Teacher:	What do you know about transformation? What is the meaning of transformation? Can you tell me? When we are talking about transformation, think about the
2.		word, what is the meaning of that? Hmmm? Transformation, hmm? Anyone? Do you want to try? No one? Ok! Transform! When we are talking about transform
3.		what is the meaning of that class? In English, transform? Hmm? Okay. Transformation is a change <u>nhe?</u> {right?} Is a change in position. If I am moving
4.		here and I moving to other position, <u>nhe?</u> {right?}?
5.	Learners:	Yes madam
6.	Teacher:	So it is transformation, <u>nhe?</u> {right?} Ok! It is a change, you can write down the meaning of transformation, It is a change in position, shape and or size <u>nhe?</u>
7.		{right?} so the object change in position, size and shape.
8.	Teacher:	So there are three types of transformation, therefore three types <u>nhe?</u> {right?} there is one <u>nhe?</u> {right?} Reflection, you know one of the types of
9.		transformation? Ok! What is it?
10.	Learner:	Rotation
11.	Teacher:	Ok, rotation. Good and what else?
12.	Learners:	Translation
13.	Teacher:	Hmmm?
14.	Learner:	Translation
15.	Teacher:	Translation, Good and what else? Hmmm?

16. Learner: Reflection.
17. Teacher: Reflection nhe? {right?} three of them, rotation, translation and reflection, nhe? {right?} There are three of them. Ok better idea {writes on the board}
18. 3 types are, 1 Reflection, 2 Translation, 3 Rotation. But today we are going to be concentrating on translation nhe? {right?} 3 types of
19. Rotation (2) Reflection (3) Translation
20. Teacher: When we are talking about translation, put down the pen, Translation is when you achieve, ahh, when you slide an object, in fact, when you slide an object from
21. one position to the next without changing ahh, without changing any shape of the object. Like if I got this duster here and then I want to
22. move it to another position, maybe I want to move it down nhe? {right?} I'm take down, nhe? {right?} want change position of duster so it's therefore
23. translation. Don't change anything in there. Translation, so you achieve by sliding the figure to a new position without turning it. Are we together?
24. Learners: Yes
25. Teacher: So you achieve by sliding the ... of a figure to a new position without turning it. So you don't change the position of object you just move up or down or to
26. the right or to the left. Ok? Ok! Firstly we got ipoint {a point} here. Point P, Ok? Put down your pen and listen nhe? {Ok?} we've got a point Nhe?
27. {right?} we want to do the translation for point P. Point P is the figure, nhe? {right?} It is the figure and it is an original, you can also say it is an original object
28. that you are given and now you want to do what? You want to do the translation on the object Ok? Ahh, to move that. You can move that.
29. The question will come and says that translate that P by 5 units to the right. So you are here, this is your starting point and count 5 units to the right nhe?
30. {right?} Count.
31. Learners + Teacher: 1 2 3 4 5
32. Teacher: So it will be here, but you knew that it is a image by that side. When you translate you showing the P by that side that it is an
33. Learners + Teacher: Image.
34. Teacher: Here is an original object and here it is an----
35. Learners & Teacher: Image
36. Teacher: Are we together?
37. Learners: Yes
38. Teacher: You translate P from here to there y 5 units to the right, nhe? {right?}
39. Learner: Yes.
40. Teacher: Are we together bethuna? {people}
41. Learners: Yes
42. Teacher: Ok! I've got another example here, on page 109. Turn to page 109. Page 109 in the red work Book, nhe? {right?} Here number 2, let us go to number 2 there, nhe? {right?} alright I'm going to put here, nhe? {right?} so that everyone must see. I have to do that on the board so that everyone can see,
43. nhe? {right?}

S P2 L3 EE2: Plotting co-ordinates (Homework)

Time # Speech

44. Learner: Yes.
45. Teacher: Oh then I'll just take the 1st object on number 2, nhe? {right?} What is the questions says?
46. Learners + Teacher: Translate these shapes 3 units right
47. Teacher: Ok then, we do that shades in here. Is that clear to everyone?
48. Learner: Yes.
49. Teacher: Translate this shape three units down, nhe? {right?} to the right nhe? {right?} Ok you are here, nhe? {right?}
50. Learners: Yes
51. Teacher: So you will count how many units?
52. Learner + Teacher: 3 units
53. Teacher: 3 units to the right, nhe? {right?}
54. Learners: Yes
55. Teacher: So where are you going to start? Here, nhe? {right?} And count how many?
56. Learner + Teacher: 1 2 3, nhe? {right?}
57. Teacher: So it means that object will be here, nhe? {right?}. From here you count 1 2 3. Are we together bethunana {people}?
58. Learners: Yes, Miss.
59. Teacher: Okay the last one at the bottom. Is that clear in fact?
60. Learners: Yes
61. Teacher: Let me do the other one at the bottom. And also that one you need to do. I mean to move, translate by 3 units to the right. Who can try and tell me where it
62. can stand? The object. Count on the board and show me where. Where do you start? But the object ends here. There is supposed to be 2 blocks for this.
63. Yes. But the object, all the object ends here, so you start where?
64. Learners: Here
65. Teacher: Hmmm? So count again.
66. Learners: 1 2 3
67. Teacher: Is he correct class?
68. Learner: Yes/ No. {class mumbles}
69. Teacher: Okay, hands up who can help him? Where's he supposed to start? Ha-ah sukuhamba, sukuhamba {Don't go, don't go} where do you start?
70. Learners: Here
71. Teacher: Count to 3
72. Learners: 1 2 3

S P2 L3 EE2: Plotting co-ordinates (Homework)

Time	#	Speech
	73.	Teacher: Then draw. You were correct. Draw it. Okay, so the said it will be here. Do we all understand?
	74.	Learner: Yes
	75.	Teacher: You count from the end of the object <u>nhe?</u> {right?} and count 1 2 3 <u>nhe?</u> {right?}
	76.	Learner: Yes
	77.	Teacher: Are we together class?
	78.	Learner: Yes
	79.	Teacher: Okay, same object nothing change to the object. Any questions, <u>nhe?</u> {right?}
	80.	Learner: No.
	81.	Teacher: No questions so far? Okay, can we do this for me now, with the whole question and all. Can we do it in a group? Ahh on page 126 <u>nhe?</u> {right?}
	82.	where it says there, it says translate this shape 4 units to the right. You are number 5, <u>nhe?</u> {right?} on your books. Number 5 <u>nhe?</u> right?}
	83.	{ <u>Khawulezisa kutheni niske nicinge nje?</u> {why do you always have to be deep in thought?} <u>iphi I ruler yakho Nontombi? Yhomntanam abanye sebegqhibile</u>
	84.	<u>ngoku ulibele kukucinga.</u> {where is your ruler, Nontombi? My child all the other children are finished while you are deep in thought.}
	85.	Teacher: <u>Khawulezani.</u> {Hurry up!}

SCHOOL P3**LESSON 1 EE1-EE3**

S P3 L1 EE1: Definition of a prime number

Time	#	Speech
00:30	1.	Teacher: What is the prime number? Explain what is a prime number? We are busy with prime number, composite number, low common factor and highest
	2.	common factor. What is a prime number? (Calling a learner by name What is a prime number?
	3.	Learner: <u>Ndiyilibele Miss</u> {I forgot it teacher}
	4.	Teacher: <u>Uyilibele. Ubuyazile kuqala?</u> {You forgot. Did you know it in the first place?}
	5.	Learner: Yes Miss. Another learner explains. Inaudible
01.10	6.	Teacher: Prime numbers are the numbers that have only two factors. <u>Andithi?</u> {Is that so?}
	7.	Learners: Yes
	8.	Teacher: That have only two factors, examples <u>zethu?</u> {our examples?} What are the examples of prime numbers? Are numbers that have only two factors?
	9.	What are the examples of prime numbers?
	10.	Teacher: <u>Uthi ngu-nine</u> {he/she says it is nine} Is nine a prime number?
	11.	Learner: No
	12.	Teacher: Is this a prime number? Pointing to the number.

S P3 L1 EE1: Definition of a prime number		
Time	#	Speech
01.31	13.	Learners: No Miss {teacher}
	14.	Teacher: <u>Uthi ngu-twelve</u> {she says it is twelve} What is a prime number? A prime number is a number that has only two factors. What is a factor first? <u>Yintoni i-factor?</u> {What is a factor?} So that we can be able to identify....
	15.	Learner: A factor is a number that gets into another number and left without a remainder.
02.08	16.	Teacher: Is a number that you divide by a number that have no remainder. What are the examples? Five, five is one of the prime number, what are the factors of five? <u>Pha ngasemva?</u> {At the back?} Pointing at learners seated at the back.
	17.	Learner: One
	18.	Teacher: <u>Ngu-one</u> {it is one}. <u>Ngeyiphi enye i-number?</u> {What is another number?} <u>Ngu-five</u> {it is five} <u>andithi?</u> {Isn't it?}
03.46	19.	Learners: Yes
	20.	Teacher: <u>Ithini enye i-prime number?</u> {Give me another prime number?}
	21.	Teacher: Example of prime numbers, don't forget a prime number is a number that has only two factors, only two factors.
	22.	Teacher: Seven. Factors of seven? <u>Sima</u> {calling the learner by name to answer}
	23.	Sima: One
	24.	Teacher: One {saying the answer loudly} Learner shouting seven
	25.	Teacher: Seven! You can see that five has only two factors and seven got only two factors . numbers? At the back?

S P3 L1 EE2: Composite numbers, factors and prime numbers		
Time	#	Speech
04.05	28.	Teacher: What about the composite number? What are the composite
	29.	Learner: Are the numbers that have two factors
	30.	Teacher: Are the numbers that have more than two numbers. Example of the number that have more than two numbers, give me one.
	31.	Teacher: Yes you! <u>Uthi-four</u> {he/she says four}. <u>Uthi</u> {he/she says} four is one of the composite numbers, what are the factors of four? So that we can see that four has more than two factors, what are the factors of four? One?
	32.	Learner: No, two.
	33.	Teacher: Its one and two
04.56	34.	Learner: Four
	35.	Teacher: No, four, <u>Ikhona enye?</u> {is there another one?}
	36.	Learners: No Miss.
	37.	Teacher: Huh?
	38.	Learners: No Miss.
	39.	Teacher: <u>Zi-more than two</u> {they are more than two?}
	40.	Learners: Yes
	41.	Teacher: <u>Ithini enye ezakubana</u> more than two factors? {What is another number that is going to have more than two factors?}
	42.	Learner: Twelve Miss {teacher}
	43.	Teacher: Twelve. Is twelve a prime number or is it a composite number?
	44.	Learners: Composite number

S P3 L1 EE2: Composite numbers, factors and prime numbers

Time	#	Speech
05.40	46.	Teacher: Zukisani {calling the learner by name} <u>khanisinike i-factor ka-twelve.</u> {Give us a factor of twelve}
	47.	Zukisani: Six
	48.	Teacher: Six, two. Huh?
	49.	Learner: Four
	50.	Teacher: <u>Bani?</u> {What?} Oh! Four
06.07	51.	Learner: Three
	52.	Teacher: Three?
	53.	Learner: One
	54.	Teacher: One, <u>pha ekoneni?</u> {At the corner?} Pointing at learners seated at the corner.
	55.	Learner: Twelve
06.37	56.	Teacher: Twelve {repeating the answer}
	57.	Learner: Eight
	58.	Teacher: Is eight a factor of twelve?
	59.	Learners: No
	60.	Teacher: When we divide twelve by eight, there is a remainder?
	61.	Learners: Yes
	62.	Teacher: There is a remainder, so we are looking for a number that we can divide and be left with no remainder <u>andithi?</u> {Isn't it?} <u>yintoni umehluko</u>
	63.	{what is the difference} between those two? A Prime number and a composite? What is a major difference between those two?
	64.	Learner: A prime number have two factors, a composite has more than two factors.
	65.	Teacher: That's the difference between those two, a prime number got only two factors and a composite number gets more than two factors

S P3 L1 EE3: Highest common factor and lowest common factor

Time	#	Speech
08.09	66.	Teacher: Lowest common factor also the highest common factor. <u>Ngubani ozakusixelela ukuba yintoni i-lowest common factor?</u> {Who is going to explain to
	67.	us, what is a lowest common factor?} Differentiate between LCF and HCF? <u>Ndinike umehluko</u> {give me the difference} between those two LCF
	68.	and HCF. Is it the first time you hear those words?
	69.	Learners: No
	70.	Teacher: Ok, come on guys, let's do it. The difference between those two. What is the lowest common factor? <u>Hayi musani ukuba ngoZolani.</u>
09.15	71.	<u>Anifuni ukuthetha?</u> {No, don't be quiet. Don't you want to say something?}
	72.	Learner: I think <u>i-number</u> {the number} of the LCF is the smallest number from the factors of that number
	73.	Teacher: What about the highest? Highest is the opposite of the lowest. Let's look at this, twenty, what are the factors of twenty?
	74.	Teacher: What are the factors of twenty?
	75.	Learner: Two
10:39	76.	Teacher: <u>Ngu-two</u> {it is two?}
	77.	Learner: Five
	78.	Teacher: Five, ten
	79.	Learner: One Miss {teacher}

S P3 L1 EE3: Highest common factor and lowest common factor

Time	#	Speech
	80.	Teacher: One, twenty
	81.	Learner: Four
11.43	82.	Teacher: <u>Ngu-four</u> {it is four}. Factors of twenty <u>ngu</u> {are} 2,5,10,20,4. Which number represents the highest factor number there? From that set of factor,
	83.	which number represents the highest factor?
	84.	Learner: Ten
	85.	Teacher: <u>Uthi ngu-ten</u> {he/she says it is ten}
	86.	Learner: Twenty
12.56	87.	Teacher: Twenty. Twenty is the highest common factor of twenty. Let us look at two different numbers, different numbers, {repeated} from those two, 18 and
	88.	24, we are looking for the common factors that both appears to both numbers. In order for you to be able to do that, first you have to write down the
	89.	factors of 18 and also write down the factors of 24 and look to those two sets of factors and after that you can be able to identify which
	90.	numbers appears both to that sets of factors. <u>Andithi?</u> {isn't it?}
	91.	Learners: Yes Miss {teacher}
	92.	Teacher: We going to start with eighteen, we cannot be able to do that, to take out the factors that appears both in 18 and 24, before that, we have to take
	93.	down the factors of 18 and also the factors of 24 and after that you can be able to see which number appears in both set of factors. Let us start
	94.	with 18. <u>Masiqale kula group size ngale</u> igroup. {Let us start with that group and then come to this one}
	95.	Learner: Two
	96.	Learner: Six
	97.	Learner: Three, eighteen
	98.	Teacher: <u>Ikhona enye?</u> {Is there another number?}
	99.	Learner: Nine
13.58	100.	Teacher: 2,1,6,3,18,9 are the factors of 18. Let us go to twenty-four
	101.	Learner: Two
	102.	Learner: Four
	103.	Learner: Twelve
15.12	104.	Learner: Twenty-four
	105.	Learner: Three
	106.	Learner: Eight, six
	107.	Teacher: There are the factors of 18 and there are the factors of 24. The question was Identify the numbers which appears in both sets of factors
	108.	that appears in 18 and in 24. The common factors of those numbers <u>ngu-six</u> {it is six} and also?
	109.	Learner: Two
	110.	Teacher: Two
	111.	Learner: Three
	112.	Teacher: <u>Nanku u-three ku 18 nanku 24.</u> {Here is three in both 18 and 24}
16.25	113.	Learner: One
	114.	Teacher: <u>Ikhona enye?</u> {Is there another one?}
	115.	Learners: Yes.....No
18.52	116.	Teacher: We've got six, two and three are factors that are common in both 18 and 24. From this set of numbers, identify the highest factor, identify the highest
	117.	Factor
19.03	118.	Teacher: Give us the highest common factor

S P3 L1 EE3: Highest common factor and lowest common factor

Time	#	Speech
	119.	Learner: Six
	120.	Teacher: Six is the highest common factor. The lowest?
	121.	Learners: One
20.10	122.	Teacher: The lowest is one and the highest is common factor is six. <u>Uthini umbuzo wakho?</u> {What is your question?} <u>Ikubhida phi?</u> {Where does it confuse you?}
21.19	123.	
	124.	End video

SCHOOL P3**LESSON 2 EE1-EE3****S3 G8 L2 EE1 – EE3****S P3 L2 EE1: Types of fractions**

Time	#	Speech
00.02	1.	Teacher: <u>Nizinxibele ntoni ii-jezi kweli langa? Khulula Vuzi, khulula.</u> {Why are you wearing jerseys in this heat? Take them off, take it off Vuzi}
	2.	[inaudible]
	3.	Learner: <u>Ndithe khulula, khulula. Khulula lo jezi.</u> {I said take it off, take that jersey off}
	4.	Teacher: [Teacher going around the classroom.]
01.10	5.	Teacher: Ok! <u>Besenze ntoni kanene izolo, besine classwork nhe?</u> {what did we do yesterday? We had class work, right?}
	6.	Learners: Yes
	7.	Teacher: Then <u>senza ii-corections, vul'incwadi, vul'incwadi xa use klasini, vul'i ncwadi. Sazenza ii- corrections andithi?</u> {then we did corrections, isn't it?}
	8.	Open your book, open your book when you are in class, hitting the desk. Then we did the corrections.
	9.	Learners: Yes Miss { teacher}
	10.	Teacher: Ok! Into <u>ebesithetha ngayo basically... Wonke umntu uzenzile ii-corrections?</u> {What we were talking about basically... has everyone ...?}
	11.	Learners: Yes
02.06	12.	Teacher: <u>Ndakukhaba ukuba awuzenzanga ii-corrections.</u> {I will kick you if you have not done your corrections.}
	13.	Teacher: Ok! <u>Besi-dealishana ne-fractions, i-types of fractions, sathi zintoni kanene ii-types of fractions?</u> {We dealt with fractions, different types of fractions and we said what types of fractions are those?}
	14.	
	15.	Learners: Equivalent fractions....Proper fractions
	16.	Teacher: <u>Sathi ke ngoku siyakwazi uku-convert {a} improper fraction to a mixed number andithi?</u> {We said we can be able to convert improper fraction to a mixed number.Right?}
	17.	
	18.	Learners: Yes
	19.	Teacher: <u>Siyakwazi uku-converth a i-mixed number to intoni?</u> {What?} To improper fractions. {We are able to convert mixed numbers to what? To improper fractions}
02.54	20.	
	21.	Teacher: Andithi? {Isn't it?}

S P3 L2 EE1: Types of fractions

Time	#	Speech
	22.	Learners: Yes
	23.	Teacher: Ok! <u>Besisenza u-skill bani kanene pha encwadini?</u> {What number skill in the book were we doing?}
	24.	Learner: Skill one
	25.	Teacher: <u>Senza u-number one, two no-number three, senza u-number six, seven and eight.</u> {We did number one, two and three then we did number six, seven and eight}. Ok, <u>Makhe sijonge bethunana kengoku nhe</u> {let us now look} how to add fractions. <u>Yintoni esiyijongayo xa sidibanisa ii-fractions?</u>
03.17	26.	{What do we look at when we are adding fractions?} <u>okanye uthini umthetho wethu</u> {or what are the rules?} <u>Besitshilo mos sathi</u>
	27.	{we had already said} that we must always follow ii-rules {the rules} so definitely in adding there is <u>intoni?</u> {what?}
	28.	Learners:
	29.	

S P3 L2 EE2: Addition of fractions

Time	#	Speech
04.04	30.	Teacher: Let's look at that. <u>Ndifuna nje umntu ozaku-traya, siyibala njani?</u> {I just want someone who is going to try and work this sum}.
	31.	<u>Sizama uku-addisha i-fraction, sizakuyenza njani?</u> {We are trying to add fractions, how do we do that?} How do we calculate? <u>Siyayazi ukuba sibala</u>
	32.	<u>Njani?</u> {We know how to calculate we all know that we have to follow some rules}
	33.	Learner goes to the chalkboard to do the problem. $\frac{3}{4} + \frac{1}{2}$.
	34.	Teacher: Addition of fractions. <u>Khawutsho ke bhuti, ucinga ukuba wena singayenza njani?</u> {Tell us boy, how are we going to do that sum}
05.10	35.	Teacher: Ok! Thank you. {to the learner who was doing the sum on the board} <u>Omnje umntu uthini yena?</u>
	36.	<u>Omnje umntu uyibona njani yena?</u> {Anyone else! Anyone else who is thinking otherwise?} <u>Khawuze!</u> {come up and show us how to add fractions}
06.21	37.	Teacher: <u>Ungayicimi, bhala ecaleni, bhala eyakho, sukuyicima eyakhe.</u> {Don't erase it, write your sum on the other side, don't erase his sum}
	38.	Learner: <u>Bendingayicimi Miss.</u> {I was not going to erase teacher}
	39.	Teacher: Oh! <u>Bhala wena, thina sizakujonga nje</u> {just write the sum, we will look at it} {We will choose between those two which one is right or which one.}
	40.	is wrong
	41.	Learner does the problem on the board.
	42.	Teacher: <u>Omnje umntu?</u> {someone else} Another learner goes to the board to do the same problem.
07.22	43.	Teacher: We've got three solutions there, <u>andithi?</u> {Isn't it} <u>Bathathu abantu ababhalileyo pha, ngeyiphi i-answer e-right? Okanye not iAnswer, ngeyiphi indlela.</u>
	44.	{There are three learners who did the sum, which one is the correct one or which method is the correct one?}
08.47	45.	<u>Asiqali mos ukudibana ne-adding ye- fractions, naniyenzile e-primary mos?</u> {It is not the first time
	46.	we have come across adding of fractions, you have done them in primary, right?}
	47.	Learners: Yes Miss.
09.37	48.	Teacher: <u>Aba bantu babalile ba-thimzile apha ikubhalansisa ii-denominators zakhona, nangapha babhalansisa ii-denominators zakhona. Abanye?</u>
	49.	<u>Nithini kanye? Kutheni ii-denominators zalapha, kufuneka zifanile nje? niyabona nalapha ziyafana, kutheni? Kutheni</u>
	50.	<u>bengazidibanisanga $\frac{3}{4} + \frac{1}{2}$ Why? {The learners have done their sums, they have multiplied to balance the denominators, they have done</u>
	51.	<u>that as well on this side, what do others say? Why do the denominators here have to be the same?} siyakwazi ukudibanisa ii-fractions ze</u>
	52.	<u>-denominators ezingafaniyo?</u> {Can we add the fractions even though the denominators are not the same?}
	53.	Learners: No Miss
	54.	Teacher: Huh.

S P3 L2 EE2: Addition of fractions

Time	#	Speech
10.40	55.	Learners: No Miss.
	56.	Teacher: <u>Asikwazi</u> . We have never before, <u>udibanise i-division fractions zakho</u> , <u>funeka uthi make sure ukuba ii-denominators zakho ziyafana after</u>
	57.	<u>wenze ii- denominators zakho zifane than uyakwazi uku addisha, okanye ke if ii- denominator zakho awuzitshintshanga. Siyavana?</u>
	58.	<u>Ufune ntoni? i-lowest common multiple {LCM} yamanani amabini. Uyazi ukuba ii-denominators sifane okanye ufune i- LCM. Masifanise apha.</u>
	59.	{What are we going to do here? {We can't.} We have never add division fractions...we have to make sure that your denominators are the same,
	60.	after you have made your denominators the same then you are able to add}. What are we going to do here?
11.55	61.	Learners: Saying something but inaudible
	62.	Teacher: <u>Sizakufuna i-LCM, ngubani i-LCM yalapha?</u> {We are going to look for the LCM, what is the LCM here.}
	63.	Learners: <u>Ngu-four</u> {it is four}
	64.	Teacher: Is it?
	65.	Learners: Yes Miss.
	66.	Teacher: <u>Le division asiyishiyi. Sithini ke ngoku i-step esilandelayo?</u> {What is the next step?}
12.48	67.	Teacher: <u>Sizakuthini ngoku?</u> What are we going to do now? <u>Siyitshintshile i-denominator yethu yangu-four. Then ke ngoku oo-four baya kangaphi ku-four?</u>
	68.	{We have changed our denominator to four, then how many times does four go into four?}
	69.	Learners: One
	70.	Teacher: Times three?
	71.	Learners: Three
	72.	Teacher: <u>Plus ... oo-two ku-four?</u> {two goes into four how many times?}
	73.	Learners: Two
	74.	Teacher: Two, times one?
	75.	Learners: Two
	76.	Teacher: <u>Ngu-two andithi?</u> { it is two, is it not?}
13.10	77.	Learners: Yes
	78.	Teacher: <u>Sigqithe, three plus two?</u> {then three plus two?}
	79.	Learners: Five
	80.	Teacher: Five over four, <u>siyayibona le nto?</u> { do we see this?}
	81.	Learners: Yes Miss.
	82.	Teacher: <u>Siyibonile kengoku?</u> {can we see it now?}
	83.	Learner: Yes Miss {teacher}
	84.	Teacher: <u>Siphinde sibe nalo mzekelo.</u> {Then we have another example} Four over six plus two over three. Let's add those fractions, four over six plus two.,
	85.	over three we must add those fractions, four over six plus two over three, <u>sizakuthini pha?</u> {What are we going to do?} You must follow the rules;
	86.	you must know what you must do then you will be able to solve the equation. Any questions? What must you do, what is your first step
14.33	87.	when you get a fraction sum that you must add? What do you look for?
15.43	88.	Learner: <u>Ukhangela i-LCM.</u> {You look for the..... LCM}
	89.	Teacher: <u>So ukhangela i-LCM, eyalapha ngubani?</u> {So you look for the LCM, what is the LCM here?}
	90.	Learners: Six
	91.	Teacher: <u>Ngu-six, andithi? U-six uyangena apha, no-three uyangena apha? Sithini ke ngoku? Sithi six, sithini ke ngoku?</u> {Calling the learner to answer}
	92.	<u>Sithini ke ngoku?</u> {It is six, right? Six goes in here and three goes in here, what then? {What else do we do now?}
	93.	Zukisani: Not responding.

S P3 L2 EE2: Addition of fractions

Time	#	Speech
16.42	94.	Teacher: Zukisani?! Sithini ke ngoku? {What do we do next?}
	95.	Zukisani: Hlisa u-four. { lower the four}
	96.	Teacher: Ahliswe njani u-four? Uvele athotywe nje u-four? {How do we lower the four, why should we lower the four?}
	97.	Teacher: Sizakuthini pha? {What are we going to do there?}
17.50	98.	Teacher: Khawuphinde {repeat that}
	99.	Teacher: 8/6 is the answer, how do we simplify this to the smallest number? Let's simplify it to the point of a fraction and not a mixed number.
	100.	Learner: i-LCM Miss ngu-two, so u-two uya kangaphi ku-eight, uye kayi- four then aye kangaphi ku-six aye kayi-three. Then i-answer yethu miss
	101.	xa sesiyi simplifaya ngu-four over three. {The LCM is two, two goes into eight how many times, four times. Then two goes into six
18.43	102.	how many times, three times then our answer teacher in a simplified manner is four over three.}
	103.	Teacher: Ibe ngu-four over three {it will be four over three}, so four over six plus two over three, i-step sethu sokuqala sithini? {What is our first step?}
	104.	Sifuna i-LCM andithi? {We want the LCM right?}
	105.	Learner: Yes Miss.
20.30	106.	Teacher: Secondly sibuzene andithi? Sibuze nge-denominator zethu andithi. Sibuze kula denominator intsha iyila LCM yethu, that answer sizakuyifumana
	107.	phaya si-multiplaya nantoni? Nge-numerator. Andithi? {Now we going to ask about our denominator, now the answer that we get as our LCM
	108.	we going to multiply that with which number? With our numerator right?}
	109.	Learners: Yes Miss
21.42	110.	Teacher: And then ke ngoku sifumane i-answer yethu siphinde sibuze i-denominator yesibini, siyibuzele kula LCM andithi? {Then we get our answer than
	111.	we ask for our second denominator right?}
	112.	Learners: Yes
	113.	Teacher: And then from there ke ngoku sifumane i-answer siyi-time ze nge-numerator yala fraction yesibini. I-step esilandelayo si-addishe ii-numerator
	114.	zethu zombini andithi? {And then from then we get our answer then we multiply it by our numerator of the second fraction. The following step we add
	115.	both our numerators. {Isn't it?}
	116.	Learners: Yes Miss
	117.	Teacher: And then ke ngoku sifumane ntoni? i-fraction yethu. Then sithi look at i- fraction yethu, if i-fraction yethu iyakwazi uku- simplifayeka siyi-simplifaye
	118.	to the smallest fraction. {And then what do we get? Our fraction. Then we look at our fraction, If our fraction can be simplified then we
	119.	simplify it to the smallest fraction}. Ukhona umntu onombuzo ke? Ukhona umntu ofuna ukuphindelwa somewhere there? Zukisani uvile?
	120.	{Is there anyone who has a question? Is there anyone who wants me to repeat the answer?}
	121.	Zukisani {calling the learner} did you understand that?}
	122.	Zukisani: Yes Miss
	123.	Teacher: Uve ntoni? kuthwani pha? Khawusicacisele, yintoni ii-rules zakho? Yintoni ekufuneka uyenzile when you add i-fractions? Phakama ke, Zukisani
	124.	ukhawulezise uthethe. {What did you hear? Explain it to us, what are your rules? What do you do when you add fractions? Stand up Zukisani,
	125.	hurry up and talk}
	126.	Teacher: Yima ngenyawo, phakama {stand up}
	127.	Teacher: Khawutsho ke simamele {tell us, we are listening.}
	128.	Teacher: Yintoni ehlekisayo? Sithi siya-addisha ii-fractions, sifuna i-LCM nenani elincinci elingena kulo. What is funny? We are adding fractions,
	129.	{we want the LCM and the smallest number that can go into it}

S P3 L2 EE3: Algorithm for addition of fractions

Time	#	Speech
25.58	130. Teacher:	busy writing a sum on the board. $5/7 + 1/14$
	131. Teacher:	<u>Ndifuna sidibanise eza-fractions, ngubani ozakusenzela.</u> {I want someone to add those fractions, who is going to do it?}
27.33	132. Teacher:	<u>Khawu sicacisele ke ukuba wenze njani? Nimamele nina bebesithi anivanga.</u> {Explain to us, how did you do that. Those that said did not understand. {Listen}
	133.	
	134. Teacher:	Then you! <u>Niyayibona?</u> {Can you see?}
	135. Learners:	Yes
	136. Teacher:	If you just follow your rules or procedure, <u>ubuyayazi i-first step yakho kukukhangela i-LCM, then i denominator, then i-step esilandelayo uyabuzela kengoku andithi? Ubuze ke ngoku u-seven uya kangaphi ku-fourteen, uya kayi one, la one simthini siya-multiplaya nge-numerator, ke ibe ngubani ngoku, ibe ngu-ten</u> and then from then ten plus one is eleven over fourteen. <u>Nivile nina bantu abame ngenyawo?</u>
	137.	
	138.	
	139.	{If you know your first step is to look for the LCM, than your second step is to take that number that you have identified and
	140.	make it the denominator, then the following step is to ask yourself how many times does seven go into fourteen, one time then we multiply
	141.	that one by the numerator which is ten and then add ten plus one which is eleven then the answer will be eleven over fourteen. Those that are
33.15	142.	standing, did you hear/get that?} We are able to add the fraction only when the denominators are the same for example if they say $2/3 + 5/3$
	143.	<u>ithini i-answer yethu 5 pha?</u> {What is the answer there?}
	144. Learner:	$7/3$
36.20	145. Teacher:	<u>Ukhona omnye umntu onombuzo?</u> {anyone who has a question? You must practice maths, if you want to know and understand maths, Nditsho every day. {I say} that every day? <u>Masijonge pha encwadini yethu, sizakwenza u-skill five no-skill seven for homework</u>
	146.	
37.23	147.	{Lets look at our exercise books, we are going to do skill number five and seven for homework}.
39.04	148.	Teacher explains to the class what they need to do, but the bell rings and the class is a bit chaotic. Learners are moving chairs and going out
40.26	149.	End of lesson.

SCHOOL P3**LESSON 3 EE1****S P3 L3 EE1: Addition of fractions**

Time		Speech
00.10	1.	Teacher: <u>Khupha incwadi</u> {Take out the book} ... <u>kupha incwadi yakho</u> ... {Take out the book}. <u>Vula kwi homework</u> {Open up your homework}
	2.	<u>ihomework yakho tyhila.....</u> {Open to your homework}. <u>01.38 Phakama.</u> {Stand up} <u>Susa ubhaka esitulweni. Ihomework yam mayize.</u> {Take your school bags off the tables and bring my homework.} [Teacher walks around checking if homework has been done.] <u>02.29</u>
	3.	
	4.	Teacher: <u>Khawuyondibhalela unnumber 1 pha.</u> {Go and write down number one} <u>Kutheni ke</u> {What now?} <u>ubengekho ke ngoku?</u> You were not here, now, what?} <u>Ibingubani nabani ebabeyo kubaleka?</u> {Who else went to athletics that day?}
	5.	
	6.	Learner: <u>Nanku miss.</u> {This one miss}
	7.	Teacher: <u>Niyakuthi shu!</u> {You will suffer} <u>Niyakufunda ukubaleka oko.</u> {You will learn athletics}. <u>02.49 Khawuyosibalela u number 1</u>
	8.	{Go and do number 1 for the class} <u>wena</u> {you} u number 2, <u>Zukisani usibalele</u> {you do} u number 3. <u>Hlalani phantsi.</u> {Sit

S P3 L3 EE1: Addition of fractions

Time	Speech
9.	down} U number 1. <u>Ubuvo kubaleka nawe?</u> {Were you also in athletics?} <u>Khange uyobaleka qha undiqhela kakubi wena.</u>
10.	{You were not in athletics you are just making me a fool}. <u>Benditheni kuwe izolo?</u> {What did I say to you yesterday?} <u>Yeh?</u>
11.	<u>Benditheni kuwe izolo?</u> {What did I say to you yesterday?} <u>Ufuna ukutshona ugrade 8?</u> {Do you want to fail grade 8?}
12.	<u>Njengokuba unyuswe ngekiliva pha kwa grade 7 nje.</u> {Just like you were promoted in grade 7} <u>Ucinga nam ndizakutshova</u>
13.	<u>nge kiliva?</u> {Do you think I will also do that?} <u>Akukho kiliva ke apha uzakutshona qha ke.</u> {You will not be promoted here.
14.	You will fail} <u>Uzakutshona uphinde ugrade 8 ukhale.</u> {You will fail and repeat grade 8 and cry} <u>Ubuze pha ko Zukisani naba</u>
15.	<u>bayaphinda oku kwabo.</u> {Ask Zukisani, he is repeating as well} <u>04.04 Ukuba awukwazi ukwenza umsebenzi uzotshona ubelapha</u>
16.	<u>kwa grade 8 nakulo nyaka uzayo.</u> {If you cannot do your work you will fail, and you will be in grade 8 again next year}
17.	<u>Uzakuba lapha nakulo nyaka uzayo.</u> {You will be here again next year} <u>Zukisani khawube uyokufuna iduster.</u>
18.	{Zukisani go and look for the duster}
19.	[Learner writes on the chalkboard]
20.	Learner: <u>Iphi?</u> {Where is it?}
21.	Teacher: <u>Hamba uye kuvifuna nditsho nje kuwe.</u> {I have just said to you, go and look for it}. <u>Hamba nokuba uyifune phi.</u> {Just go and find
22.	one, I do not care where you get it}. <u>04.15 Undixelele ngomntu ongagqhibanga.</u> {Tell me about the person who did not finish?}
23.	<u>Ungakulinge uvule incwadi ungagqibanga okanye ndizobetha wena</u> {Do not even open your book if you did not finish or I will
24.	beat you up} <u>Uyazi ukuba ndiza kukhomba wena mntu ongagqibanga, uphethe incwadi ungagqibanga.</u> {I will hold you
25.	responsible if there is someone who has their books open, but their work is not done} <u>05.55 Le ibonakala</u>
26.	<u>nje ngendlela eyenziwe ngayo ukuba khange yenziwe nguwe.</u>
27.	{I can just see this is not your own work} <u>Zezikabani ezi?</u> {Whose is this?} <u>Heh? Bekuthiwe ziya edishwa i</u>
28.	{Did someone say we a dd the.....} <u>Bekuthiweni ngoku bekusenziwa ifractions pha izolo?</u> {What did we say when we
29.	were doing fractions yesterday?} <u>Besi edisha ifractions.</u> {we were adding fractions} <u>bekuthiwe yenza ntoni?</u> {what did they say
30.	we must do?} <u>Yhe? Intoni?</u> {what?} <u>Bezingaphi kanene?</u> {how many were there again?}
31.	<u>Ziyi 4 nhe?</u> {there's 4 right?}
32.	Learners: Yes miss.
33.	Teacher: <u>Azikho 5 ziyi 4</u> {it's not 5, it's 4} ... <u>Oobhaka mabasuke esitulweni!</u> {Bags, off the chair!} <u>Ubhaka makasuke esiyulweni.</u> {Bags,
34.	off the chair!} <u>Ngenisa isitulo, ngena, ngena.</u> {Bring inside the chair, inside inside} <u>Umele ntoni wena?</u> {Why are you standing?}
35.	<u>Kutheni ungagqibanga?</u> {Why did you not finish?} <u>Khange ndithi hlala phantsi?</u> {Did I not say sit down?} <u>Hamba uyokwenza u</u>
36.	<u>number 4.</u> {Go and do number 4} Zukisani?
37.	Learner: <u>Uphumile. Usayoboleka iduster.</u> {He is gone to look for the duster}
38.	Teacher: U 11 plus 6 ngu? {is?} ... <u>ugqhibile pha Zukisani ukubala?</u> {Zukisani are you finished writing?} <u>Ugqhibile?</u> {are you finished?}
39.	Zukisani! 11 plus 5 ngubani? {is?} Tyhini!
40.	Learner: Miss
41.	Teacher: La {that} 1 plus 6 usezantsi umbekela ntoni kengoku yena? {at the bottom there, why did you write it?} <u>08.35 U 7 over 7 x a sim</u>
42.	<u>simplifaya ngubani?</u> {When we simplify it is?}
43.	Learner: Yi whole
44.	Teacher: <u>Ngubani kaloku iwhole?</u> {What is the whole?}
45.	Learner: <u>Ngu</u> {it's} 1

S P3 L3 EE1: Addition of fractions

Time	Speech
09.34	<p>44. Teacher: Skill 7. unnumber 1, <u>khawuleza</u>. {hurry!} [Learner writes sum on the board.] <u>Khawukhe ubhale kakhulu ntombazana ndini</u>. {Write clearly you girl} 10.34 <u>Icacile le nto uyibhalayo, ibhalwe encwadini?</u> {What you are writing is clear; you are getting it from the book} <u>u-7 uyangena ku 3 no 9?</u> {does 7 go into 3 and 9?} <u>Uyangena ku 3?</u> {does it go into 3?} <u>Besingathanga sifuna inani elingena kuwo omabini la manani pha?</u> {Did we not say we are looking for a number that goes into both numbers?} <u>I denominator zethu zizosinika ilantuka ... i.....remainder.</u> {Our denominators will give us a remainder} <u>Uyangena u 3?</u> {Does it go into 3?} <u>u-7 uyangena?</u> {Does it go into 7?} <u>No 9 uyangena ku 3?</u> {Does it go into 9?} <u>Sukusiqhela kaloku.</u> {do not make us fools} <u>Khona ukuba bekufunwa inani elingena ku 9.</u> {if they are looking for a number that goes into 9} <u>u 3 uyangena ku 7?</u> {does 3 go into 9?} <u>Yhe ntombazana ndini wenza u 2 okanye wenza u 1?</u> {Hey you girl are you doing number 2 or number 1?} 11.29</p> <p>51. Teacher: <u>Tshini! Wenza ntoni wena ntombazana ndini kulo bhodi?</u> {Hey you girl what is it exactly that you are doing on that board?}</p>
11.29	<p>52. Teacher: <u>Tshini! Wenza ntoni wena ntombazana ndini kulo bhodi?</u> {Hey you girl what is it exactly that you are doing on that board?}</p>
12.37	<p>53. <u>Yincwadi kabani le ufunda kuyo?</u> {Whose book are you reading from?}</p> <p>54. Learner: <u>Yeyam</u> {It is mine}</p> <p>55. Teacher: Huh?</p> <p>56. Learner: <u>Yeyam</u> {It is mine}</p> <p>57. Teacher: <u>u9 ... u9 no 8 bayangena ku 2?</u> {Does 8 and 9 go into 2?} <u>Yhe? Ke ngoku ubabhalela umbhalela ntoni u2?</u> {Why did you write 2?} {Now why did you write them?}... <u>Ngubani inani elingenayo u 9 no 8?</u> {What number goes into 9 and 8?}</p> <p>58. Learner: Ngu {It's} 1</p> <p>59. Teacher: u 9 uyangena ku 1? {Does 9 go into 1?} No 8 angene ku 1? {and does 8 go into 1?} Hmmm?</p> <p>60. Learner: No miss</p> <p>61. Teacher: <u>Ngoku kutheni usithi ngu 1 ke?</u> {Now why did you say its 1?} <u>Khawusuke kulo bhodi</u> {Move away from that board}. 13.00</p> <p>62. <u>Khawusuke.</u> {move} <u>Ume pha ngenyawo.</u> {You must stand there} <u>Akekho omnye umntu onokulungisa pha?</u> {Is there someone who can fix this?} <u>He bethunana besingathanga xa idenominators zingafani sifuna inani</u> {People did we not say when our denominators are different we look for a number} ... <u>Sijonga inani eza denominators zethu zingena kuzo zombinizingashiya i remainder?</u> {We look for a number that both our denominators can go into and that will not leave remainders}</p> <p>63. Learners: Yes miss</p> <p>64. Teacher: <u>Besingatshongo?</u> {Did we not say so?}</p> <p>65. Learners: Besitshilo {we said so}</p> <p>66. Teacher: <u>Ke ngoku aba benza ububhanxa babenzela ntoni?</u> {So, those who are doing this nonsense, why are they doing it?} He? <u>Okanye bafuna ukukhatywa ndim?</u> {or do they want a beating from me?} He? <u>Ngaba bafuna ndibakhabe?</u> {they want a beating from me?}</p> <p>67. <u>Aba bangamameliyo?</u> {Those who do not listen} 14.55 <u>Bangatshoyo naxa bengakhangabe beve</u> {and do not even say when they do not understand} <u>Yhe? Ntombazana ndini kutheni uzobhala u 2 wena?</u> {You little girl why did you write 2?} U 2 no {and} 9, u 9</p> <p>68. <u>uyangena ku 2?</u> {does 9 go into 2?} <u>u-8 uyangena ku 2?</u> {does 8 go into 2?} <u>Ke ngoku uyenzela ntoni lonto?</u> {now why did you do that?} He? <u>Tshini! Bonisa abanye uyenze njani.</u> {show the others how you did it} Hmm? <u>Yhe? Niyenze ngololahlobo?</u> {did you all do it like that?}</p> <p>69. Learners: Yes miss</p> <p>70. Teacher: <u>u-8 uyangena ku 18?</u> {Does 8 go into 18?} <u>Kengoku?</u> {now?} <u>o-9 bangena kangaphi ku 72?</u> {How many times does 9 go into 72?}</p> <p>71. Learners: 8</p> <p>72. Teacher: 8 times 5?</p> <p>73. Learners: 40</p> <p>74. Teacher: o-8 <u>ku</u> {in} 72?</p>

S P3 L3 EE1: Addition of fractions

Time	Speech
15.13	82. Learners: 9
	83. Teacher: 9 times 7?
	84. Learners: 63
	85. Teacher: 63. <u>udibanise</u> {add} u 40 no 63 <u>ufumana bani?</u> {What do you get?}
	86. Learners: 103
	87. Teacher: 103 over?
17.02	88. Learners: 72
	89. Teacher: U number 2. <u>Khawulezisani!</u> {hurry} [Learner writes sum on the board, while teacher is walking around class marking homework.]
	90. Teacher: <u>Uyabala apha.</u> {You count here} <u>Ubale apha ubonakalise la ndlela ukuba uyifumene njani not nje ubhale nje i answer.</u> {You count and show how you got to your answer not just the answer} <u>Kudala ume pha kula ...</u> {It's been a while since you have been stuck here} ... <u>kudala usenza.</u> {You have been doing this for a while} <u>Ububhale ntoni kanti encwadini yakho?</u> {What did you write on your book?} <u>Njengokuba ungakwazi ukuqala apha encwadini.</u> {Now that you cannot even start from your book} <u>Yhe? Kutheni?</u> {Why?} <u>Njani?</u> {How?} [Learner goes to explain to the teacher.] 18.02 u 7 <u>uyangena ku 9?</u> {does 7 go into 9} <u>Bekutheni ukuze ufune</u> {then why did you } ... <u>usebenzise u 9?</u> {use 9?} <u>u-7 uyangena ku 9?</u> {does 7 go into 9}
	96. Learner: [Shakes head]
	97. Teacher: <u>Besingathanga ukhangela inani elingena kuwo omabini amanani?</u> {Did we not say you look for a number that goes into both number?} <u>Yhe? Ngoku uyenzele ntoni wena lonto?</u> {Why did you do that?} <u>Kutheni?</u> {why?} <u>Kutheni ungamameli nje?</u> {Why do you not listen?} <u>Yhe? Ngubani igama lakho?</u> {What is your name?} <u>Yhe Aviwe kutheni ungamameli?</u> { Aviwe why do you not listen?} Hmm? <u>Ufuna ndikukhabe?</u> {do you want a beating?} <u>Ndizokukhabe ke.</u> {I will beat you up} <u>Ndizokukhaba nyani. I will beat you up, really}</u> <u>Yiya, hamba yenze.</u> {Go, go and do it}
	101 [Pointing to another learner]
	102 <u>Hamba uyokwenza u number 2 lona.</u> {go and do this number 2} <u>Iworse ukuba ucime leyo iright.</u>
	103 {why do you erase the right one?} <u>Yhe Aviwe wayithini ke ngoku itshokwe?</u> {Aviwe what did you do to the chalk?}
	104 <u>Khawubhale lento apha wena, apha kule ndawo icacayo.</u> {write that thing here where it can show} <u>Ewe kaloku wenza yona.</u> {yes
	105 you are doing it} <u>Yhe wethu.</u> {hey you} <u>Ufuna inani elingena ku 9 no 7 andithi?</u> {You are looking for a number that goes into 9 and 7 right?}
	106 Learners: Yes miss
	107 Teacher: <u>Inani elingena ku 9 no 7 ngubani?</u> {Which number goes into 9 and 7?}
	108 Learner: <u>Ngu</u> {it's} 63
	109 Teacher: <u>Akukho elinye elincinci?</u> {is there no smaller one?}
	110 Learners: No
	111 Teacher: Mmm?
	112 Learners: No
	113 Teacher: So, <u>siyabuza o 9 ku 63 bangaphi?</u> {So we ask how many times does 9 go into 63?}
20.20	114 Learner: <u>Bayi</u> {there are} 7.
	115 Teacher: <u>o 9 ku</u> {in} 63 ?
	116 Learner: 7

S P3 L3 EE1: Addition of fractions

Time	Speech
21.13	<p>117 Teacher: <u>Bayi</u> {there are} 7, times 5?</p> <p>118 Learners: 35</p> <p>119 Teacher: <u>Bayi</u> {there are} 35, <u>simbeke apha anditsho?</u> {We put it here, right?}</p> <p>120 Learners: Yes</p> <p>121 Teacher: <u>Faka iaddition sibuze</u> {you put in addition and ask} <u>o7 bangaphi ku 63?</u> {How many times does 7 go into 63?}</p> <p>122 Learners: <u>Bayi</u> {they are} 9</p> <p>123 Teacher: <u>Bayi</u> {they are} 9 times 6?</p> <p>124 Learners: 54</p> <p>125 Teacher: It is 54 <u>andithi?</u> {right?}</p> <p>126 Learners: Yes miss</p> <p>127 Teacher: 36 plus 54?</p> <p>128 Learners: <u>ibengu</u> {it will be} 89</p> <p>129 Teacher: 89 divided by <u>bani?</u> {what?}</p> <p>130 Learners: 63</p> <p>131 Teacher: <u>Basimamele aba bebeyodla?</u> {there one's that were out playing must listen} Or <u>bebaleka?</u> {or running} <u>Yhe?</u></p> <p>132 Learners: Yes</p> <p>133 Teacher: <u>Uphi unumber 3?</u> {where is number 3?} <u>Sizokwenzelwa ngubani inumber 3?</u> {who is doing number 3 for us?} <u>Kutheni kunga krwelanga?</u> {why are there no lines?} 22.12 <u>Kutheni kungakrwelanga apha?</u> {why are there no lines here?} <u>Yincwadi kabani le?</u> {whose book is this?} <u>Yhe?</u> <u>Kutheni ungakrweli migcha?</u> {why do you not rule lines} <u>Kutheni ungakrweli migcha?</u> {why do you not rule lines} <u>Ubeka classwork nantoni.</u> {you put classwork with} <u>Kutheni ungakrweli migcha?</u> {why do you not rule lines} <u>i Umsebenzomdaka apha.</u> {Your work is dirty} <u>Kuqhubeka ntoni apha?</u> {what is happening here?} 22.34 <u>Kwenzeka ntoni apha?</u> {what is happening here?} <u>Yhe bethuna!</u> {people!} <u>Pha ngu</u> {there it's} <u>3 no</u> {and} 7, <u>ibengu</u> {it becomes} 21 <u>andithi?</u> {am I right?}</p> <p>134 Learners: Yes miss</p> <p>135 Teacher: <u>Inani elingena kuwo omabini u7 no 3 ibengu 21.</u> {a number that goes into both numbers 7 and 3 is 21} <u>ubeke u21 apha,</u> {you put 21 here} <u>ubuze o3 ku 21 bangaphi?</u> {and ask how many times does 3 go into 21?}</p> <p>136 Learners: 7</p> <p>137 Teacher: 7. 7 times 1?</p> <p>138 Learners: 7</p> <p>139 Teacher: 7. o7 <u>ku</u> {into} 21?</p> <p>140 Learners: 3</p> <p>141 Teacher: 3. 3 times 5?</p> <p>142 Learners: 15</p> <p>23.39</p> <p>143 Teacher: 15. 7 plus 15?</p> <p>144 Learners: 22</p> <p>145 Teacher: 22 over?</p> <p>146 Learners: 21</p> <p>147 Teacher: unumber 4.</p> <p>148 Learner: Uxolo miss {excuse me miss} ukuba lento ibingu 42 ogqhiba wena waphinda wayitshintsha {if this was 42 and you change it} ...</p>

S P3 L3 EE1: Addition of fractions

Time	Speech
24.24	149 Teacher: Khawume ndibone {let me see} [Learner gives teacher her book and she marks it.]
	150 Learner: Xa eyisimplifayile m'am? {and if you simplified it m'am?}
	151 Teacher: uright kaloku xa esimplifayile. {she/ he is right if the simplify} Oh she is he is very good one. Uright kanye {just correct}. Yintoni le imdaka ubuyenza apha? {What is the dirty thing you were doing here?} Ubuzama ntoni? {What were you trying to do?} Uyiyenzele ntoni le nto ubuyiyenza apha? {Why did you do what you were doing here?} Utsho ngomsebenzi omdaka. {Your work is so dirty} Uyiyekela ntoni? {Why did you stop?} Uyiyekela ntoni kodwa iright? {Why did you stop when it is right?} Ha?? nabani {and?}? 7 no 5. u7 ungena ku 35? {Does 7 go into 35?}
	152 Learners: Yes miss
	153 Teacher: 7 abe ngubani ku 35? {How many times does 7 go into 35?}
	154 Learners: ngu {it's} 5
	155 Teacher: 7 times bani? {what?} 7 times bani? {what?}
	156 Learners: 5
	157 Teacher: isinike bani? {Which gives us what?}
	158 Learners: 35
	159 Teacher: 35. so 35, u7 no {and} 5 bangena ku {go into} 35 bobabini {both}, so ilantuka yethu izakuba ngubani? {what is our...?} 35. o7 ku {into} 35?
	160 Learners: Bayi {they are} 5
	161 Teacher: 5 times 3
	162 Learners: 15.
25.42	163 Teacher: 15. o5 ku {into} 35?
	164 Learners: 7
	165 Teacher: 7 times 2?
	166 Learners: 14
	167 Learner: Apha ianswer ngu 29 {here the answer is 29}
	168 Teacher: Hmm?
	169 Learner: Ianswer miss ngu 29 {the answer is 29 miss}
	170 Teacher: 15 plus 14?
	171 Learners: 29
	172 Teacher: 29 over?
	173 Learners: 35
	174 Teacher: Unike umntu incwadi yakhe sizokwenza icorrections. {Give back the books to the owner so people can make corrections} [Learners write corrections on their workbooks. Teacher goes around marking learner's books] Yhe bethuna sine {people, we have} (inaudible) ezayo ine fractions. {that is coming and it has fractions} Abantu bagqibile ukubhala icorrections? {are people finished writing down correction?}
	175 Learners: Yes miss
30.25	176 Teacher: <u>Yhe bethunana masiyeni ku page 43.</u> {people let's go to page 43} yho 48. page 48
	177 Learners: <u>Uxolo miss</u> {excuse me miss} <u>bayathethe aba.</u> {they are conversing}
	178 Teacher: <u>Uphosisa njani nithetha.</u> {How is he lying when you really are conversing?} <u>Nithetha nyani ninga kuphathanga ncwadi.</u> {you are conversing without any books open} Page 48 <u>andithi?</u> {am I right?}

S P3 L3 EE1: Addition of fractions

Time	Speech
179	Learners: Yes
180	Teacher: Skill number 7. <u>niyabona pha ku</u> {do you see in} skill number 7?
181	Learners: Yes miss
32.38	182 Teacher: “adding fractions with unequal denominators”
183	Learners: Yes
184	Teacher: <u>Siphinde sifumane more fractions andithi?</u> {We get more fractions, am I right?} Because <u>abanye abantu</u> {other people} ... most <u>abantu abakayibambi, so umntu azenzele umsebenzi wakhe.</u> {Most people have not grasped this so a person must do their own work}
185	Learners: Yes
186	Teacher: <u>Athini?</u> {They must do what?}
187	Learners: <u>Azenzele</u> {do their own work}
188	Teacher: <u>Azenzele umsebenzi wakhe azoku zibona ukuba undawoni ngoku.</u> {they must do their own work so they can see where they are} <u>Siphinde xa siyiyenza sizokwenza u number.....</u> {When we do this we will do number}.... <u>U number 3. Page 48, skill 7 sokwenza</u> {we will do} unumber 3.
189	Learners: Yes miss
190	Teacher: No number 1, number 3. Number 1, no number 3 no number 6, 1,3 6. no 10. 1,3 , 6, 10. 33.40 [Learners start discussing and doing the work, while teacher explains to other learners who do not fully understand how the work is done.]
191	Teacher: [Explaining to a group of learners] Come closer ... <u>xa si edisha</u> {when we add}... (cannot clearly make out what the teacher is saying) <u>lento siyenzayo sosuke sikhangele inani elingena ku 7.</u> {We will look for a number that goes into 7} angene no 3 kulo niyeva ke? {And 3 must also go into it, do you hear?} <u>Inani elitheni?</u> {a number that is?....} <u>elingena u 3 no 7 angene kulo kunga phumi nto igqitha phaya.</u> {a number that goes into 7 and 3 goes into it as well and there is no remainder} So if <u>siyajonga ke ngoku</u> {if we look now} <u>u 7, 14, 21. u 3 uyangena ku</u> {goes into} 21 <u>niyabona? Asikwazi ukwenza idenominator u7, ezi ku 3 niyeva?</u> Because u7 ... 3, 6, <u>niyabona u 7 akangeni.</u> {Do you see 7 does not go} <u>So sisebenzisa eziku</u> {we use} 7, 14, 21 ku {in} 21 <u>uyangena u7,</u> {7 can go into 9 it} <u>uyangena u 3</u> {3 can also go into it} so that means <u>yi</u> {is our} LCM <u>yethu ke leyo.</u> <u>Nokuba mayi</u> {even if there is} <u>two ukuba</u> ayangena omabini. {As long as both numbers can go in} Sometimes sikhe sifumaniseke into yokuba awangeni omabini xa iyi one {sometimes we find that when there is just one number not both number can go into that one number} [Teacher is trying to explain something to the learners. Inaudible] <u>sizo buza ukuba o3 ku 21 bangaphi?</u> {we will ask, how many times does 3 go into 21?} <u>Bayi 7 nhe?</u> {there are 7 right?} 36.26 <u>Uphinde uthi</u> {and again you say} 7 times 1. <u>ibengu</u> {it becomes} 7. <u>o7 ku</u> {into} 21 <u>bangaphi?</u> {How many are there?} <u>Bathathu</u> {three}. Times 5 andithi? {Am I right?} Then <u>sidibanise u7 sidibanise no</u> {we add 7 and} ... <u>ibengu</u> {it becomes} 22. <u>nalapha</u> {here} u7 no {and} 5 <u>sizokhangela inani</u> {we look for a number} ... <u>imultiple engena ku</u> {that goes into} 7 <u>engena nakuye u</u> {and will also go into} 5 <u>izakuba ngu</u> {it will be} 35. <u>u7 uyangena ku</u> {goes into} 35, no {and} 5 <u>uyangena ku</u> {goes into} 35, <u>so o7 ku 35 bangaphi?</u> {how many 7 into 35?} 38.23 <u>Bayi 5 simultiplaye ngale.</u> {we multiply by this} <u>O5 ku 35 bangaphi?</u> {how many 5 into 35?} <u>Bayi</u> {there are} 7. 7 times 2 <u>ibengu</u> {will be} 14. <u>udibanise u 15 kunye</u> no 14 <u>ufumane u</u> {you add 15 plus 14 and you get} 29 over 35. <u>So funeka uyazi into yokuba xa ndijonge pha utshekishe into yokuba ngeliphi inani omabini lamanani elinye lalo elingenayo.</u> {you must know that when you look there you must know which number goes into both numbers} 39.40 <u>La manani omabini elinganayo</u> {a number that goes for both numbers} <u>la manani angatshintshi</u> {the numbers must not change} <u>then eli nani uzoli sebenzisa ngalo.</u> {You will use this number}

S P3 L3 EE1: Addition of fractions

Time

Speech

Uku divider, once udivide singafani. {You divide once you divide and they are not the same}

SCHOOL P6

LESSON 1 EE1-EE3

S P6 L1 EE1: Recap of the rules for addition and subtraction of integers

- 1 Teacher: Okay let's start. This is a (cannot make out the word) in the rules of adding and subtracting integers. So let's quickly reflect what are those
2 rules. When adding one take the rule, when adding integers of the same signs what do you do kanene? {again} Yes bhuti {boy}, add like
3 normal and put the sign over. Huh? Same sign you add then you put the common sign e.g. let's start with the normal 4 plus 5 both are positive
4 so you add them which is?
5 Learners: 9
6 Teacher: Positive?
7 Learners: 9
8 Teacher: Minus 5 and minus 4, yes bhuti {boy}! Negative 9. Remember we add the common sign which is negative, minus 9 and then for different
9 signs...and then izandla ziphakanyisiwe, {put up your hands} yes? you subtract the smaller digit from the bigger digit okay? We
10 subtract the smaller digit from the bigger digit and put the sign of the bigger digit. Masenzeni i-example {Let's make an example} start from
11 what we know from past days. 5 minus 3, which is the bigger digit here?
12 Learners: 5
13 Teacher: What is the sign there? What is the sign of 5?
14 Learners: Positive
15 Teacher: Positive, so we subtract the smaller digit from the bigger digit and put the sign of the?
16 Learners: Bigger digit
17 Teacher: Yes, Ithini ibigger { what is your big digit} digit apha {here}?
18 Learners: 6
19 Teacher: Then you subtract the smaller digit. What is the sign of the bigger digit?
20 Learners: Positive
21 Teacher: Ngubani ianswer yalapha? {What is your answer here?}
22 Learner: 1
23 Teacher: Nantsiya, yes sisi? {Yes, Nantsiya?}
24 Learners: Minus 2
25 Teacher: Minus 2. Remember 7 is the biggest digit, subtract the smaller digit and put the sign of ...
26 Learners: Bigger digit.
27 Teacher: Okay ke, so pha sihamba sonke? {So, we all agree on it?}
28 Learners: Yes
29 Teacher: Ukhona umntu onombuzo? {Anyone else have questions?}

S P6 L1 EE1: Recap of the rules for addition and subtraction of integers

- 30 Learners: No.
- 31 Teacher: Okay. So now let's turn to page 30, page 30. let's look at unnumber 2 {number 2} there, actually explaining lanto sigqhibo
- 32 kuyiyenza apha kuthiwa pha {work that we have just done here, said there} let's look at them examples with adding integers. Minus 2 plus
- 33 minus 4 is equals to minus 6, so ngubani icommon {what's the common} sign there?
- 34 Learners: Minus.
- 35 Teacher: NguMinus {It is a minus}so of which you add them and put isign le common phakati kwazo, so u6 ngubani icommon sign? Ngu
- 36 negative. Ngubani ozokwenza u minus 1 plus negative 8? Isign ecommon pha? {Put a sign that's common in between the two. So, at 6 – what's the common sign? It's a negative. Who is going to do/write minus 1 plus negative 8? What sign is common here?}
- 37 Learners: Ngu {It is a} negative.
- 38 Teacher: So we add them 1 plus 8 gives us 9. Then the common sign is?
- 39 Learners: Negative
- 40 Teacher: Following one.
- 41 Learners: Minus 2 plus 4
- 42 Teacher: Plus positive 4. Which is the bigger digit pha?
- 43 Learners: 4
- 44 Teacher: and what's the sign of the bigger digit?
- 45 Learners: Positive
- 46 Teacher: So are we going to subtract the smaller digit which is?
- 47 Learners: 2... positive 2
- 48 Teacher: Hayi ngu minus 2 kaloku u2 yidigit. So ngubani isign ye answer yethu pha? {No it is minus 2 because 2 is a digit. So what is the sign of the answer here?}
- 49 Learners: Positive 2
- 50 Teacher: Okay, so ithini pha iexplanation ecaleni? Ithi {What is the explanation on the side? It says: }“when you add negative integers the answer will always be?”
- 51 Learners: Negative
- 52 Teacher: Niyabona lonto? {Do you all see?}
- 53 Learners: Yes
- 54 Teacher: Can you explain why? Khumbula pha kula number lining besiyiyenza? {Remember, that number on the number line that we were doing?} Let's
- 55 continue. Let's look at number P. Minus 1 plus 8 ngubani ianswer pha? {what is the answer there?}
- 56 Learners: Positive 7
- 57 Teacher: Positive 7, ngeyiphi ibigger integer pha? {which is the bigger integer there?}
- 58 Learners: 8
- 59 Teacher: Ngubani isign yayo? {What is the sign?}
- 60 Learners: Positive
- 61 Teacher: Positive. Let's look at the last 1. Ngeyiphi ibigger integer pha? { Which is the bigger integer there?}
- 62 Learners: Ngu 8 {It is 8}
- 63 Teacher: Ngubani isign yayo? {What is it's sign?}
- 64 Learners: Negative
- 65 Teacher: So ngubani ianswer kengoku pha? {So what is the answer there?}
- 66 Learners: Minus 7

S P6 L1 EE1: Recap of the rules for addition and subtraction of integers

- 67 Teacher: Si ithini iexplaination pha ecaleni? Ithi {What does the explanation there say?} “when you adding a negative and a positive integer ignore the
68 signs, find the difference between the number and the answer will have the sign of the larger number.” So basically bayibeke
69 ngenye indlela apha, niyayibona lonto? {They’ve put in a different way.} Now kengoku {then} what I want you to ngu
{is}. number 3 kuthiwa pha {which says} when we try to explain izinto ezi okharisha apha {things that occur here}

S P6 L1 EE2: Operations (addition and subtraction of integers)

- 70 around us we use models and theories to help us,
71 you can do the same in mathematics. One way to illustrate the addition and subtraction of integers is to use the poker-chip model.
72 This may help you to understand. Let’s look at that example kuthiwa pha {which says} for all those positive ones use an
73 open circle and then a solid circle represents a? Negative one. So kengoku lamzekelo bayiyenzile {the example they have made} for us. Jonga
74 pha sino {Look there we have} positive 4, how many circles represent a positive 4?
75 Learners: 4
76 Teacher: Then how many solid circles represent a negative 3?
77 Learners: 3
78 Teacher: Niyabona ukuba {You all see that} each and every case (cannot make out words) ,ingu {is} minus 4 plus 3 so niyabona ukuba {you see that}
79 every case ezi ziya perisha then kengoku le ishiyekileyo {those that pair up and what is left} is your answer. Then kengoku ngubani? {What is
it?}
80 Learners: Negative 1
81 Teacher: Negative 1, eziya ziyaperisha kushiyeke la {the rest pair up and this one is left} negative 1
82 Learners: Ngu {It’s} Positive
83 Teacher: Sithe pha... yho! Sorry ndilibeke, siyabona sonke? {We say ... I forgot, can we all see?}
84 Learners: Yes
85 Teacher: So siyabona ziyaperisha then le ishiyekileyo yi answer yakho and then xa iyi open circle ithetha ukuba ipositive. {So we see that if we pair up
and this one that is left is your answer and when it is an open circle it is positive.}
86 Now kengoku njengoba nihleli nobabini I want you to do A B and C using le method bayisebenzisileyo apha. {Now as you are seated, I want
you to do A,B and C using the method given}
87 To show le example yakho masitshoneni khona ke, 10 minutes. Faka isandla sakho nhe? Niyayibona phofu lento? {To show the example,
let’s jump right in, 10 minutes. I want your input. Can you guys see this?
88 So uzokwenza pha unnumber 3 A B C using la mizekelo niyini kiweyo pha. {You are to do Number 3 A, B, C using the prescribed method}
89 Teacher walking around the class checking on learners’ work.
90 Teacher: Wonke umntu uclear nge {Is everyone clear on} instructions?
91 Learners: Yes
92 Teacher: Phakamisa isandla ukuba awukho {Raise your hands if not} clear. Teacher explains to a learner who does not understand clearly.
93 Teacher: You have 10 minutes nhe {right}? Okay class, a quick question from the last one. Xa une {When you have} pairing what are you going to do?
94 From the last one. Xa une {When you have} co-pairing like ngolohlobo {like this} what are you going to do of the last one? Yes, bhuti {boy}?
95 Learner: Uzobhala {You will write} the amount of ezizinto zikhoyo {what you have }.
96 Teacher: Uzobhala {You will write } the amount of ezizinto zikhoyo, umzekelo pha { what you have for example }, how many solid circles for number C?

S P6 L1 EE2: Operations (addition and subtraction of integers)

- 97 Learners: 5
- 98 Teacher: 5?, So what's the answer there?
- 99 Learners: 12 5 (argument, some learners say 12 and others say 5)
- 100 Teacher: Negative 5, because there is no pairing pha uzobhala ezinto zipha. {You will write what there is.} So wonke umntu uclear kulondawo leyo? {So is everyone clear on this?}
- 101 Learners: Yes
- 102 Teacher: Lungisa umntu wakho bhuti, wonke umntu u-understandile nhe? {Correct yourself, everyone understands, right? Basically le exercise bendininika yona bendifuna iexplaine lento besithetha ngayo yokuba if isigns ziyafana like if idots ziyafana like lena ka minus3 plus minus 2 if isigns ziyafana you just add them, sihamba sonke? {Basically with the exercise I gave you, I wanted to explain what we spoke about, like if the signs are the same, like if dots are the same like the one 3 plus minus 2. If the signs are the same you just add them, We're clear? So these are all solid circles so we are going to add them and are going to give us?
- 103
- 104
- 105
- 106 Learners: Minus 5
- 107 Teacher: But xa kungena {if you place them together}, are any of them the same sign?
- 108 Learners: No
- 109 Teacher: So can we add them?
- 110 Learners: No
- 111 Teacher: No, funeka sithini? Sibhale le ine amount enintsi si cancelishe kweziya sibale le ishiyekileyo, sihamba sonke? {What should we do? We write the one with the most amount and cancel it from the remaining ones, we're clear?} So ishiyeka phi le ishiyekayo? {Where does the remainder go?}
- 112
- 113 Learners: Negative 1
- 114 Teacher: Ngubani ianswer yethu? {What is our answer?}
- 115 Learners: Negative 1
- 116 Teacher: Negative 1, Sihamba sonke? {We're still good?} Siphinde sifunde eza rules zethu zithini eza rules zethu? {Let's read all our rules again, what are they again?} When you add negative integers the answer will always be a negative. Sibonile mos apha because sisebenzise ntoni? {We saw here right? Because, what did we use?} A solid circle to represent a negative nazo zi negative zoyi {they are both negative} 2 and the answer is negative. Then there is a second explanation - When adding negative plus positive integers ignore the sign, le into besithetha ngayo {what we spoke about}. Ignore the sign and find the difference between the numbers, the answer will have the sign of the larger number or larger integer. Sibonile umzekelo apha this is minus 4 plus 3, then ngeyiphi idigit unkulu pha? {We saw this example here.... This is minus 4 plus 3, then which is the bigger digit?}
- 117
- 118
- 119
- 120
- 121
- 122 Learners: Ngu {It's} 4
- 123 Teacher: Ngu 4 so simayinase bani? {It's 4, so what do we minus?} u-3 ku 4 sobhala isign engubani? Engu negative. {We minus negative 3 from 4 and write which sign? Negative.} Nalena iyabonisa ukuba kushiyeke bani? {What is left?} inegative sihamba sonke? {Negative, right?}
- 124
- 125 Learners: Yes
- 126 Teacher: So wonke umntu uclear ngoku ngezi rules zethu? {Are you clear on the rules?}
- 127 Learners: Yes
- 128 Teacher: Uclear ngezi rules zethu? {So are we clear on our rules?}
- 129 Learners: Yes

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 130 Teacher: No, kengoku {then} let's talk about imultiplication {multiplication} as well as idivision {division} because the rules are more
131 or less the same. Multiplication and division of integers. Okay, from pha kwi {over the} past lessons zethu {all} we know that 4
132 times 5 ngubani ianswer yethu? {What is the answer?}
- 133 Learners: Ngu {It's} 20
- 134 Teacher: Ngu {It's} 20, so ngubani isign ka 4 apha? {What is 4's sign?}
- 135 Learners: Positive
- 136 Teacher: Positive, ngubani isign ka 5 pha? {What is 5's sign?}
- 137 Learners: Positive
- 138 Teacher: So ngubani ianswer ye {What is the answer?} answer? Isign ye answer yethu? {And our answer's sign?}
- 139 Learners: Ngu {It's} positive
- 140 Teacher: Ngu {It's} positive, now kengoku {then} what happens ukubana uno {if you have} minus 4 times minus 5? Are we going to have
141 the same thing?
- 142 Learners: No!!! ... Yes!!!
- 143 Teacher: Bangaphi abathi {How many say} yes? Negative answer? Oh, unfortunately no.
- 144 Learners: Hayi! {No!}
- 145 Teacher: Mamela ke {Listen}, when we have a negative multiplied by a negative the answer is a ...? Positive. Masiphinde 375ense
146 Omnnye. {Let's do another.} Izakuba ngubani ianswer apha? {What'll the answer be?} So both zinegative ianswer izakuba ngubani? {So if both
are negative, what will our answer be? }
- 147 Learner: Minus
- 148 Learners: Hayi {No} positive
- 149 Teacher: So ngamanye amazwi {in other words} it's a rule, ithi irule yethu {our rule says} , when you multiply a negative multiple by negative we get a
150 positive. When you multiply a positive by a positive you get a?
- 151 Learners: Negative
- 152 Learners: Positive
- 153 Teacher: Now kwenzeka ntoni xa umultiplier inegative ne positive? {What happens when you multiply a negative with a positive?}
- 154 Learner: You get a positive
- 155 Learners: Positive
- 156 Teacher: Bangaphi abathi {How many say} positive? Nge {With} show of hands, bangaphi abathi {how many say} negative? Sofumana {We'll get}
negative
- 157 Learners: Oh!
- 158 Teacher: Negative 18. Nalena {This one too} it's another rule. Masenzeni iother {Let's do} example. Is this a negative or positive?
- 159 Learners: Negative ... Positive (other learner say negative and others say positive)
- 160 Teacher: Bangaphi abathi {How many say} positive? By show of hands. Abathi {Who says} negative? Okay khawume ndini trepe kengoku. [Let me
161 catch you out now] Ukhona umehluko phakathi kwalento, idifference nalento? {Is there a difference between this and that?}
- 162 Learners: No... Yes
- 163 Teacher: Ngubani ianswer yalapha? {What is this answer?}
- 164 Learners: 18... 24
- 165 Teacher: Mandisuse ezisigns. Ngubani ianswer yapha? {Let me remove the signs. What is the answer?}
- 166 Learners: Ngu 24 {It's}
- 167 Teacher: Ngubani ianswer yalapha? {What is that answer?}
- 168 Learners: 4

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 169 Teacher: 4, now masitshekeni le ngubani ianswer yale iphezulu? {Let's check what the answer above is.}
- 170 Learner: Negative 24
- 171 Teacher: Le isezantsi? {And the one below?}
- 172 Learner: Negative 24
- 173 Teacher: Negative 24 ngamanye amazwi {in other words} it does not matter where the negative is as long as you are multiplying the positive by negative, you get a negative as well as negative multiply by a positive you get a negative. Where as another rule which
- 174 is if a positive multiply by a negative a? negative or a negative multiply by a positive gives us a?
- 175
- 176 Learners: Negative
- 177 Teacher: Negative. Masiphinde siwele back singazukushiya abantu {Let's go back and not leave people behind}, a positive multiply by a positive gives us what?
- 178 Learners: Positive
- 179 Teacher: Sonke siyayazi lonto nhe? {We all know that, right?}
- 180 Learners: Yes
- 181 Teacher: Entsha yeyiphi {What is the new one}? Yile ye {It's the one we} multiply by negative. So a negative times negative gives us a what?
- 182 Learners: Positive
- 183 Teacher: Positive, sihamba sonke? {Are we all clear?}
- 184 Learners: Yes
- 185 Teacher: So when the signs are the same you get a positive answer uyabona {you see}? When the signs are the same you get a positive answer,
- 186 but when the signs are not the same you get a?
- 187 Learners: Negative
- 188 Teacher: Negative answer, but now kwenzeka ntoni apha xa kunjena {What happens here when this happens}?
- 189 Learners: Negative ... Positive
- 190 Teacher: Bangaphi abantu abathi {How many people say} negative? By show of hands. Bangaphi abathi {How many say} neg? Oh bantu
- 191 {people} remember kaloku {that} a positive times a positive gives us a what?
- 192 Learners: Negative ... Positive
- 193 Teacher: A positive times a positive gives us a what?
- 194 Learners: Positive
- 195 Teacher: Positive times a negative which is a?
- 196 Learners: Negative
- 197 Teacher: Negative, okanye inje lento {or is it?}. Positive times a negative?
- 198 Learners: Negative
- 200 Teacher: Negative times positive?
- 201 Learner: Negative
- 202 Teacher: Niyayibonani lento {Do you all see it}?
- 203 Learners: Yes
- 204 Teacher: So when the signs are the same that is very important. Negative negative or positive you get a?
- 205 Learners: Positive
- 206 Teacher: But when the sign are different when you are multiplying you get a ?
- 207 Learners: Negative
- 208 Teacher: Negative, sihamba sonke {are we clear}? Now what about idivision {division}? Yes.

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 209 Learner: 5
- 210 Teacher: Ngu 5 otheni {What kind of 5}? Positive or negative?
- 211 Learners: Positive
- 212 Teacher: Positive, okay but kengoku {then} what about unegative {negative} 20 divided by negative 4
- 213 Learner: Positive 5
- 214 Teacher: And negative divide by negative gives you a ?
- 215 Learner: Positive
- 216 Teacher: Nalapha {Here too} the very same you are given. The signs are the same you get a
- 217 Learners: Positive
- 218 Teacher: Positive answer. Ngubani {Who} who is going to help me xakunje {in this situation}?
- 219 Learners: Negative
- 220 Teacher: Xa kunjena {And in this situation}?
- 221 Learners: Negative
- 222 Teacher: So irules ze division ne {the rules of division and } multiplication are the same, Sihamba sonke? {Are we still following}
- 223 Learners: Yes
- 224 Teacher: Are the same, so when we are dividing alike sign we get a?
- 225 Learners: Negative
- 226 Teacher: iSign zethu {Our sigs} are not the same so we get a ?
- 227 Learners: Negative
- 228 Teacher: But when isigns zethu zifana {our signs are the same} we get a?
- 229 Learners: Positive
- 230 Teacher: Do I need to write that down okanye {or}?
- 231 Learners: No! ... Yes!
- 232 Teacher: do I write it down for your notes?
- 233 Learners: No! ... Yes!
- 234 Teacher: Mandiyibhale, then kengoku umntu azakwazi ukuzifunda xa elibele. {Let me write it all down so that, should you forget, you can re-read.}
- 235 Learner: Bhala {Write} exercise
- 236 Teacher: Oh nifuna i-exercise? {You want an exercise?}
- 237 Learners: No! ... Yes!
- 238 Teacher: Okay let's do an exercise
- 239 Learners: Yes ... No!
- 240 Teacher: Before senze le exercise remember mos sasithe ibrackets, what did ibrackets {we spoke about the brackets}, when we see ibrackets
- 241 {brackets} what do we do? Okay umzekelo {an example} what is the answer there?
- 242 Learner: 20
- 243 Teacher: So ibrackets ziminisha {brackets mean} multiply
- 244 Learners: Multiply by
- 245 Teacher: So let's look at the following example
- 246 Learners: Negative 12 minus positive 12
- 247 Teacher: Nisakhumbula u-BODMAS wenu? {Do you still remember your BODMAS?}
- 248 Learners: Yes
- 249 Teacher: Wenza ntoni na pha {What do you do here?}

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 250 Learners: Brackets of division
251 Teacher: Yimani, yimani, yimani, yimani, sijonge le. Sinantoni apha? {Wait, wait, wait, look at this}
252 Learners: Sine {We have} brackets
253 Teacher: Nantoni enye? {Anythng else?}
254 Learners: Of subtraction
255 Teacher: There is a difference between lento, nalento {this and that one}. There is a sign before ibrackets {brackets}, there is a number before ibracket
256 {bracket}. So apha lento nalento are {here this and that one} not the same. Sihamba sonke? {Do we all agree?}
257 Learners: Yes
258 Teacher: So we have a bracket and a subtraction sign so athini u-BODMAS wethu {what does BODMAS say}? Soqala ngantoni {What are we going to start with?}
259 Learners: Nge bracket {With a bracket.}
260 Teacher: Nge bracket, so sizakuthini? Mamela ke there is a 1 pha so ngubani elanani {With a bracket. What are we going to do? Listen then, there is a one there, so what is that number?}? Ngu {It is a} negative multiply by negative 3 gives us?

261 Learners: 4
262 Teacher: Gives us?
263 Learners: 4
264 Teacher: Mamela {Listen} there is a bracket sign
265 Learner: Negative 3
266 Teacher: Negative times negative?
267 Learners: Positive
268 Teacher: So ngubani ianswer apha {What's the answer here}?
269 Learners: Positive 1
270 Teacher: 4 plus ? Ndinisihyile {Have I lost you guys}?
271 Learners: Yes
272 Teacher: Mamela, kaloku ndithi masiqaleni apha sithi ngubani ianswer pha? {Listen, I'm saying let's all start here, what should we say the answer is here?}
273 Learners: Minus 20 minus positive
274 Teacher: Positive what?
275 Learners: 20
276 Teacher: Apha {Here}?
277 Learners: Negative
278 Teacher: Huh?
279 Learners: Negative 20
280 Teacher: Now kengoku ndithi apha ngubani ianswer kengoku {what should I say the answer is here}?
281 Learners: Positive
282 Teacher: Positive bani {what}?
283 Learners: 2
284 Teacher: Niyayibona {Do we all understand it?}
285 Learners: Yes

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 286 Teacher: Kutheni ingu positive pha {Why is it positive there?}? There is a 1 phayana esingambhaliyo {there that we did not write}, before la {that} bracket so it's negative 1 times negative 2 equals positive 2
- 287 Learners: Positive 2
- 288 Teacher: Niyayibona lento? Senze omnye umzekelo? {Should we make another example?}
- 289 Learners: Yes
- 290 Teacher: What about lena {this one}? Ngubani ianswer pha {What is your answer there?}
- 291 Learners: Negative 2
- 292 Teacher: Because there is a positive pha {there}, remember when there is a negative times a positive?
- 293 Learners: Negative
- 294 Teacher: Sibuyele kengoku kulo, ngubani ianswer apha? {So now we're going back to it. What is the answer here?}
- 295 Learner: Ngu {It's} positive 12
- 296 Teacher: Kukho ntoni apha? Brackets kubekho isubtraction sign. {Then the subtraction sign.} So ndizoqala ngezphi? {Which one of those am I going to start with?}
- 297 Learners: Brackets
- 298 Teacher: Brackets, negative times negative?
- 299 Learners: Positive
- 300 Teacher: Positive what?
- 301 Learners: 3
- 302 Teacher: now 4 plus 3?
- 303 Learners: 7
- 304 Teacher: iyazama ukubhadla kancinci? {Now you are starting to understand.}
- 305 Learners: Ewe {Yes}
- 306 Teacher: Yhe? {Yes?}
- 307 Learners: Yes
- 308 Teacher: Ngubani kengoku ianswer pha? Uzoqala ngantoni qala. {What's the answer here? Where will you begin?}
- 309 Learners: Brackets
- 310 Teacher: So positive times la {that} negative
- 311 Learners: Negative
- 312 Teacher: Lena yona sizoyithoba yona {That one we will leave}, it's not part of the ?
- 313 Learners: Multiplication
- 314 Teacher: Multiplication, so we are going to go with it as it is. So negative 5 minus 2?
- 315 Learners: Positive 7
- 316 Teacher: Huh? Negative 5 minus 2?
- 317 Learners: Negative 7
- 318 Learner: But isign ziyafana nje {the signs are the same though?}
- 319 Teacher: Mamelani, mamelani, mamelani, mamelani, besitheni? {Listen, listen, listen, listen, what did we say? Iphelile iperiod. {The period is over.}}
- 320 Apha ziyi 2 ioperations zethu ezikhoyo apha, if ujongile lento, zezphi neziphi {Here there are two operations that are present, here if you look at this, which one is this?}
- 321 Learners: Brackets ... Multiplication

S P6 L1 EE3: Recap of the rules for multiplication and division of integers

- 322 Teacher: Multiplication plus?
323 Learners: Subtraction
324 Teacher: But there is a negative sign before ibackets {brackets} . We have to multiply that side sihamba sonke {do we all agree}?
325 Learners: Yes
326 Teacher: So negative plus negative?
327 Learners: Positive
328 Teacher: Positive, itshintshile ngoku {it changed now.} Niwujongile lamzekelo beningawo pha {Did you all look at the examples that were given here}?
329 Learners: Yes
330 Teacher: Le injena nifumene bani ianswer pha? {Did you all get an answer like this?} Ibengubani ianswer? {What was the answer?}
331 Learners: Negative 5
332 Teacher: Negative 5, kwenzeke {the same thing happened} the same thing. Positive times negative?
333 Learners: Negative
334 Teacher: Equals negative 5, sihamba sonke {do we all understand}?
335 Learners: Yes
336 Teacher: Ndizakuthumela le exercise, isithathe iperiod yonke {I am going to give you this exercise. It will take the whole period.}. So kengoku
337 ndizonibona ngomso kengoku nhe {So then I will see you all tomorrow, right}? So I shall see you tomorrow.

SCHOOL P6

LESSON 2 EE1-EE2

S P6 L2 EE1: Recap of the rules for multiplying and dividing integers

- 1 Teacher: Positive times positive equals?
2 Learners: Negative
3 Teacher: Negative times negative equals?
4 Learners: Positive
5 Teacher: Remember if the signs are positive, we get a positive sign, sihamba sonke? {are we altogether?}
6 Learners: Yes
7 Teacher: Positive times negative equals?
8 Learners: Negative
9 Teacher: Negative times positive equals?
10 Learners: Negative
11 Teacher: Remember sithe {we said} if the signs are not the same we get a?
12 Learners: Negative answer
13 Teacher: That is for multiplication. Than for i-division {division} sithe {we said} positive divide by positive equals?
14 Learners: Positive
15 Teacher: Negative divide by negative?
16 Learners: Positive
17 Teacher: Remember sithe {we said} if the signs are the same kwi-division {in division} it's a positive. A negative divide by positive?

S P6 L2 EE1: Recap of the rules for multiplying and dividing integers

- 18 Learners: Negative
19 Teacher: Than positive divide by negative?
20 Learners: Negative
21 Teacher: Remember mos division you can also write it as division over division that form. Are we all familiar with this form?
22 Learners: Yes
23 Teacher: Should I recap on this form? Are you all familiar with this form?
24 Learners: Yes

S P6 L2 EE2: Integer arithmetic

- 25 Teacher: Lets do an example. Hundred over twenty-five [writing on the board], so what's the answer?
26 Learners: Four
27 Teacher: So its going to be positive. Remember, you may also use the same format for Negative signs, so negative divide by divide equals positive,
28 so minus sixty divide by minus fifteen, whats the answer?
29 Learners: Negative four....positive four
30 Teacher: Remember when there are same signs you get a ...?
31 Learners: Positive answer
32 Teacher: Positive answer. Than we can also use this form...negative twenty-four divide by three, whats the answer? Minus twenty-four divide
33 by three? Whats the answer?
34 Learners: Negative eight
35 Teacher: Negative eight. Than the last one, positive divide by negative which would be eighteen divide by negative three, whats the answer there?
36 Eighteen divide by negative three, whats the answer?
37 Learners: Negative six
38 Teacher: Negative six. So we all familiar with this? {pointing to the sums on the board}
39 Learners: Yes
40 Teacher: Ok. Now before we attempt that last problem we did yesterday, I want us to do this exercise, there is an exercise sheet in front of you,
41 look at this side the one that has exercise twelve written on it, uyayibona? {can you see it?}
42 Learners: Yes
43 Teacher: Kuthiwa {they say} try and work out as quickly as you can on this exercise, it is important to be able to work quickly and accurately
44 in Maths, siyayiva lo nto? {Do you hear that?}. Kengoku { Now } we going to spend fifteen minutes, inintsi gqithi kodwa {although it
45 is a lot} lets do from number one to twenty-two, masikhawulesiseni {let us hurry up}. We using these formats
46 Learners are busy doing the exercise.
47 Teacher: You don't have to copy this down, you can answer apha kuyo {on the worksheet} and you write your name on this sheet and keep it in a
48 safe place because we are going to be using this. Learners continue with their work on the worksheet.
49 Teacher: If you do not know, raise your hands and I will come and assist you.
50 Teacher walking around the classroom checking on the learners if they are doing the sums correctly.
51 Teacher: And remember rule number one, kuthwa {they say} use the rules. You don't have to copy it down, write on the worksheet, save time
52 {talking to one of the learners}. Niyagqiba phofu? {are you finishing?} and also ningazi libali i-rules ebesithethe ngazo nhe?
53 {don't forget about the rules we have talked about, ok?} Learner calling the teacher, showing something, asking something referring to
54 the worksheet.
55 Teacher: Ok! Class mamelani {listen up} pha ku-number nine {number nine says} they want us to..... { rest of the sentence cannot make sense}

56 Ndiyiphinde? {should I repeat that?}. Niyivile lo nto? { did you hear that?}. When we are adding to a positive number that number is going
 57 to increase, but xa u-addisha kwi-positive number than i-answer yakho iya-decreasa {when you are adding to a positive number than your
 58 answer will decrease} that means u-addisha what? {what are you adding?}. What did you add? A negative number Siyahamba sonke?
 59 {are we altogether?} Teacher walking around the classroom helping the learners
 60 Teacher: Be very careful of you sign. Kuyavukwa man! { you must wake up! Telling the late comers who have just entered the classroom}.
 61 Akukho stulo? {is there no chair?}, yizo hlala apha {come sit here} Teacher going around helping the learners.
 62 Teacher: Kushiyeke five minutes ngoku { five minutes left, looking at his watch}. Teacher helping a learner but cannot make sense of what
 63 he is saying
 64 Teacher: Uggibile? { have you finished?} (Asking one of the learners.) Bangaphi abangakagqibi? {How many of you have not finished yet?}
 65 Kawulezisani! {Hurry up!}
 66 Teacher: Oho xa ubhala i-answer yakho yi-checke kakhona ukuba yi-final answer. {Everytime you write your answer check it, check your final
 67 answer}. Ngobani abangakagqibi? {Who has not finished yet?} [A few learners raise their hands]. Teacher writing corrections on the
 68 board
 69 Teacher: Ukhona umntu osabhalayo? {Anyone still writing?} Ok! Umntu aphakamise isandla sakhe {Raise your hand if you have an answer}.
 70 Minus eight plus minus two? Yes bhuti? {yes boy?}
 71 Learner: Minus ten
 72 Teacher: La minus ten simfumene njani? {How did we get that minus ten?} Let's explain this part. Remember sithe {we said} when you see that
 73 bracket that means what? It means times out and then you have a sign before the bracket, so what we do usually, when we see
 74 a sign before the bracket we multiply the sign inside the bracket nhe? {right?} so what is positive times negative?
 75 Learners: Negative
 76 Teacher: Negative, so its exactly the same as minus two which is?
 77 Learners: Minus ten
 78 Teacher: Minus ten. Mandiphinde ndithi { let me repeat by saying} when we see brackets that means what? Multiplication, but when there is a sign
 79 before a bracket we need to multiply whatever is inside the bracket by that side before we can add, you must always know its
 80 bracket first than addition. Sihamba sonke? { are we altogether?}
 81 Learners: Yes
 82 Teacher: Negative times negative?
 83 Learners: Positive
 84 Teacher: Positive eight times two?
 85 Learners: Sixteen
 86 Teacher: Sixteen. Sihamba sonke? {Are we altogether?} Elandelayo! {the following sum}. Nalapha {and here also} there is a bracket, can we add
 87 if there is a bracket first?
 88 Learners: No
 89 Teacher: So we have to?
 90 Learners: Multiply
 91 Teacher: Multiply, start with a bracket. Masiyeni ke {lets go}. Negative times a negative?
 92 Learners: Positive
 93 Teacher: Positive, so its minus eight plus two, which is?
 94 Learners: Answer but cannot make sense.
 95 Teacher: Ok! Before.....aba bathi minus ten {those who say minus ten} are these two digits have the sane signs?
 96 Learners: No

97 Teacher: No. There is a negative and a positive, so sithini xa i-signs zingafani? { what do we do when the signs are not the same?} What do we do?

98 We subtract the smaller one from the bigger one, eight is the bigger digit so we going to minus two from eight, which will give us?

99 Learners: Six

100 Teacher: Six. Ngubani i-sign yethu? { what is the sign?}, its negative six, sihamba sonke? { are we altogether?} When we have different signs look

101 at your digits, if your digits don't have the same signs we subtract the smaller one from the bigger one and write the sign of the

102 bigger digit. Masigqitheni kengoku { lets move on to the next sum}. A negative divide by a negative?

103 Learners: Positive

104 Teacher: Positive. Eight divide by two?

105 Learners: Four

106 Teacher: Than size kwakhona kwi-bracket {than we come to a bracket again} Yintoni ekufuneke siyenzile xa sibona i-bracket? { what must we do

107 when we see a bracket?}

108 Learners: Multiply

109 Teacher: Multiply. So negative times a positive?

110 Learners: Negative

111 Teacher: Minus ten minus two?

112 Learners: Eight

113 Teacher: Minus ten minus two?

114 Learners: Minus eight

115 Teacher: The signs are the same, senza ntoni xa i-signs are the same? { what do we do when the signs are the same?}

116 Learners: Positive

117 Teacher: Huh?! {What?!} Minus ten minus two, the signs are the same senza ntoni? {what do we do?} xa i-digits zine signs ezifanayo? {when the

118 digits have the same sign?}

119 Learners: Add

120 Teacher: We add, what is the common sign here?

121 Learners: Minus

122 Teacher: Minus ten minus two

123 Learners: Minus eight

124 Teacher: Huh?! Minus twelve. Mamelani kalok guys { listen up guys} you have to read those books because we have done this on Friday

125 and yesterday and the day before that, now a negative times a positive?

126 Learners: A negative

127 Teacher: A negative two times ten?

128 Learners: Twenty

129 Teacher: Huh?

130 Learners: Twenty

131 Teacher: There is a bracket, what do I need to do first?

132 Learner: Multiply

133 Teacher: Multiply. A negative times a negative?

134 Learners: Positive

135 Teacher: Ngubani kengoku i-answer in this case? {what is an answer?}

136 Learner: Minus eight

137 Teacher: Huh?!

138 Learners: Minus eight

139 Teacher: Ndicela umntu kengoku asi-explainele simfumene njani u-minus eight. { can someone please explain to us how did we get minus eight?}

140 Learners: Azifani eza-signs { the signs are not the same} so which means izakuba ngu-negative {which means it will be negative}, so ten minus two, ngu-eight {gives us eight}

141

142 Teacher: Niyamva? {Do you hear him?} Eyakhe i-explanation ithi {his explanation is} sine {we have} different signs and that digit is bigger so

143 i-answer yakhe izakuba ngubani? {so what will the answer be?} Negative so uzakuthini? {what you going to do?} ten minus two which is?

144 Learners: Negative

145 Teacher: Negative, remember kalok sithe {we said} if there is no sign esiyibhalileyo {written} in front of a digit, that's what?

146 Learners: Positive

147 Teacher: That's a positive. So theres a negative and a positive?

148 Learners: Negative

149 Teacher: Ten divide by two?

150 Learners: Five

151 Teacher: Negative five. Now lets do this one. Twenty plus something to give us twelve. Its negative eight which gives us twelve.

152 {Teacher explaining the sum but cannot make sense} This one is obvious, ngubani pha? {What is the answe?}

153 Learners: Six

154 Teacher: Six....positive six.

155 Teacher: What about this one? {pointing to the board}, eight minus something to get ten?

156 Learners: Minus two

157 Teacher: Minus two. Why minus two? A minus times minus?

158 Learner: Positive

159 Teacher: So its eight plus two which gives us ten . Now that's something interesting. {teacher explaining but cannot make sense}. Sihamba sonke?

160 Learners: Yes

161 Teacher: Sihamba sonke? {Do we understand?}

162 Learners: Yes

163 Teacher: Are we together so far?

164 Learners: Yes

165 Teacher: Then this one, what is the answer?

166 Learners: Minus four

167 Teacher: Size kwelandelayo {The next one} sixty seven plus a number gives me fifty. What is the sign there?

169 Learners: Negative

170 Teacher: Negative. So its negative seventeen to give fifty. Sihamba sonke? { are we altogether?}

171 Learners: Yes

172 Teacher: Next one, twenty one plus something to give us fourteen?

173 Negative seven, so twenty one mi nus seven gives us tenty-five?

174 Learners: Negative five

175 Teacher: Minus times a minus?

176 Learners: Positive

177 Teacher: Twenty plus five gives us tenty-five. Size ku-number sixteen {we than go to number sixteen}. Minus eight times a number to give us

178 sixteen?

179 Learners: It's a negative two
 180 Teacher: Negative eight times negative two gives us sixteen. Remember kaloku sithe {we said} to get a positive answer the signs must be the same,
 181 that is very very important to get a positive answer, the signs must be the same. It can be positive-positive or it can be negative-negative
 182 Teacher: Negative what times four to give us negative four? Teacher doing the next problem.
 183 Learners: Negative one
 184 Teacher: Negative times a positive?
 185 Learners: Negative
 186 Teacher: One times four?
 187 Learners: Four
 188 Teacher: Now in future when we doing i-exercise zethu {our exercises} I want to see this [pointing to the signs on the board] because
 189 zi-tools zethu, umzekelo {they are our tools} [For example, opening one of the learners exercise book showing to the class how he wants
 190 it to be]. When ever you going to do your calculations I want to see your tools on this side [left hand side] and than your calculations on
 191 this side [right hand side]. Why ndifuna nibe nazo ezi-tools {why do I want you to have these tools} I want you to remember these
 signs, this is very important [pointing to the signs on the board] Sihamba sonke class? {Are we altogether class?}
 193 Learners: Yes
 End of class.

SCHOOL P6

LESSON 3 EE1-EE6

PERIOD 1

S P6 L3 EE1: Recap of the rules for multiplying and dividing integers

1 Teacher: Akukho mntu {Doesn't anyone have a} one stapler apha {here?} Stapler? Okay. Uyabona kengoku kudala liphelile ixesha le lunch
 2 ngoku. {Can you see that the lunch break time is finished?} Ngekabani le? {Whose is this?} Ngubani ongena phepha? {Who does not
 have paper?
 3 Learner: Ndicela abemabini tishala. {Please, can I have two, teacher?}
 4 Teacher: Okay, before sigqhibezele eziya apha, {Before we finish up} let's quickly recap. Sisebusy {Are we still busy} with multiplication ne
 5 {and} division nhe {right}? sathi ke {as I was saying a} positive multiply by positive equals a what?
 6 Learners: Positive.
 7 Teacher: Negative times a negative?
 8 Learners: Positive.
 9 Teacher: Positive times a negative?
 10 Learners: Negative.
 11 Teacher: Negative times a positive?
 12 Learners: Negative.
 13 Teacher: Division. Positive divide by positive?
 14 Learners: Positive.
 15 Teacher: Negative divide by negative?

PERIOD 1

S P6 L3 EE1: Recap of the rules for multiplying and dividing integers

- 16 Learners: Positive.
17 Teacher: Negative divide by positive?
18 Learners: Negative.
19 Teacher: He?
20 Learners: Negative.
21 Teacher: Positive divide by negative?
22 Learners: Negative.
23 Teacher: Now kengoku masigqhibezele la exercise yethu. {Let's finish off our exercise.} Sukuyihoya le in front of you now, sukuyihoya
24 ndizokuxelella ngoku ukuba ngeya nini. { Don't worry with what you have in front of you now. Don't bother about it. I'll tell you
now when it's due for.} Ngubani le number ke? {What number is this?}
- 25 Learners: 19
26 Teacher: 19, okay. 19 minus 3 times negative 3. Minus time a minus? Times 2?
27 Learners: Positive.
28 Teacher: 3 times 3?
29 Learners: 9
30 Teacher: 9, twenty. Times 8 equals negative 8. Now let's look at that one. 8 is positive but the answer is negative, so pha {here} what is,
31 ngubani isign yela nani? { What is the sign of that number?} What is the sign of that number?
32 Learners: Negative
33 Teacher: So ngubani elanani? {So what was that number?}
34 Learners: Ngu {It is} 1
35 Teacher: Ngu {It is} 1. Negative 1 times 8?
36 Learners: 8
37 Teacher: Negative times positive?
38 Learners: Negative
39 Teacher: 1 times 8?
40 Learners: 8
41 Teacher: Next one. Divided by positive 2 and you get a negative answer, what is that number?
42 Learners: Negative 4
43 Teacher: When divided by positive 2 gives us a negative 4? Yes sisi {student}.
44 Learner: Negative 8
45 Teacher: Negative 8, Remember negative divide by positive gives us a what?
46 Learners: Negative
47 Teacher: So 8 divide by 2?
48 Learner: 4
49 Teacher: Last one. Let's look at this one. What number must be multiplied by negative 2 then multiply by negative 2 we get negative 20?
50 Learners: Positive 4 tishala, {teacher} positive 5
51 Teacher: Bathi {Some say} positive 5, abanye? abanye? {Anyone else?} Remember kaloku negative times negative? {Remember that a
negative times a negative is?}

PERIOD 1

S P6 L3 EE1: Recap of the rules for multiplying and dividing integers

- 53 Learners: Positive.
- 54 Teacher: So what, ngubani isign yeli nani? {What is the sign of this number?}
- 55 Learners: Positive.
- 56 Teacher: It can't be positive because that one ithini? {What is it?}
- 57 Learners: Negative.
- 58 Teacher: Negative, so what is the answer of that number?
- 59 Learners: Negative.
- 60 Teacher: Negative 5, remember kaloku negative times negative? {Remember that a negative times a negative is?}
- 61 Learners: Positive.
- 62 Teacher: Positive, but ianswer yethu injani? {But how does your answer look?}
- 63 Learners: Negative.
- 64 Teacher: So funeka isign yalapha ibenjani? {So what should the sign be here?}
- 65 Learners: Negative.
- 66 Teacher: Ibe {Let it be} negative. Negative?
- 67 Learners: Negative 5.
- 68 Learner: But zizoba negative zonke njani? {But how is it all going to be negative?}
- 69 Teacher: Yima ke {Let's wait} let's check it together. Negative times negative?
- 70 Learners: Positive
- 71 Teacher: Positive, that means ukuba eziya zoyi two zibe positive {if it becomes positive} that positive multiply by a negative gives us a
- 72 negative.
- 73 Learners: Negative
- 74 Teacher: Sihamba sonke? {We agreed} You don't only look at this, also look at that one nayo {also}. Remember kaloku sithe {we said that} a
- 75 positive times a negative times a positive gives a? Positive times a negative?
- 76 Learners: Negative
- 77 Teacher: Negative, ezi ziyi {these are} two. Negative times positive?
- 78 Learners: Negative
- 79 Teacher: Negative, siyabona? {Do we all see?}
- 80 Learners: Yes
- 81 Teacher: Negative times a positive times a negative? Negative times a positive?
- 82 Learners: Negative
- 83 Teacher: Negative. Negative times a negative?
- 84 Learners: Positive
- 85 Teacher: Positive, siyayibona lonto? Siyayibona lonto? {Do we all see?}
- 86 Learners: Yes
- 87 Teacher: Masijonge enye. {Let's look at another one.} Positive times a negative times a positive? Negative times negative times negative.
- 88 Negative times negative?
- 89 Learners: Positive
- 90 Teacher: Positive times negative?
- 91 Learners: Negative

PERIOD 1

S P6 L3 EE1: Recap of the rules for multiplying and dividing integers

- 92 Teacher: Negative, sihamba sonke? {Do we all understand it?}
93 Learners: Yes
94 Teacher: No, ukhona onombuzo kwezi sum zoyi 22? {Anyone have questions from these 22 sums?}
95 Learners: No
96 Teacher: Sonke sihambani kunye? {Do we all get it?} Look at the one that you think gave you a problem then we can quickly settle it out.
97 Let's put this aside sizophinda sibuyele kuyo {we'll get back to it} quickly. Now in front of you, you have a new sheet of paper.

PERIOD 1

S P6 L3 EE2: Order of operations

- 98 Teacher: Ngubani ongena phepha? {Who doesn't have paper here?} Ukhona? {Anyone?} So in front of you there is a ... shhhhh.... piece of
99 paper. So I want all of us to check kulendawo ibhalwe {look at where it is written} order of operations, order of operations. Can we all see that?
100 Learners: Yes
101 Teacher: Okay, wonke umntu? {everyone}
102 Learners: Yes
103 Teacher: Now kengoku {then} let's read that so that sizo attempter ezasums ziku {we can all attempt these sums of} 5 B.
104 Learner: When a ...
105 Teacher: Yimani, yimani, yimani, yivekeni ndizofunda ngokwam {Wait, wait, I'll read it myself}. Nantsiya, {So there it is} when a calculating
106 instruction, that is i-instruction {instruction}, a calculating instruction includes more than one operation. You need to use the
107 following rules to calculate the answer. Nantsi irule yokuqala, went from left to right. do multiplication and division
108 before addition and subtraction. You may find it helpful to put in brackets to show what to do first sihamba sonke? {Are we following?}
109 Learners: Yes.
110 Teacher: Example efresh nantsiya {Another example}. 12 minus 3 plus 4. now let's look at those operations, how many operations are we
111 having there?
112 Learners: Yes
113 Teacher: Iyintoni i-operation phofu? {What is the operation actually?} Give me an example of an operation.
114 Learner: Subtraction.
115 Teacher: Subtraction.
116 Learners: Division.
117 Teacher: Division.
118 Learners: Multiplication.
119 Teacher: Multiplication.
120 Learners: Addition.
121 Teacher: Addition, okay so how many operations are we having here?
122 Learners: 2
123 Teacher: It's subtraction plus?

PERIOD 1

S P6 L3 EE2: Order of operations

- 124 Learners: Addition.
- 125 Teacher: Now kengoku kuthiwani pha? {Then what is said there?} Kuthiwa {It says} let's work through the left going to the right.
- 126 Learners: Right
- 127 Teacher: Now 12 minus 3?
- 128 Learners: 9
- 129 Teacher: 9 plus 4?
- 130 Learners: 13
- 131 Teacher: Sihamba sonke? {Do we all agree?} So that was the first rule, ithini kengoku icomment? {So what does the comment say?} It's only a
- 132 positive and a subtraction so went from left to right. lento sigqhibo yiyenza {So what we have just done}, right. Masiyenzeni kengoku
- 133 {let's go to} the following example. 15 plus 5 divide by 3. Now let's look at that. How many operations are we having there?
- 134 Learners: 2
- 135 Teacher: Let's name them
- 136 Learners: Division
- 137 Teacher: Division and ?
- 138 Learners: Addition
- 139 Teacher: Now kengoku kuthiweni phayana {The what did we say there?} I yethu yokuqala {Our first one} went from the left to the right.
- 140 Can you use that one apha {here}?
- 141 Learners: No.
- 142 Teacher: No, because the following statement ithini? {says what?} Ithi {it says} do multiplication and division before addition and subtraction
- 143 Learners: Subtract...
- 144 Teacher: So soqala ngantoni pha {So what are we going to start with here?} in this case?
- 145 Learners: Division
- 146 Teacher: Nge division seziphi? { With which division?} Ngeziphi inumber {What are the numbers } we should be dividing?
- 147 Learners: 9
- 148 Teacher: 9 and?
- 149 Learners: 3
- 150 Teacher: So kengoku ithi phayana, {Then what does it say there} you may find it helpful to put in brackets, so let's put in brackets zethu {our
- 151 brackets} between which numbers?
- 152 Learners: 9
- 153 Teacher: 9 and?
- 154 Learners: 3
- 155 Teacher: Why we put in brackets? We are trying to show that we are going to start with our division
- 156 Learners: Division
- 157 Teacher: Then addition
- 158 Learners: Addition
- 159 Teacher: then 15 plus you write it down because you have not dealt with, nhe {okay}?
- 160 Learners: Yes
- 161 Teacher: Right, now 9 divide by 3?
- 162 Learners: 3
- 163 Teacher: We have dealt with ibrackets zethu kengoku {our brackets} now we can add. 15 plus 3?

PERIOD 1

S P6 L3 EE2: Order of operations

- 164 Learners: 18
165 Teacher: Sihamba sonke? {Do we all understand?}
166 Learners: Yes
167 Teacher: Kuthweni kengoku xa iexplainwa pha? {What is said when it is explained there?} Kuthiwa. {It is said} do division before addition.
168 Let's do the next one. 2 times 4 minus 1? How many operations are we having there?
169 Learners: 2
170 Teacher: Which ones?
171 Learners: Multiplication and subtraction
172 Teacher: Now which one are we going to start with?
173 Learners: Multiplication.
174 Teacher: So we put in our brackets.
175 Learners: Brackets.
176 Teacher: To show that we are going to start with multiplication.
177 Learners: Multiplication
178 Teacher: So 2 times 4?
179 Learners: 8
180 Teacher: Siyabhala, {We are all writing} you write it down because we have not subtracted yet. So kengoku {then} 8 minus 1?
181 Learners: 7
182 Teacher: Ithi iexplaination pha { The explanation there}, kuthiwa {says} do multiplication before subtraction.
183 Learners: Subtraction.
184 Teacher: Masenzeni ngokuthe {Let's do the last one} last one. 5 plus 1 times 3 naphaya {there also} how many operations?
185 Learners: 2
186 Teacher: So which one are we going to start with?
187 Learners: Multiplication
188 Teacher: Always use brackets, nhe {right}? so it's 5 plus, 1 times 3?
189 Learners: 3
190 Teacher: 11, 5 plus 3?
191 Learners: 8
192 Teacher: Now kengoku kuthiwa pha {it says there}, okay, let's do la activity 5B. Calculate and show the different steps in your calculation by putting in brackets to if necessary, sihamba sonke {do we all agree}?
193 Learners: Yes
194 Teacher: So ke eza sums zezakwa {those sums from} grade 7 so to check you have at least 10 minutes.
195 Learner: Wenza njani xa yi multiplication and division kweza sums? {How do you do it when its multiplication and division in those sums?}
196 Teacher: Oh that is the last one, 24 divide by 3 times 4. Now a quick question, which one are we going to start with?
197 Learners: Multiplication
198 Teacher: Huh?
199 Learners: Multiplication
200 Teacher: Bangaphi abathi division? {How many say it's division?} By show of hands. Bangaphi abathi multiplication? {How many say it's multiplication} Now kengoku {then} and quick question are we going to have a different answer if we start with division?

PERIOD 1

S P6 L3 EE2: Order of operations

- 203 Learners: No
- 204 Teacher: or Multiplication. Let's check. Let's start with division so we put ibackets {brackets} there. 24 divided by 3?
- 205 Learners: 8
- 206 Teacher: 8. 8 times 4? Times 2
- 207 Learners: 32
- 208 Teacher: Now starting with division, let's start with multiplication. Kengoku masenzeni engoku {So let's all do } multiplication first 3 times 4?
- 209 Learners: 12
- 210 Teacher: 24 divide by 12?
- 211 Learners: 2
- 212 Teacher: Ithiwani kengoku xa injena? {What do we do in this situation?}
- 213 Learners: BODMAS tishala. {teacher}
- 214 Teacher: uthini Ubodmas lo nimbizayo? {What does BODMAS stand for that you are all talking about?}
- 215 Learners: Brackets of division, multiplication, addition, subtraction
- 216 Teacher: So kengoku ngeyiphi ecorrect? {Which one is correct?} Which is correct?
- 217 Learners: Division
- 218 Teacher: I agree with division
- 219 Learners: yes
- 220 Teacher: But it also looks tempting le yesibini. {The second one looks tempting.}
- 221 Learner: Ingathi yiyo tishala le irlight. {I think this one is right teacher.}
- 222 Learner: Hayi iyabhida. {Now this is confusing.}
- 223 Teacher: I'm confused. Kunyanzelekile kaloku. {It is most definitely important} we can be given any sum. We need to know what to do at that
- 224 particular moment. So xa sisithi sonke sivumelana naleyana ka BODMAS then let's use leyana ka BODMAS {we all agree with BODMAS?} then.
- 225 Learners: Yes
- 226 Teacher: Khawume ke ndiyijonge. {Wait and I'll see.}
- 227 Learners: Hayi mna ndiphuma nale yesibini. {No I agree with the second one.}
- 228 Teacher: Eyiphi? {Which one?}
- 229 Learners: Eyokuqala. {The first one} {Learners disagree on which problem is correct}
- 230 Teacher: Masiyifundeni ngamazwi ukuba ithini lento. Ithini lento? {Let's all read and see what it means exactly.} Ithi {It says} 24 divide by 3
- 231 times 4. how many sets of 3 can I get from 24?
- 232 Learners: 8
- 233 Teacher: 8, and then ezozinto zoyi {those} 8 ndizithini {what do we do with them?} Ndizi multiplaye by ? {I will multiply them by}
- 234 Learners: 4
- 235 Teacher: But ithini le? {What does this say?}
- 236 Learners: 24 ...
- 237 Teacher: So that means which one is correct now?
- 238 Learners: Division ... multiplication
- 239 Learner: Zi right zombini. {Are both of them correct?}
- 240 Teacher: Hayi nantsi eright. {No, here is the correct one.}
- 241 Learners: yiDivision. {It is division.}

PERIOD 1

S P6 L3 EE2: Order of operations

- 242 Teacher: Mamelani, masiphindeni siyifunde. {Listen, let's read again} I have 24 items, now how many sets of 3 can I get from those 24?
- 243 Learners: 8
- 244 Teacher: 8, multiply by?
- 245 Learners: 4
- 246 Teacher: That's how we did it, so yeyiphi ezobe iright? {which one will be correct?}
- 247 Learners: Division. {Learners try to explain to those who still do not get why the sum that start with division is the correct one.}
- 248 Teacher: Okay, ngubani one calculator apha kuni? {who has a calculator with them now?}
- 249 Learners: Ndim {Me.}
- 250 Teacher: Masikhupheni icalculator. {Who has a calculator with them now?} Let's check kwi calculator {on the calculator} and see which... isinika eyiphi ianswer. {What answer does it give us?} Let's do nge {with} technology. Let's use technology going to help us. Calculate as you see it ibhaliwe pha ebhodini {on the board}. 24 divide by 3 times 4. So which one is correct there?
- 251 Learners: Division
- 253 Teacher: Yimani kaloku masibuyeni {Wait, let's go back.}. I-calculator {The calculator} is using this thing BODMAS. So ke {then} people the first one is correct. So it's always division before multiplication. Okay ke masibeni sixoxa lento {Let's all work together and discuss this}. Let's look at activity 5, let's look at activity 5. I-10 minutes will be sufficient for nina? {Will 10 minutes be sufficient for you guys?}
- 257 Learners: Yes
- 258 Teacher: Masikhawulezeni ke. {Let us hurry up then} 10 minutes, 10 past. Encwadini yakho {In your books} quickly.
- 259 Learners do activity 5 and they help each other out while the teacher walks around checking if learners are doing the activity.
- 260 Learner: Tishala ukuba awuzibekanga ibrackets urongo? {Teacher, if you did place brackets, is it wrong?}
- 261 Teacher: Mamela ithi iquestion {Listen, the question says}, yho i-instruction ithi {no, sorry the instruction says} "calculate and show the different steps in your calculations by putting in brackets if necessary." If you think ukuba ndiyisebenzisile apha {I used it here too}
- 262 put in.
- 263 Learner: Xa yi {It is an} addition and subtraction uqala ngeyiphi? {Which one do you start with?}
- 264 Teacher: Hmm?
- 265 Learner: iaddition ne subtraction {Addition and subtraction}
- 266 Teacher: Ujonga pha {You look there}, ungaqala nokuba ngeyiphi kwi {you can start the} addition and subtraction because now we have done ilaws ze {the laws of} integers, nhe {right?}? With addition and subtraction you can start with any of the 2 because we have done ilaws ze {the laws of} integers, but be very careful xa kufika kwi {when we get to} addition and multiplication.
- 267 Learners continue to do the activity.
- 270 Learner: Tishala ke leyokugqibela ndibeke ibrackets kwi subtraction ndashiya idivision ecaleni. {Teacher, I put the last one in brackets for subtraction and left the division alone.}
- 271 Teacher: Yima, kanene besitheni nge brackets? {Wait, what did we say about brackets?} Which one do we start with?
- 272 Learners: Division
- 273 Teacher: Hayi kaloku uzodivider ntoni, ngantoni? {What will you divide by what?} Uyabona? {See it?} Uzodivider ntoni ngantoni? {What will you divide and by what?} You must have something ozoyi {you will divide by}.
- 274 Teacher: Now jonga pha, uyafumana pha phezulu kuthi {Look there, you will find above that} we order of operation. We start work from left to izengapha. {Our left side will go to this side when we're on this side.}
- 275 Learners continue doing the activity.

PERIOD 1

S P6 L3 EE2: Order of operations

- 282 Learner: {Asks a question but I could not make out what it was.)
- 283 Teacher: Remember kaloku sithe {because we said} order of numeration always start from left to right. How far are we?
- 284 Learner: Sigqhibile. {We are} Finished
- 285 Teacher: Bangaphi abangeka gqhibi? {How many have not finished?}
- 286 Learner: Leyana unumber 9. {Number 9.}
- 287 Teacher: Khawulezisani kaloku into elula kangaka? Ndingacima mos apha? {Hurry up, this is easy. Can I erase this?} This side?
- 288 Learner: Yes. {Teacher writes the activity on the board.}
- 289 Teacher: Okay, bangaphi abangekagqhibi? Nigqhibile ngoku nhe? {How many are not done? You're all finished, right?}
- 290 Learners: No tishala {sir}
- 291 Teacher: Khawulezisani kaloku nani. {Hurry up.} Niyabona ngoku siye kwi {You see it's taken} 15 minutes. Niyabona iyaphela ne first period
- 292 kengoku. {Our first period is ending.} Hayi kugqhityiwe masiyeni ke. {No, let's move on.} Nantsiya 8 minus 5 plus 2. {there is 8 minus 5
- 293 plus 2.} which one are we going to use apha? {here} To the left to the right?
- 294 Learners: Left
- 295 Teacher: Yes?
- 296 Learners: Left
- 297 Teacher: Left to the right so 8 minus 5?
- 298 Learners: 3
- 299 Teacher: 8 minus 5, 3. equals to 3 plus 2?
- 300 Learners: 5
- 301 Teacher: Now masiyeni apha {let's do this}. Multiplication or addition first?
- 302 Learners: Multiplication,
- 303 Teacher: Multiplication, so we use brackets to show that we are going start with? Multiplication. So 3 times 4?
- 304 Learners: 12
- 305 Teacher: 12 plus 5?
- 306 Learners: 17
- 307 Teacher: Sihamba sonke? {Do we agree?}
- 308 Learners: Yes.
- 309 Teacher: Now the next one, subtraction or multiplication first?
- 310 Learners: Multiplication
- 311 Teacher: Put in brackets to show 2 times 3?
- 312 Learners: 6
- 313 Teacher: Then 13 minus 6?
- 314 Learners: 7
- 315 Teacher: Yimani kengoku before sigqhitheni {wait, before we move on} a positive times a positive?
- 316 Learners: Positive
- 317 Teacher: Nantsiya ibhodi. {There's the board.} Positive times a positive?
- 318 Learners: Positive
- 319 Teacher: Positive times negative?
- 320 Learners: Negative
- 321 Teacher: Negative, niyayibona lento bendithetha ngayo? {You see what I mean?} Yokuba {That} whenever there's a sign before ibrackets

PERIOD 1

S P6 L3 EE2: Order of operations

- 322 {brackets} you multiply whatever is inside those brackets ngalo {with that} sign, sihamba sonke {do we agree}?
- 323 Learners: Yes
- 324 Teacher: Sizeni apha {We're now on this one}. Multiplication or division first?
- 325 Learners: Division
- 326 Teacher: 40 divide by 2?
- 327 Learners: 20
- 328 Teacher: Ndiyibeke rongo lena {I put this incorrectly.}. 20 multiply by 5?
- 329 Learners: 100
- 330 Teacher: Si sure ngalo mcimbi sonke? {Are we all sure of this operation?}
- 331 Learners: Yes
- 332 Teacher: Masiyeni ke {Let's go}, 24 divide by 4?
- 333 Learners: 6
- 334 Teacher: Yho {Oh} sorry, which one are we going to start with? Lena? Okanye leyana? {This? Or that?}
- 335 Learners: Left
- 336 Teacher: Always from the left then the?
- 337 Learners: Right
- 338 Teacher: So sizogala ngaleyana {We'll start with that one}, so 24 divide by 4?
- 339 Learners: 6
- 340 Teacher: Then we divide this ngo {with} 2 we get?
- 341 Learners: 3
- 342 Teacher: Masijongeni kengoku le, ukuba besiqale ngalena {Let's look at it because we decided to start with it} are we going to get the same answer?
- 343 Learners: No.
- 344 Teacher: Masiyitshekeni {Let's check}, 24 divide by, 4 divide by 2?
- 345 Learners: 2
- 346 Teacher: Which is?
- 347 Learners: 12
- 348 Teacher: Niyayibona kengoku {Do you see it}? Always move from the?
- 349 Learners: Left
- 350 Teacher: To the?
- 351 Learners: Right.
- 352 Teacher: Because nawe if ubesebenzisa icalculator ubuzo qala ngo 24 divided by 4 divide by 2 ibizogala ngezi zokuqala icalculator.
- 353 {if you used the calculator, you would have started with 24 divided by 4 divided by 2, that's what the calculator would have started with.} Sihamba sonke? {Do we understand?}
- 354 Learners: Yes
- 355 Teacher: Mandinge apha ke, iobvious le 12 minus 8? {Let me start here. Is 12-8 obvious? }
- 356 Learners: 4
- 357 Teacher: Plus 1?
- 358 Learners: 5

PERIOD 1

S P6 L3 EE2: Order of operations

- 359 Teacher: That one is exactly the same. 100 divide by 10 divide by 2. Start from?
- 360 Learners: Left
- 361 Teacher: Left, 100 divide by 10?
- 362 Learners: 10
- 363 Teacher: 10 divide by 2?
- 364 Learner: 5
- 365 Teacher: Ndiyayithanda lena. {I like this one}, 2 plus 1 times 3 plus 2. Which one am I going to start with?
- 366 Learners: 1 times 3
- 367 Teacher: 1 times 3, remember kuthiwani? {what do we do?} Kuthiwa {We should} we start with?
- 368 Learners: Multiplication
- 369 Teacher: Before iaddition. {Addition}
- 370 Learners: Subtraction
- 371 Teacher: So ezi singeka diilishani nazo {So those we haven't dealt with } I'll write it down. 1 times 3?
- 372 Learners: 3
- 373 Teacher: Plus which gives me?
- 374 Learners: 7
- 375 Learner: Uyobona tishala ndiqale ngo 2 plus 1 kodwa ndifumene u7 {You see sir, I started with 2 plus 1 but I got 7.}
- 376 Teacher: Yimani ke, masiyitsheke. {Wait, let's check} Uthi yena uthe {It says he took} 2 plus 1 times 3 plus 2. Jonga {Look} 2 plus 1, 3
- 377 Learners: 3.
- 378 Teacher: times 3?
- 379 Learners: 9
- 380 Teacher: So ngu 11. {It's 11.} Azifani. {It's not the same.} Always kaloku remember irule yethu {Our rule.} start with multiplication first then you
- 381 add. Sihamba sonke? { Do we understand?}
- 382 Learners: Yes
- 383 Teacher: Nantsiya {There it is}. 10 plus 8 divide by 2. So which one am I going to start with apha? {here}
- 384 Learners: Division
- 385 Teacher: Division, so I'll use brackets to show lento ndizogala ngayo {what I will start with.} . Lena ndizoyibhala ngoluhlobo ilulo ingeka
- 386 dilishani nayo, {The one I'm about to write, I will write like this as we haven't dealt with it.} 8 divide by 2?
- 387 Learners: 4
- 388 Teacher: 10 plus 4?
- 389 Learners: 14. ngu number bani lowo? {What number is it?}
- 390 Teacher: Ngu number 10 lona. {It's number 10.}
- 391 Learner: Number 10
- 392 Teacher: Now masenzeni lena, nalapha {Let's do this then this} we can use brackets to show ukuba sizoqala ngantoni? {what will we start with?}.
- 393 We have multiplication, addition and multiplication. We can use 2 brackets apha. {here.}
- 394 Learners: Yes
- 395 Teacher: Nhe? pha na pha {Yes, her and there}. So we are going to start with the multiplication first then the addition. So...
- 396 Learner: Xolo tishala ukuba wenze ibrackets eyi 1. {Sorry teacher, you wrote one bracket.}
- 397 Teacher: Kwathini ? {What happened?}
- 398 Learners: Like ukuba wenze ku 4 times 3 qha. {The way you wrote it, 4 times 3 only.}

PERIOD 1

S P6 L3 EE2: Order of operations

- 399 Teacher: Waphinda wathini? {And then you did?} Given awuqalanga wa fumana ianswer wa edisha u1? {Didn't you get the answer and then
- 400 added 1?} ukuba wenze lonto it's wrong. {If you did so it is wrong.} Yimani nantsi enye imethod. {Let's wait, here's another method.}
- 401 Learner: Ndithe 4 times 3 nhe, ndafumana u12 then kengoku ndathi plus. Ndaphinda ndafaka ibrackets leyana la 1 times 2
- 402 {I said 4 times 3 and I got 12, then I said 1 times 2 in brackets.}
- 403 Teacher: Oh ndiyakuva uthi wenze ngoluhlobo, 4 times 3 plus {Oh I hear you, you said you did it like this 4 times 3 plus } ... uthe qala 4 times 3?
- {Did you say that you first start with 4 times 3?}
- 404 Learners: 12
- 405 Teacher: Wathi plus 1 times 2 wathi {You then plus 1 times 2.}}
- 406 Learner: Yilena ndiyiyenzileyo {This is the one I did.}
- 407 Teacher: 12 plus 2 is equals to?
- 408 Learners: 14
- 409 Learner: Thina siyiyenze ngenye indlela tishala. {We did it in a different way teacher.} Sithe 4 times 3 safaka ibracket. {We said 4 times 3 then placed the bracket.}
- 410 Teacher: 4 times 3
- 411 Learner: Sathi plus 1 times 2 tishala. {Then we said 1 times 2 teacher.}
- 412 Teacher: Oh niyayibona kengoku imistake yakhe? Still awukwazi ngokuba kutheni? {Oh so do you all see his mistake? You still can't
- 413 because...} Kukho imultiplication before iaddition. {There is multiplication before addition.}
- 414 Learner: Xolo tishala. Khayibhale tishala. {Sorry teacher, can you count it again.}
- 415 Teacher: Masiyibhaleni. {Let's all count it.} 4 times 3?
- 416 Learners: 12
- 417 Teacher: 12 plus 1 times 2, bathi bona 12 plus 1 ngubani? {They say 12 plus 1 is?}
- 418 Learners: 13
- 419 Teacher: Times 2
- 420 Learners: 26
- 421 Teacher: Which is different to ianswer yethu. Idifferent from le siyiyenze ngayo thina? {Is our answer different from the way they did it?}
- 422 Hayi. {No} Okay ke, {Okay, then} this one 3 plus 6 divide 3. which one are we going to deal with first?
- 423 Learners: Division
- 424 Teacher: Division. Brackets zethu. {Our brackets} 6 divide by 3
- 425 Learners: 2.
- 426 Teacher: Yes?
- 427 Learner: 2.
- 428 Teacher: Which is?
- 429 Learners: 5.
- 430 Teacher: Now this one which is quite interesting. Masijongeni le. {Let's all} Siyinikwe {We were given } with brackets, so ndizokwenza ntoni
- 431 kengoku? {so what am I going to look at in this one?}
- 432 Learners: Brackets.

PERIOD 1

S P6 L3 EE2: Order of operations

- 433 Teacher: 11 minus 5?
434 Learners: 6
435 Teacher: Divided by 3?
436 Learners: 2
437 Teacher: Masibuyeni siyenze la ABC sakwenza ezinye {Let's all go back to that A, B,C, we are going to do integers tomorrow} integers

S P6 L3 EE3: Description of the commutative property

- 438 tomorrow, we are going to start doing ezinye integers {other integers} tomorrow and see ... yimani ke {let's all stop}. Masiyeni.
439 kengoku pha masiyeni kula {Let's all go through the} properties of operations. Masibuyeni, kulena ke... ndingacima apha? {Let's all get back to that one then ... can I erase here?}
440 Learner: Cima konke tishala. {Erase it sir.}
441 Teacher: Masenzeni kengoku, kuthiwa pha from page 2. {Let's all start, it says there from page 2} kuthiwa {it says the} properties of operation.
442 masimameleni {let's all listen}, you can simplify calculations by using the commutative, association and distributive properties of
443 operation. In this session you will learn how to recognise these properties and apply them to simplify calculations.
444 Let's look at this one right, 4 plus 5, 5 plus 4. 4 plus 5 ngubani? U 4 plus 5? {What is 4 plus 5?}
445 Learners: 9
446 Teacher: What about 5 plus 4?
447 Learners: 9
448 Teacher: 9. 3 times 6, 6 times 3. Ngubani u 3 times 6? {What is 3 times 6?}
449 Learners: 18
450 Teacher: Ibengubani u6 times 3? {What will be the answer of 6 times ?}
451 Learners: 18
452 Teacher: Now kengoku {then} the question is, now kengoku {then} when you are adding is it important where the order...
453 iplace yelo nani? {the order of this number?}
454 Learners: No
455 Teacher: Cause we see apha siqale ngo 4 saza ngo 5 but apha siqale ngo 5 saza ngo 4. {We start with 4 then with 5, but here we started with 5 then with 4.} Let's look at multiplication. The order is important?
457 Learners: No
458 Teacher: No, because siqale ngabani? {what do we start with?} Ngo 3 saza ngo 6 kanti pha siqale ngabani? {With 3 then 6 but what did we start with there?} Ngo 6 saza ngobani ngo 3. {With 6 then with what 3.} Kengoku xa siyibiza le yi {so when we call this, it is a}
459 commutative property. Ithini? {What does it say?} Ithi {It says} when we add or multiply 2 numbers you can change the order of the
461 number. Multiply and addition are sometimes easy to do when you change the order of numbers you are working with. So that means le
462 property yokwazi ukuwaguqulela amanani {the property can swop numbers} we call it a? associ... or commutative property. The order of numbers is not important when we are adding or multiplication but if...

SCHOOL P7

LESSON 1 EE1.1-EE5

S P7 L1 EE1-EE5

S P7 L1 EE1.1: Factors

Time	#	Speech
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- | | | |
|-------|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 00:00 | 1. | Teacher: [Writing on board] Now you can easily start that way. Now if you look at those numbers, you see those numbers are all even numbers. So therefore, what will also be affected? |
| | 2. | Learners: Two. |
| | 3. | Teacher:: [Writes / 1; 1;/] There we put in the two .. for each one .. Right. Now is three a factor of thirty-six? |
| | 4. | Learners: Yes, Sir. |
| | 5. | Teacher: So let's put the three in. Let's try forty. Is three a factor of forty a number more than four? That is a fact that the factor of twenty and the factor of twenty-four and the factor of forty. ... Hmm? .. |

S P7 L1 EE1.2: Generating the highest common factor

Time	#	Speech
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- | | | |
|-------|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | 6. | Teacher: ... No. So that's the biggest one. So the highest common factor of those there .. The highest common factor |
| | 7. | Learner: Ten. |
| | 8. | Teacher: The highest common factor is 4. |
| | 9. | Learner: Four. |
| 01:00 | 10. | Teacher: [Writes on board but the board is obscured.] The highest number that can divide into all three. Right. Now take this down and see if you can get the answers for those two. |
| | 11. | Learner: [Inaudible] |
| 01:13 | 12. | Teacher: Yes. Start again. Draw a line and start again. [01:30] [Break in transmission] ... [Writes /Cube Roots /] ... |
| | 13. | [Break in Transmission]. |
| 01:55 | 14. | Teacher: ... common factor? Now you look at all the factors of thirty, forty eight and seventy two. Who can tell me, what's the highest [02:00] common factor of those three numbers? What do you see? Here are thirty factors, here are forty eight factors and there we have seventy two factors. Which is the highest one? Now you need to examine thirty's factors. Let's look at ten. Is ten a factor of forty eight? |
| | 15. | Learners: No.
Teacher: No, ok. Which one do you think fits in by all three? [chair noise makes word unclear]. Yes my girl? |
| | 16. | Learners: Sir, um thirty, thirty eight, seventy two the highest common factor is six. |

S P7 L1 EE1.2: Generating the highest common factor

Time	#	Speech
	17.	Teacher: Is six. That means the biggest number that can divide each one of them, so six. Now, thirty six and sixty. ...Thirty six and sixty. ... Yes?
	18.	Learners: Six.
	19.	Teacher: I think there's a bigger number than six that can divide into [03:00] both of them.
03:02	20.	Learners: Twelve.
	21.	Teacher: Twelve. Okay there we got it. Now we've just done this; I just want you to get the feel of this and understand what that's all about. Right! .. I want you ... next time when we get a chance we're going to mark each other's work and check each other's work. Check that all those answers are right.

S P7 L1 EE2: Calculating cube roots and cubes

Time	#	Speech
	22	Teacher: Right, make another heading; [Writes /Cube Roots / on board] cube roots. [Activity in class]... ... [04:00] Right, class. We've done cubes and cube roots already. The first answer, the cube root of eight. What's the cube root of eight? Hmm?
04:26	23	Learner: Two.
	24	Teacher: Two why two?
	25	Learner: [Inaudible].
	26	Teacher: No it doesn't go into eight, three times because two times two times two gives you the eight. What's the cube root of one hundred and twenty
	27	Learner: Five.
	28	Teacher: Five because? Five times five times five, brings you back to one twenty five. Hands up. Who can tell me what is the cube root of a thousand?
	29	Learner: Ten.
	30	Teacher: Ten. Why? Because ten times ten times ten ...
	31	Learners: Because ten times ten times ten.
	32	Teacher: Brings you back to?
05:00	33	Learners: A thousand.

S P7 L1 EE2: Calculating cube roots and cubes

Time	#	Speech
	34	Teacher: A thousand. Now we've got these big numbers here. ... Okay. ... Those people with the scientific calculators, I want you to not use that special
	35	facility...How are we; we must look for a number which will multiply by itself. Now. Here you've got to try. ... Try twenty times twenty times
	36	twenty and see what you get. Eight thousand
	37.	Teacher: What do you get? Eight thousand. Now that number is smaller than eight thousand so the cube root is going to be less than twenty. Now find a
	38.	what number do you think it's going to be? Now we know our cubes already. [06:00] We know our cubes from one to nine. What's one
	39.	Class, whole class. one cubed?
06:07	40.	Learners: One.
	41.	Teacher: Two cubed?
	42.	Learners: Four [one learner]
	43.	Teacher: Two cubed?
	44.	Learners: Eight.
	45.	Teacher: Three cubed?
	46.	Learners: Twenty seven.
	47.	Teacher: Four cubed?
	48.	Learners: Sixty four.
	49.	Teacher: Five cubed?
	50.	Learners: One hundred and twenty five.
	51.	Teacher: Six cubed?
	52.	Learners: Two hundred and sixteen.
	53.	Teacher: Seven cubed?
	54.	Learners: Three four three.
	55.	Teacher: Three four three. Eight cubed?
	56.	Learners: Five hundred and twelve.
	57.	Teacher: Five one two and nine cubed?
	58.	Learners: Seven two nine.
	59.	Teacher: Right, now notice eight cubed give you five hundred and twelve ends in a two so this number here ... [points at board] this number here is

S P7 L1 EE2: Calculating cube roots and cubes

Time	#	Speech
	60.	going to end in an eight. The answer here is going to end in an eight because of that two.
	61.	Learners: Sir.
	62.	Teacher: So now what number; now we must try and find it. Yes? {Answer by learner is unclear}.
	63.	Learner: Eighteen times eighteen times eighteen.
	64.	Teacher: That's right so the cube root of that is eighteen. Now this ends in a nine. [07:00] Which number cubed gives you a nine from the numbers one
06:54	65.	to nine? Which one gives you a nine? At the end.
07:10	66.	Learners: Nine.
	67.	Teacher: Nine, so now, you must try a number maybe you try twenty or thirty times thirty times thirty. Write it down. What does thirty times thirty
	68.	times thirty make?
	69.	Learners: Twenty seven thousand.
	70.	Teacher: Twenty seven thousand. Is that number bigger or smaller than twenty seven thousand?
	71.	Learners: Smaller.
	72.	Teacher: Smaller, so the cube root is going to be less than thirty and is going to end with a nine. Now find the numbers between twenty and
	73.	thirty that end in a nine, there's only
	74.	Learners: Sir
	75.	Teacher: One number.
	76.	Learners: Sir, Sir
	77.	Teacher: Yes?
	78.	Learners: Twenty nine
	79.	Teacher: Twenty nine. Check: multiply twenty nine by twelve and get
08:00	80.	Learners: Twenty nine
	81.	Teacher: Get twenty-nine. ... Ok. The next one, you have to try and get a starting point. What starting point can we use here
	82.	Learner: Seven.

S P7 L1 EE2: Calculating cube roots and cubes

Time	#	Speech
	83.	Teacher: Hmm? Wait, its going to end in a, ... your answer is going to end in a?
	84.	Learners: Three
	85.	Teacher: Three. But get a two digit number, an easy number, to multiply by itself and it ends by itself. ... What number can we try? .. Try fifty. .. Try
	86.	fifty and what do you get? A hundred and twenty five thousand. That's too much. So your answer is less than fifty. Now you try forty times forty times forty.
08:49	87.	[Break in transmission]
08:50	88.	Teacher: two hundred and twenty four. ... We've already seen that one hundred and three thousand gives us four seven, now we must try and, maybe seventy [09:00] thousand. Seventy times seventy times seventy.
09:05	89.	Learners: Thirty four thousand
	90.	Teacher: Three hundred and forty three thousand. That's more than thirty thousand. Now what about eighty times eighty times eighty? ... Five hundred and twelve thousand. Your answer will be between seventy and eighty and because there's a four there it's got to be which one?
	91.	Now try, try it on your calculators. Yes?
	92.	Learners: Seven four.

S P7 L1 EE3: Types of numbers: Recap

57	Teacher:	Seventy four. ... Right. Now let's stop for a minute. Let's look at all the work you've done because all of these have been done so far; it's all going to be tested. Let's start off with, we started off with whole and natural numbers, even numbers, odd numbers, multiples,
58		
		prime numbers, composite numbers carried on and then we've got all the prime numbers less than ten; then we went on to prime
59		
		factors. We went on to factors, squares, square roots, right. Why are we there?
09:43	60	What's the difference between these two? [10:00] The number one the answer is something and by number two something else. Now, what's the answer by number one?

10:09 61 Learners: One's the square root.
 62 Teacher: That's the square root. What's the answer by number one? What's the answer by number one? Hm?
 63 Learners: Eight.
 64 Teacher: Eight. What's the answer by number two?
 65 Learners: One.
 66 Teacher: The cube root of sixty four.
 67 Learners: Four.
 68 Teacher: It's four. Notice the difference here. So sixty four is a cube and it is a square; both a square and a cube.

S P7 L1 EE4: Exponents

Time	#	Speech
	105.	Teacher: Right, now the next thing we're going on to is some more factorization, okay? ... Let's first take this [11:00] heading down, powers. ... We've got powers of two. The first one. Now anything .. now I'm not going to explain this to you I'm just gonna tell you, anything to the power of zero
	106.	always comes to one. That you're gonna learn more about in grade nine and ten.
	107.	Now, two to the power of one is just plain two. Now what is two squared? Take out two fingers, and it's two times? Two. And what do you get?
11:55	108.	Learners: Four.
	109.	Teacher: I want you to expand this here. Write it out like that. [12:00] Then this one, two times two times two and then use your calculators. I want you to do this
11:56		here. Complete them and try to keep the equal signs in a line even when you come to these long ones. Start off by putting your equal signs at the end there. Right. complete all of these powers here. The two, what do we call the one, two, three, four and so on? What do we call that here? ..
	110.	That small number? .. Hello?
	111.	Learners: Exponents.
	112.	Teacher: Exponents. Or another word? Another word for exponents? Is? Index. Right, I want you to complete all of those.
	113.	[Activity in class] ... [13:00] ... [Teacher writes on board]. ... [14:00] ...
14:14	114.	Teacher: First here .. Those questions .. take them down. Then you answer all of them. ...

S P7 L1 EE4: Exponents

Time	#	Speech
	115.	Learners: [Working in their books]. ...
14:44	116.	Teacher: What you don't complete in class today, you're gonna finish at home. Right now. The answers here. Equal signs. [Inaudible] each other. ... [15:00] ...Okay, put your pens down. Put your pens down. Pay attention. .. Yes, my boy? .. Hello! ... Right. Now. You see the answers of these
	117.	questions are very important when you get to grade nine and ten. You must recognise the answers. Don't write now please. Answer first that one.
	118.	Hands up. Two to the fourth power.
	119.	Learner: Four.
	120.	Teacher: Yes?
	121.	Learner: Two times two times two times two
	122.	Teacher: Yes, what's the answer?
	123.	Learner: Equals ..
	124.	Teacher: Gives you? Yes?
	125.	Learner: Four.
	126.	Teacher: No man.
	127.	Learners: Sir?
	128.	Teacher: You just said two times two times two times two.
	129.	Learners: Four ... Four ... Six ...
	130.	Teacher: So what's the answer?
	131.	Learners: Sixteen .
15:56	132.	Teacher: Sixteen. Okay. What's the answer here, the final answer class? [16:00] Hands up. Put your pens down. No writing now, no checking now, you can finish that just now. Two to the fifth power? Yes?
16:07	133.	Learners: Thirty-two.
	134.	Teacher: Thirty-two. Two to the sixth power?
	135.	Learners: Sixty four.
	136.	Teacher: No, no shouting out please. Yes?

S P7 L1 EE4: Exponents

Time	#	Speech
	137.	Learners: Sixty-four.
	138.	Teacher: Sixty four. Two to the power seven?
	139.	Learners: Hundred and twenty-eight.
	140.	Teacher: Hundred and twenty. Tw-eight. Two to the power of eight? Don't shout here. Yes?
	141.	Learners: Two hundred and fifty-six.
	142.	Teacher: Good. Okay. Now we go to these ones. three to the power of nought? Now this one you must just remember; anything to the power of naught or zero is always?
	143.	Learners: One,
	144.	Teacher: And anything to the power of one stays the same. This one, whole class, three squared?
	145.	Learners: Nine.
	146.	Teacher: Three cubed?
	147.	Learners: Twenty seven.
	148.	Teacher: Three to the power of four?
	149.	Learners: Eighty-one.
	150.	Teacher: Eighty-one. Three to the power five?
	151.	Learners: Two four three.
	152.	Teacher: Two forty-three. Now we go to four to the power of zero?
	153.	Learners: One.
	154.	Teacher: One. Four to the power of one?
	155.	Learners: Four.
	156.	Teacher: Four. It stays the same. Four squared?
17:00	157.	Learners: Sixteen.
	158.	Teacher: Sixteen. Four cubed?
	159.	Learners: Sixty four.
	160.	Teacher: Four .. Right, now we come to five. Five to the power of zero?

S P7 L1 EE4: Exponents

Time	#	Speech
	161.	Learners: One.
	162.	Teacher: You are writing there! ... Five to the power of zero?
	163.	Learners: One.
	164.	Teacher: One. Five to the power of one?
	165.	Learners: Five.
	166.	Teacher: Five squared?
	167.	Learners: Twenty-five.
	168.	Teacher: Five cubed?
	169.	Learners: One hundred and twenty five.
	170.	Teacher: And five to the fourth?
	171.	Learners: Six hundred and twenty-five.
	172.	Teacher: Good, okay. ... Now just check your work and get that work complete. Right, now I want those whose work is all complete to come here. ...
	173.	Learners: Take work up to the teacher, for it to be marked.
	174.	Teacher: ...Right, now I'm going to try this. Now you must get someone to check all the wrong answers for you, okay? ... You must get someone to check all those individual answers. ...[18:00] .. Make your... [inaudible] signs properly right, make them nicely like that. You must get someone to check all these answers for you, ok.
	175.	Learners: Sir [Inaudible]
	176.	Teacher: Where? Ok, you must get someone to check all these individual answers. Get your friend to do that for you, ok? ... Check all those answers; check all those answers. Mark it with that, with that pencil. Somebody must check all these. You must get somebody to check if everything is right. .. Thank you. Somebody must check all of this for you, right? .. Times three's, four's ok? three's, four's, five's. Somebody must check [19:00] your work, good okay. Somebody must check all these small answers here. Ok, hmm? I'm now going to do it. I am going to do it later. Not I am now gonna do it. Now and later. Don't speak English like that. That's Afrikaans. Ek gaan dit nou doen. ... Get somebody to do it. ... Four. Okay. Somebody must check it, okay?
	177.	Learners: [Inaudible].

S P7 L1 EE5: Classification of numbers

Time	#	Speech
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19:57	178. Teacher:	I want you to write down these numbers now. [20:00] ... Right, I'm gonna put the numbers here; One, two, four, five, six, .. eight, .. twenty four, thirty-three, thirty-seven, ,,, and ,,, forty. Write down all those numbers. ... [21:00] ...
21:20	179. Intercom:	All teachers and learners please. The Spades. All those belonging to the Spades group and those that are interested, there will be a meeting in the hall; all grades ten to grades twelve after school at two thirty. Report to Miss Jacobs in the hall at two thirty. Thank you. [Inaudible.]
	180. Teacher:	You still have about one or two minutes before you go. Now I want you to write this down and choose from these numbers which ones fit that description. Which of these numbers are prime numbers. Which of them [22:00] are odd, odd composite numbers, which ones are squares and which ones are cubed. ...
22:04	181. Learners:	[Copy down the homework.].
	182. Learner:	If we're finished can we go?
	183. Teacher:	Then you can pack up and finish that for homework. ... [23:00] ... Wait, wait, wait. If you have got everything finished then you will be dismissed. In the meantime stand, those of you ... [inaudible] ... Good afternoon, class.
23:26	184. Learners:	Good afternoon, sir.
	185. Teacher:	Thank you. ... You can go.
23:34	186. End of Lesson.	

SCHOOL P7

LESSON 2 EE1-EE5

S P7 L2 EE1: Powers

Time	#	Speech
00:00	1.	Teacher: [Marking worksheets.] [Inaudible] [00:26] Now two. ... First. Okay. Part two, ... number one. Read. Now can we read that?
	2.	Learners: [Inaudible]
	3.	Teacher: One squared or one to the power of two. Right. Put your pens down. ... The next one. Three squared. The next one. Read it.
	4.	Some Learners: Four cubed.
	5.	Some Learners: Four to the power of three.
	6.	Teacher: Four?
	7.	Learners: Cubed.
	8.	Teacher: Cubed. Which is the better way? Four cubed or four to the power of three.
	9.	Some Learners: Four cubed.
		Some Learners: Four to the power of three.

S P7 L2 EE1: Powers

Time	#	Speech
01:00	10.	Teacher: Four cubed is better. Okay? Next one. And what does four cubed mean. ... Take three fingers.
	11.	Learner: Four...
	12.	Teacher: And now?
	13.	Learners: Four [01:00] times four times four.
	14.	Teacher: If you want to use your calculators or if you have a calculator like me. Right. Next one. What have we got there? Two to the power of four.
	15.	Take your four fingers and take.
	16.	Teacher and Learners: Two times two times two times two.
	17.	Teacher: And that gives you eight. Right? ... Right?
	18.	Learners: Yes, sir.
	19.	Teacher: Two to the power of four is eight. Yes?
	20.	Learner: Yes sir.
	21.	Teacher: Two times two times two times two is eight.
	22.	Learners: Yes sir.
	23.	Teacher: Two times two times two times two. ... Yes?
	24.	Learners: Sixteen.
	25.	Teacher: Sixteen. Next one. Ten to the power of three. Thirty. Okay?
	26.	Learners: Yes. Yes.
	27.	Teacher: Now where do we come with. What is it supposed to be? [02:00] Three fingers.
02:02	28.	Teacher & Learners: Ten times ten times ten.
	29.	Teacher: Is that thirty?
	30.	Learners: No sir.
	31.	Teacher: No man. You must wake up man. Next one. Five squared. Ten. Right?
	32.	Learner: No.
	33.	Teacher: Five squared? No. Okay next one. Three to the power of four. Twelve.
	34.	Learners: [Inaudible]
	35.	Teacher: Hmm? What is it? Take four fingers. Three. Or your calculator. Three times three times three times three. Okay. I want you to finish that one then we go
	36.	to part two. The square root of one. This you calculate again, do you? ... But you should know it straight away like that. The square root of one?

S P7 L2 EE2: Square roots and cube roots

Time	#	Speech
	37.	Learner: One.
	38.	Another Learner: Is one sir.
	39.	Teacher: I told you earlier on, check. ... Come and get the right one here. Next one. Square root of three, of nine.
	40.	Learner: Three.

S P7 L2 EE3: Finding the first two digit number, the digits of which have a sum of 12 and finding the first three digit number, the digits of which has a sum of 14

Time	#	Speech
02:55	41.	Teacher: Three. And so on, okay? Then we go on to part four. All [03:00] two-digit numbers. What do we mean by two-digit numbers.
	42.	Your first two-digit number is ten. From naught to nine, we have one digit. Take that. [Hands worksheet to a learner and goes to the board.]
	43.	The first two-digit number is ten and which is the biggest two-digit number? What's the biggest two-digit number?
	44.	Learner: [Inaudible]
	45.	Teacher: Hmm?
	46.	Learner: Ninety-nine.
	47.	Teacher: What's it?
	48.	Learners: Ninety-nine.
	49.	Teacher: That's right. And your first three-digit number is
	50.	Teacher & Learners: One hundred.
03:33	51.	Teacher: And your last three-digit number is nine-hundred and ninety-nine. And so it goes, okay. Right. Let's look at these things again. Right .
	52.	A two-digit number where the sum of the digits is twelve. ...
	53.	What's the sum of the digits in ten. Sum means when you add. [04:00] So the digits are one and zero. What's the sum of one and zero.
	54.	One. The next two-digit number is eleven. What's the sum of the digits in eleven? Sum of the digits in eleven is two. One plus one. The next number
	55.	is twelve. What is the sum of the digits in twelve.
	56.	Learners: Three.
	57.	Teacher: Three. Now where is your first two-digit number where the sum of the digits is twelve.
	58.	Learner: Six.

S P7 L2 EE3: Finding the first two digit number, the digits of which have a sum of 12 and finding the first three digit number, the digits of which has a sum of 14

Time	#	Speech
05:00	59.	Teacher: That's not that..even part of the.. one of the answers. It's a two-digit number. Where's the first one?
	60.	Learner: Six.
	61.	Teacher: Six is not a two-digit number.... What's the first two-digit number where the sum of the digits is twelve. ...
	62.	Learner: Six.
	63.	Teacher: Hm?
	64.	Learner: Six. [05:00]
	65.	Teacher: But it's a one-digit number. It must be a two-digit number. What's the first two-digit number where the digits will add to twelve.
	66.	Learner: [Inaudible].
	67.	Teacher: Let me see who can get that. Instead of me giving you those kind of chocolates I'll give you a real chocolate. Hmm? Twelve? No, my boy. The digits
	68.	twelve are one and two which add up to three.
06:00	69.	Learners: [Inaudible].
	70.	Teacher: What's the first two-digit number where the digits add up to twelve. It will be thirty-nine. It can't be in the twenties. The biggest number that you can
	71.	get is twenty-nine, which will give you eleven. The first one will be thirty-nine. Three and nine adds up to twelve.
	72.	Then in the forties you find one. Which number will be in the forties.... Two numbers that add up to twelve in the forties. [06:00]
	73.	Learner: Forty-eight.
	74.	Teacher: Forty-eight. In the fifties you'll find such a number. Which number will that be?
	75.	Learner: Fifty-seven.
	76.	Teacher: Hmm?
	77.	Learner: Fifty-seven.
	78.	Teacher: Fifty-seven. And she's already said in the sixties. Right. You can work out the rest just now. Okay. The first four three-digit numbers where the sum
	79.	of the digits is fourteen. Three-digit numbers. The digits add up to fourteen . What's the first three-digit number? One hundred. What do the digits
	80.	add up to in a hundred..... What are the digits in a hundred. One.
	81.	Teacher & Learners: Zero and zero.
	82.	Teacher: What do those digits add up to?
	83.	Learner: One.
	84.	Teacher: One. Now I'm looking for the first of those three-digit numbers. Remember your three-digit number. Numbers go from a hundred up to nine
	85.	hundred and ninety-nine. [07:00] I want the only the first four. The first four. That means the smallest four. The lowest four.
	86.	Where the digits add up to fourteen. Right. So it's going to be a hundred and something.... Which one? To add up to fourteen.
	87.	Learner: One hundred and seven.
	88.	Teacher: One hundred and?

S P7 L2 EE3: Finding the first two digit number, the digits of which have a sum of 12 and finding the first three digit number, the digits of which has a sum of 14

Time	#	Speech
	89.	Learner: Seven.
	90.	Teacher: Seven. The digits don't add up to fourteen my boy. The digits add up to eight. Yes.
	91.	Learner: One hundred and four.
	92.	Teacher: One hundred and four. The digits add up to five. Where's the first one. Okay. Is it in the one. Is it in the one one something? No because the biggest
	93.	number you can add on to one one is nine and that will give you a total of.. a sum of eleven. [08:00] One one nine. So maybe in the one twenty? One and
	94.	the biggest number you can add there is nine. One and two and nine add up to twelve. So it's not in the one-twenties. In the one thirties, the biggest number
	95.	can add up to .. add on to one and three will be nine. And those digits will add up to? They're waiting for me.
07:36	96.	Learner: One hundred and forty-nine.
	97.	Teacher: Hundred and forty-nine she said. Is she right? That's the first one. One and nine at the end makes ten and four in the middle makes fourteen. Now you
	98.	find the next one. It will be in the one-fifties. Maybe. Not sure. But it is. Okay. The next one. The first four
	99.	four-digit numbers. Four-digit numbers. Where's the first four-digit number?
	100.	Learner: One thousand.
	101.	Teacher: One thousand. [09:00] And that will take you up to nine thousand, nine hundred and ninety-nine. Okay. And then the first four five-digit numbers where
09:00	102.	the sum of the digits is twenty. Okay class. I'm giving you some time now, to do this sum. [Inaudible] This sum.
	103.	Learners: [Inaudible murmurs.]

S P7 L2 EE4: Recap of the meaning of 5²

Time	#	Speech
	104.	Teacher: Five squared. What does five squared mean. You see, this is something you mustn't do. [Teacher writes on the board.] Five squared equal ten.
	105.	He is saying five times two. What is he supposed to say?
10:00	106.	Learner: [10:00] Five times five.
	107.	Teacher: Five times five. Five squared is two fives being multiplied together. Come on man. Let's
	108.	Learner: [Inaudible].

S P7 L2 EE5: Listing odd numbers, even numbers greater than 30, prime numbers and composite numbers greater than 30

Time	#	Speech
	109.	Teacher: That's alright what we just saying. [Inaudible] the question. Then three and nine. Number three and nine. Oh yes. ... Can't look like this enough time to [drowned out by classroom noise]. Let's go back to the other side. Go back to the first page. ... Now where are my answers? They are there somewhere.
	110.	Learners: [Inaudible].
	111.	Teacher: Hmm?
10:59	112.	Learners: Sir gave it to them. [11:00] ...Giggling.
	113.	Teacher: [To the learner to whom he had previously given the sheet.] But you [inaudible] got the answers. Why didn't you give it back to me? You must start again
	114.	afresh. And you must give me that back now. Give that back to me now. Get up. [Inaudible]. Okay. The odd numbers. Now come.
	115.	Let's. Let's do this quickly. We're not going to finish everything, but I don't. I want you to mark those that you got right.
	116.	{Inaudible} got the answers and you still taking it. There. Take this one. That's nice. Right. The odd numbers. What are they? Oh.
	117.	{You only mark afterwards. If it's all correct. If it's not right, [12:00] leave it. We'll fix it up afterwards. Yes.
12:05	118.	Learner: One.
	119.	Teacher: Yes.
	120.	Learner: Five.
	121.	Teacher: Yes.
	122.	Learner: Nine.
	123.	Teacher: Mmm mm.
	124.	Learner: Twenty-five.
	125.	Teacher: Yes.
	126.	Learner: Twenty-seven.
	127.	Teacher: Yes.
	128.	Learner: A hundred and twenty-five.
	129.	Teacher: That's right. That was easy, wasn't it?
	130.	Learners: Yes.
	131.	Teacher: Now why leave it. Why leave a sum like that. Now if it's wrong. If you get anything wrong there, don't mark it wrong and you can't mark it right.
	132.	You got to fix it. Okay. The next one. Even numbers greater than thirty. You know this girl here. She tries so very hard to answer all the time. What's
	133.	wrong with the rest of you. Let me ask this boy. Now he's another one who answers a lot. Yes?
	134.	Learner: Thirty-two, thirty-six, sixty-four.
	135.	Teacher: Louder. Thirty-two.
	136.	Learner: Thirty-two, thirty-six .
	137.	Teacher: Thirty-six.
	138.	Learner: Sixty-four , a hundred, two hundred and sixteen [inaudible] a thousand.

12:54 139. Teacher: There we go. Let me go over it again. Two, sorry, more than thirty. [13:00] More than thirty is thirty-two, thirty-six, sixty-four, one hundred,
140. two one six and one thousand. Prime numbers. One. Right?

141. Learners: Yes sir. No. No sir.
142. Teacher: Huh?
143. Learners: No sir. No.
144. Teacher: Why?
145. Learner: Yes sir. [Inaudible. All talk together.] One only had one factor, sir.
146. Teacher: Is one a prime number? So where's your first prime number.
147. Learners: Two.
148. Teacher: Two. Then?
149. Teacher and Learners: Five.
150. Teacher: Then?
151. Learners: Then nothing. [Inaudible]. That's all.
152. Teacher: Hmm?
153. Learners: That's all.

154. Teacher: At least you're starting to understand prime numbers. Then composite numbers less than thirty. Yes my girl. Tell them quickly now.
155. Learner: [Inaudible].
156. Teacher: Hmm?
157. Learner: [Inaudible].
158. Teacher: Less than thirty.
159. Learner: [Inaudible].
160. Teacher: Hmm?
161. Learner: [Inaudible].

13:51 162. Teacher: Or you took the wrong one. Okay. Composite numbers less than thirty [siren goes off] are four, six, [inaudible], okay okay. I'm gonna stop there. now.
Look here. Go home. Go over everything. And show your mommies and daddies what you've got to do.

163. Learners: No sir. [Inaudible].
14:14 164. End of lesson.

SCHOOL P7

LESSON 3 EE 1

S P7 L3 EE1: Bar graphs

Time	#	Speech
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- | | | |
|-----|----------|-------------------------------------------------------------------------------------------------------------------------------------------|
| 1. | | Activity and talking in class. |
| 2. | Teacher: | What's this talking for? My girl, you're asking me questions. Now what did I say? I'll give you an excellent chance to go over this work. |
| 3. | | Right? Now don't ask me any questions today. I'm giving you a chance today. Alright? Let's get to the board now. I said start on a |
| 4. | | fresh page I want you to write your heading and you'll do okay. Yours is almost a fresh page so you might as well start there. |
| 5. | | Right. Your heading. will be ... okay /Writes on board/ bar graphs. Some people say graphs. Others say tomatoes ... others say |
| 6. | | tomatoes. I say graph you say graph whichever you wanna say it as long as you spell it correctly. Okay? Now pay attention first |
| 7. | | Let me explain. That's the work that we're doing today ... bar graphs. Now any graph [points to bar graph on board] is a picture of a |
| 8. | | ... of a set of numbers. Leave that for the other part. [Erases detail from the graph on board.] Okay? Now I want you ... you're going |
| 9. | | to take down this ... and that's the heading [Points to heading on board]. After bar graph .. that's your heading. Spine Road High |
| 10. | | School Enrolment two thousand and eight Grade eight, nine, ten, eleven and twelve. And those are the number of learners. |

S P7 L3 EE1: Bar graphs

11. That was in two thousand and eight. You take down that table. We call that a .. table. Then you're going to set out your graph. Now
12. listen carefully.. You work five centimetres away from the margin. We will see why. Then you make these spaces here one centimetre
13. apart. This will be your vertical axis and that will be your?
14. Learners: Horizontal axis
15. Teacher: Horizontal axis. Some of you learnt that last year already
16. Learners: Yes, sir
- 03:00 17. Teacher: Good. Right, then just set out your ... you take your .. your graph .. just set it out so long and we'll come back to it now. Work
18. quickly and neatly and accurately. [Learners busy preparing grid.] Oh .. sorry ... listen .. all of you ... pay attention. You use two
19. spaces here. Okay? Just let's .. let's just put in something else so long. This point is going to be nought. After two spaces
20. we're going to have fifty ... after another two spaces .. one hundred. And then that will be a hundred and fifty and so
21. we go along. Alright? So how many spaces are you going to need altogether here? Come, come, come, come! We need
22. two spaces from nought to fifty. Another two .. another two .. another two. Okay? Then we come to two hundred .. then to two
23. Fifty .. then three hundred .. then three hundred and fifty. How many spaces? That would be four hundred
24. Learner: Four hundred
25. Teacher: You don't need four hundred. Two .. four .. six ..
26. Learner: Sixteen
27. Teacher: Eight .. ten .. twelve .. fourteen ..
28. Learners: Sixteen
29. Teacher: Sixteen. So you need sixteen spaces. There will be enough after you've done your table. Then you still leave a few lines between
30. You must have a title for this graph. Okay? Now then you do that quickly... then we can do this graph together.
31. [Learners continue preparing grid]. Okay, this is your ... this is your?
32. Learners: Bar graph
33. Teacher: Vertical ...
34. Learners: Vertical axis
35. Teacher: Vertical axis. And it must be ...

S P7 L3 EE1: Bar graphs

36. Learners: Horizontal
37. Teacher: Wait .. and it must be five centimetres away from the ..
38. Learners: Margin
39. Teacher: Margin. So you measure five centimetres, make another mark and that will be your vertical axis. You'll see why just now.
40. [To a learner] You don't need to do that. You don't need to do this here. What you're going to do is ..
41. goes to board and writes over the grid l[in]es of the graph] you're going to draw those lines. You don't need to make that mark there.
42. Come .. hurry up. Get down your table and set out the graph. Always the table comes first, man. Rub that out and start again.
43. [To another learner] Three spaces open then you start setting out your graph. [To another learner] What you doing with so
44. many spaces? That's not right
45. Learner: [Inaudible. Asks teacher something]
46. Teacher: Pay attention. Put your pens down and pay attention. Right. Now on our vertical axis we mark it off.
47. We are using a scale here. Every two spaces on the book or page we take is equal to fifty ... fifty ...?
48. Learners: Learners
49. Teacher: Learners. Now ev .. each axis must be labelled. What is this axis showing us? You'll see it in the table.
50. Learners: The number of learners
51. Teacher: The number of learners. So here you're going to write number of learners and underline it. And on the horizontal axis
52. .. what do those numbers represent?
53. Learners: Grade
54. Teacher: Grades ... and you're going to write that there. The grades .. you're going to write that there. So you're labelling both axes. The
55. plural for axis is axes. Okay? Now get that set out nicely as .. exactly as I told you to. You're starting five centimetres from the...
56. [Brief break in transmission.] [To a learner] Five centimetres from the margin. [Speaks directly to a learner.]
57. Inaudible exchange between teacher and a learner
58. Teacher: Where's yours now?

S P7 L3 EE1: Bar graphs

59. Learner: [Inaudible]
60. Teacher: Somebody got a spare ruler for her?
61. Learner: (Inaudible)
62. Teacher: Thanks you. [To another learner] Is this right or wrong? How many spaces... how .. how far apart must these lines be .. going up?
63. What did I tell you? These lines going up here. ... How far .. how far apart must that be? What did I say? ... I want it one centimetre
64. apart. And that's not one centimetre. Spaces from the bottom. So it's going to set you sixteen spaces from the top. Now come
65. man! Now you are not marking off every space. We're only marking every second space. Okay? ... Ready? She's ready.
66. I don't know about the rest of you. How many people are ready? ... We have to mark off our vertical axis. Mark it off the value and
67. label it.. [Demonstrates on board] We are using one centimetre spaces here. We're marking it off and that eight comes
68. exactly below that line. Not next to, exactly below that line. This ... is the label. Each axis has a ...
69. Learners : Label
70. Teacher: Label. And each axis is marked off. Now please get that done. We're waiting for you. Some people are there, some are far from
71. there. ... [To a learner] She must tell you what's wrong. ... What did she ... Two spaces
72. [Addresses whole class] Right, all the pens down. Pencils down. Pay attention. You're working too slowly. Others are
73. talking .. not even paying attention. Pens down. Right. I told you to label each axis. Right? What label do we have here?
74. Learners: Number of learners
75. Teacher: Right. Now I also said below your ... before you come to your graph .. leave space open. Now what's that space for? That space is for
76. your title of the graph. Now every graph must have a title. So on top there .. what do you think is the title for this graph?
77. Learners: Spine Road High School Enrolment
- 12:00 78. Teacher: Spine Road High School Enrolment two thousand and eight. So that's your title. Then you have your vertical axis marked and
79. labelled. Horizontal axis marked and labelled. [Stops learners from writing] No wait .. wait .. wait. Get that done just now.
80. Pay attention. Now let's start with the first ... let's do the first bar. The bar for grade eight. .. How many grade eight's are there?
81. Learners: Two hundred and fifty

13:00 82. Teacher: Now you can easily see... now I want you to do it in a special way now. You've drawn these lines. I want you to mark a millimetre on each
83. side. A millimetre there with your pen .. rulers and draw that line there and there's your first bar which you're going to shade
84. in nicely for me. Okay? The first one is easy .. two hundred and fifty. Now we come to grade nine. Three hundred and ten.
85. Where does that one go? ... How far does that bar go? You see here's your three hundred. There's your three hundred and ...
86. Learners: Fifty
87. Teacher: Fifty. Now what...how much is that point there? Halfway between three hundred
88. Learner: Three hundred and twenty five
89. Teacher: ... and three hundred and fifty?
90. Learners: Three hundred and twenty five
91. Teacher: Three hundred and twenty five. Now is three hundred and ten coming halfway?
92. Learners: No
93. Teacher: No. Now what's half of twenty five?
94. Learner: Fif ...
95. Another learner: Twelve
96. Teacher: Twelve and a half
97. Learners: Twelve and a half
98. Teacher: Now ten is just a little less than ten and a half. That means each space here .. each space in your book is twenty five. Half of twenty five is?
99. Learners: Twelve and a half
100. Teacher: Twelve and a half. So ten will ... now you must picture this in your mind. Alright? One space there will take you to three twenty five. Half
101. of that distance there. Oh not there. Half of that space will be three twenty five and half of three twenty five will be three hundred and
102. twelve and a half. And then just below half of that space ... then one millimeter on each side... so your bars...
103. I want your bars to be two millimeters and there is space between your bars. Is that clear? That there is space between your bars. Right?
14:00 104. Now the next one. Number... grade ten. Two hundred and five. Where does two hundred and five go?
105. Learners: Just above ...
106. Teacher: Just above the two hundred ... but not just above it. Work it out in your mind where it comes. And then we've got grade eleven and
107. grade twelve. Okay? Now get this completed. [Learners carry out instruction.]
108. [Holds up a learner's book] We said two spaces for every fifty and therefore it should be sixteen spaces.

109. What did this boy do? Did he especially close his ears? Then we got other people that are using two centimetres here like that girl
 110. there at the back. Now I want you to follow instructions. It's not a joke. Sit up. I don't know. You people must learn to follow
 111. instructions. In maths you've got to learn to go by the rules. If you don't go by the rules then you're gonna make mistakes.
 112. If you don't pay proper attention then you're gonna get the wrong answers. You must learn to listen to the teachers. That's what this
 15:00 113. ...now, one of the things about this lesson is that one of these days you're getting a project. That's why I want to teach you about bar
 114. graphs because you're going to have to do it all on your own. Is that clear?
 115. Learners: Yes sir
 116. Teacher: Now after this is finished, I've got some questions there, which we'll carry on tomorrow. But on the next page I want you to take down
 117. this table. Then you do this graph again. It'll be nearly the same. This time you do .. as some of these people made a mistake ..
 118. you still use two cen .. two spaces for every fifty. But this time you use two centimetre gaps here.
 119. Okay? For this one here .. for today's one .. for this one it's one centimetre spacing along your horizontal axis and two spaces on your
 120. vertical axis. This one also two spaces for your vertical axis ... and two centimetres for your horizontal axis. But the first thing you've
 121. got to do is your table. But what do you have to do with this table here? This is not going to help you You have to do
 16:00 122. something first. What do you have to do here? You have to do something before you can start your graph. ... Yes, my girl?
 123. Learner: You must lay it out
 124. Teacher: Yes, you must lay it out .. but here the table is incomplete. What's incomplete about the table? Yes, my girl?
 125. Learner: Um .. get the total
 126. Teacher: You've got to get these totals. Over here I gave you the totals. Here I gave you eight A, eight B, eight C, eight D. Which class are
 you?
 127. Learners: Eight A
 128. Teacher: Eight A ... and you are forty three?
 129. Learners: Yes, sir
 130. Teacher: This year's enrolment. Enrolment of the school is how many learners there are at the school. Now these are this year's figures. Now when
 131. we're finished we're gonna compare this year's figures with last year's figures. We gonna also see certain things. What do you
 132. see here? In grade eight there are seven classes .. seven groups here. Here we got six .. here we got eleven .. here we
 133. only got four. Now what happens? You're in grade eight this year and what happens to the grade eight's by the time
 18:00 134. they're getting to grade twelve?

135. Learner: They drop out of school
136. Teacher: They drop out. Are you going to be one of those drop-outs?
137. Learners: No, sir
138. Teacher: Okay. Another thing is ... take notice here .. take note here. This is very info .. interesting information. You see that three hundred and
139. ten? How come there are such a lot of grade nine's?
140. Learner: Maybe because some of them failed.
141. Teacher: Because the grade eight's went up and in grade eight .. let me tell you .. it's very easy to pass. If you fail grade eight then you really did not
142. work. And all those grade eight's went up and some grade nine's failed. I'll tell you now, grade nine is not so .. going to be
143. as easy as grade eight. Then they ... look here. Two hundred and five only in grade ten. Such a lot failed. It went up. And then
144. grade eleven .. a hundred and ninety. There's more failures here. Too many failures. We go to grade eleven and then between
145. grade eleven and grade twelve they drop more. Why? They dropped out. You see? So you're going to be ... when you get there
- 19:00 146. there mustn't be ... there must be two hundred and fifty grade twelve's when you come here. You are this year in two thousand and?
147. Learners: Nine
148. Teacher: Nine. In two thousand and ten you are in grade ..
149. Learners: Nine
150. Teacher: In two thousand and eleven?
151. Learners: Ten
152. Teacher: In two thousand and twelve?
153. Learners: Eleven
154. Teacher: Two thousand and ...
155. Learners: Thirteen
156. Teacher: Thirteen, you are going to be doing matric. That's what you have to say to yourself. Right? Now get this work completed.
- 20:00 157. [Addresses a learner] Are you going to be in grade twelve in two thousand and thirteen?
158. Learners: Yes, sir
159. Teacher: You?
160. Learner: Yes, sir

161. Teacher: You sure?

162. Learner: Yes, sir. I'm positive

163. Teacher: You?

164. Learner: No sir. After Grade ten I'm going to college for Grade eleven and twelve

165. Teacher: What's that? You're going to complete grade ten and then?

166. Same learner: Go to college

167. Teacher: And what do you do at college? You do grade eleven

168. Learner: (Inaudible)

169. Teacher: Just table it down on the next page. Make sure you have this table down on the next page. Remember here we've got a heading, High School Enrolment two thousand and ..

21:00 170. Learners: Nine

171. Teacher: nine. And then you come and finish this bar graph right here. Make sure you've got that down so you can go and do it at home. ...

172. ... You need to have your own eraser. You can't borrow. Those people who have ... I don't want you to lend anybody

173. your stuff. You don't lend .. you don't borrow. [To a learner] Um...let's see .. you've got to take it down straight away. You don't

174. need to write it down in your notebook. [To another learner] I see you've

175. Learner: Yes sir

176. Teacher: Good ... those lines ..

177. Another learner: [Inaudible]

178. Teacher: Yes .. centimetres from the ..

179. Learner: Margin

180. Teacher: Margin.

181. Teacher: [Inaudible] [Goes around the class, checking learners' work].

182. Learner: Sir ...

183. Teacher: Yes, for this table. [Walks around classroom checking learners' work.] [Takes a ruler from learner who is drawing the grid one block

184. at a time, instead of drawing columns and rows.] Why don't you just draw one line one time? Why do you draw little bits like that? Hmm?

185. Just mark how many spaces you need here. One .. two .. three .. four .. five .. six .. seven .. eight. Okay? You could have done eight
186. centimetres One centimetre at a time and then go to eight. Mark it here and then you draw this line one time I don't want

187. multi-coloured bars. All in pencil. I don't want all that colouring. That is just in grade R and grade one. All those rainbow colours.
188. [Still going around the class.] You don't need all that. No, man ... [inaudible.]

24:00 189. Leave that now. You might as well...[inaudible]. Right, once you've got your table complete for the next graph then you go
190. and get this one complete. Just sit up everybody. Sit up. Grade twelve ... a hundred and thirty five. Where is a hundred and thirty five?
191. Put your pens down. Do you know where's one thirty five? This line is a ...

192. Learners: Hundred
193. Teacher: A hundred. This line is ...
194. Learners: A hundred ..
195. Teacher: A hundred and fifty. And that line running between ...
196. Learners: A hundred and twenty five
197. Teacher: Is a hundred and ..
198. Learners: Twenty five
199. Teacher: Twenty five. What are we looking for? A hundred and ...

28:00 200. Learners: A hundred and thirty five
201. Teacher: A hundred and thirty five. So that line there that's a hundred and twenty five. You want to go a little bit above that line. That will take you
202. to ...
203. Learners: A hundred and thirty five
204. Teacher: A hundred and thirty five. Grade eleven .. one hundred and ninety. There's a hundred and fifty ... there's two hundred. So ten below
205. two hundred. That's how far that one goes. Okay? Now get this graph finished. Tomorrow we'll go on. Now get this
206. graph and get that one finished and then we're gonna compare [Learners busy in books.]
207. rry .. sorry .. one last question. Could we have done both these graphs together on one graph? Hello?

208. Learners: Yes, sir
209. Teacher: How could we do that?
210. Learner: Sir, if you do another line there up ...
211. Teacher: You could have two bars next to each other. The one bar could be two thousand and eight. The other ... then you can use colours. Then

212. the two thousand and eight graph in one colour and the two thousand and nine bar in a ...

213. Learners: ... another colour.

214. Teacher: Different colour. So we could have made one graph and done both of them.

215. Learners: Yes, sir

216. Teacher: Right, now that's what we must also think of. We're coming there {Speaks to learner} No..no.. [Addresses class. Short break in

30:00 217. transmission.] .. slightly different to that one. Here I want you to make two centimetre spaces. This graph has got ...

218. Learners: One centimeter

219. Teacher: One centimeter spaces. Is that clear?

220. Learners: Yes sir

221. Teacher: You must please learn to follow instructions and listen to the teacher.

222. [Addresses a learner] Is that the first one? How many centimetre space have you got there?

223. Learner: Two

224. Teacher: Two centimetres? How many is supposed to be there?

225. Learner: One

226. Teacher: Okay rather leave it now.

31:02 227. Lesson ends

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